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AN INTRODUCTION TO WEAKER AND STRONGER FORM OF SOFT OPEN SETS BY γ-OPERATION

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Abstract: In this paper, we investigate the weaker form of soft sets using τ_γ-Int. We show relation between different kind of weak forms of soft open sets. In process to find the weaker we found its stronger form which we have discussed by the help of τ_γ-Cl. Finally, we give different kind of Separation Axioms by the help of these weak form and strong form of soft sets.

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1. Introduction

The term Soft Set was coined by Molodtsov [4] in 1999 and developed a new approach for modeling uncertainties. Soft set has a rich potential for applications in several directions. One of them have been explained in Pei et al. [13]

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where the relationship between soft sets and information system have been discussed.In further development, Maji et al. [12] presented some new definitions on soft sets. Maji et al. [11] also expanded soft set to fuzzy soft set.

Njastad [9] introduced α-open sets. Ogata [10] called the operation α (respectively α-closed set) as γ-operation (respectively γ-closed set) and introduced the notion of τ_γ which is the collection of all γ-open sets in a topological space. We have introduced the work of Ogata [10] in soft sets using γ-operation given to generate some weak and strong form of Soft open sets and used them to define seperation axioms.We have used same notation as given by Hariwan [5] like soft α-γ set.

2. Preliminaries

Throughout this paper let X be any non empty set referred as universe and let E be the set of parameter for X.

Definition 1. [6] Let X be any non-empty set, E be a set of parameters and $P(X)$ denote the power set of X, then the pair (F, E) is called a soft set over X, where F is a mapping from E to $P(X)$ and denoted by :

$$F : E \to P(X).$$

Definition 2. [12] Let (F, E) and (G, E) be two soft sets over a common space X over common parameter E. $(F, E)\tilde{\subset}(G, E)$, if $F(e) \subset G(e)$ $\forall e \in E$. Otherwise if A and B are subsets of parameter E, if $A \subseteq B$ and $F(e) \subset G(e)$ $\forall e \in A$, then $(F, E)\tilde{\subset}(G, E)$.

Definition 3. [12] Let (F, E) and (G, E) be two soft sets of a set X over common parameter E. $(F, E)\tilde{\subset}(G, E)$ and $(G, E)\tilde{\subset}(F, E)$, if $G(e) \subset F(e)$ and $F(e) \subset G(e)$ $\forall e \in E$. Hence (F, E) is equal to (G, E).

Definition 4. [12] A soft set (F, E) of X is called a null soft set, denoted by $\tilde{\emptyset}$, if $\forall e \in E$, $F(e) = \emptyset$.

Definition 5. [12] A soft set (F,E) of X is called a absolute soft set, denoted by $\tilde{X}$, if $\forall e \in E$, $F(e) = X$.

Definition 6. [4] The union of two soft sets of (F, A) and (G, B) is defined by: $(H, C) = (F, A)\tilde{\cup}(G, B)$ where $C = A \cup B$ and $\forall e \in C$

$$H(e) = \begin{cases} F(e), & \text{e} \in \text{A} \quad \text{B}; \\ G(e), & \text{e} \in \text{B} \quad \text{A}; \\ F(e) \cup G(e), & \text{e} \in \text{B} \cap \text{A}. \end{cases}$$

Definition 7. [4] The intersection of two soft sets of (F, A) and (G, B) is defined by:
$(H, C) = (F, A)\tilde{\cap}(G, B)$ where $C = A \cap B$ and for all $e \in C$ and $H(e) = F(e) \cap G(e)$.

Definition 8. [6] Relative Complement for any soft set (F, A), where $A \subseteq E$ is defined by $(F, A)^c = (F^c, A)$, where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X - F(e)$ for all $e \in A$.

Definition 9. [7] let (F, E) be a soft set over X and $x \in X$. We say that $x \notin (F, E)$, whenever $x \notin F(e)$ for some $e \in E$.
whereas $x \in (F, E)$, whenever $x \in F(e)$ for all $e \in E$.

Definition 10. [7] Let $x \in X$ then (x, E) denotes the soft set over X for which $x(e) = x$, for all $e \in E$.

Definition 11. [7] Let X be any non empty set with some set of parameter E, τ be the collection of soft sets over X is said to be soft topology on X with parameter E if it satisfies the following axioms:

1. $\tilde{\emptyset}, \tilde{X} \in \tau$,
2. the union of any number of soft sets in τ belongs to τ ,
3. the intersection of any two soft sets in τ belongs to τ.

Here (X, τ, E) is called as Soft Topological space over X.

Definition 12. [7] Let (X, τ, E) be a soft topological space of X, (G, E) be a soft set of X and $x \in X$. Then (G, E) is said to be a soft neighborhood of x if there exist (F, E) s.t. $x \in (F, E)\tilde{\subseteq}(G, E)$.

Definition 13. [7] Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X. the soft interior of (G, E) is the soft set
$\text{Int}(G, E) = \bigcup\{(F, E) : (F, E) \text{ is a soft open and } (F, E)\tilde{\subset}(G, E)\}$.

Definition 14. [3] Let (X, τ, E) be a soft topological space over X, (G, E) be a soft set over X, the soft closure of (G, E) is the soft set
$\text{Cl}(G, E) = \bigcap\{(P, E) : (P, E) \text{ is a soft closed and } (G, E)\tilde{\subseteq}(P, E)\}$.

In this section we mention some preliminary definitions and theorems which we have applied in our paper. Let X be any space and E be any class of parameters on X. A soft subset (P, E) is said to be [8]:

A [1] α-soft open if $(P, E)\tilde{\subseteq}Int(Cl(Int(P, E)))$;

B [2] pre-soft open if $(P, E)\tilde{\subseteq}Int(Cl(P, E))$;

C [3] semi-soft open if $(P,E)\tilde{\subseteq}Cl(Int(P,E))$;

D [2] β-soft open if $(P,E)\tilde{\subseteq}Cl(Int(Cl(P,E)))$.

3. γ-Operation and its Application

Let X be any non empty set, E be the set of parameters for X and (X,τ,E) be any soft topological space.

Definition 15. An operation on a soft topology τ over X is called a γ-operation if a mapping from τ to the set $P(X)^E$ and defined by.

$$\gamma : \tau \to P(X)^E$$

such that for each $(V,E)\in\tau$,$(V,E)\tilde{\subseteq}\gamma(V,E)$.

Definition 16. A soft set (P,E) is said to be γ-soft open set if for each $x\tilde{\in}(P,E)$, $\exists$ a soft open set (V,E) such that $x\tilde{\in}(V,E)\tilde{\subset}\gamma(V,E)\tilde{\subset}(P,E)$.

Note : τ_γ represents soft topology generated by γ-soft open sets, τ_β represents soft topology generated by β-soft open sets,τ_α represents soft topology generated by α-soft open sets,τ_{semi} represents soft topology generated by semi-soft open sets,τ_{pre} represents soft topology generated by pre-soft open sets and τ represents soft topology generated by soft open sets.

Remark 17. $\tau_\gamma \tilde{\subset} \tau$, it can be proved easily by the definition of γ-soft open sets.

Definition 18. If X is a soft topological space and (P,E) is any soft subset of X,then the τ_γ-interior of (P,E) in X is defined by
τ_γ-Int$(P,E) = \bigcup\{(G,E) : (G,E)$ is τ_γ-open set and $(G,E)\tilde{\subseteq}(P,E)\}$.
$\Rightarrow \tau_\gamma$-Int$(P,E) \tilde{\subseteq} (P,E)$.

Definition 19. If X is a soft topological space and (P,E) is any soft subset of X,then the τ_γ-closure of (P,E) in X is defined by
τ_γ-Cl$(P,E) = \bigcap\{(G,E) : (G,E)$ is τ_γ-closed set and $(P,E)\tilde{\subseteq}(G,E)\}$.
$\Rightarrow (P,E) \tilde{\subseteq} \tau_\gamma$-Cl(P,E)

Definition 20. A soft topological space (X,τ,E) is γ-soft regular, if for each $x\tilde{\in}\tilde{X}$ and for each soft open neighborhood (V,E) of x $\exists$ a soft open neighborhood (U,E) of x such that $\gamma(U,E)\tilde{\subseteq}(V,E)$.

3.1. Weak Form of γ-Soft Open Sets

Definition 21. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be α-γ-soft open set, if

$$(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(P,E))).$$

Definition 22. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be pre-γ-soft open set, if

$$(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(P,E)).$$

Definition 23. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be semi-γ-soft open set, if

$$(P,E)\tilde{\subset}Cl(\tau_\gamma - Int(P,E)).$$

Definition 24. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be β-γ-soft open set, if

$$(P,E)\tilde{\subset}Cl(\tau_\gamma - Int(Cl(P,E))).$$

Theorem 25. *Let (X,τ,E) be a soft topological space, then the following properties holds for a soft subset (P,E) of a set X :*

1. *Every γ-soft open set is a α-γ-soft open.*
2. *Every α-γ-soft open set is a semi-γ-soft open.*
3. *Every semi-γ-soft open set is a β-γ-soft open.*
4. *Every α-γ-soft open set is a pre-γ-soft open.*
5. *Every pre-γ-soft open set is a β-γ-soft open.*

Proof. 1. If (P,E) is γ-soft open set, then $(P,E)\tilde{\subset}\tau_\gamma - Int(P,E)$. We also know that $(P,E)\tilde{\subset}Cl(P,E)$
$(P,E)\tilde{\subset}Cl(P,E)\tilde{\subset}Cl(\tau_\gamma - Int(P,E))$
$(P,E)\tilde{\subset}\tau_\gamma - Int(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(P,E)))$ Hence (P,E) is α-γ-soft open set.

2. If (P,E) is α-γ-soft open set, then $(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(P,E)))$. We also know for any soft subset (T,E) of X τ_γ-Int(T,E) $\tilde{\subset}$ Int(T,E) $\tilde{\subset}$ (T,E)
$(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(P,E)))\ \tilde{\subset}Cl(\tau_\gamma - Int(P,E))$.
Hence (P,E) is semi-γ-soft open set.

3. If (P,E) is semi-γ-soft open set, then $(P,E)\tilde{\subset}Cl(\tau_\gamma - Int(P,E))$. We also know for any (V,E) $(V,E)\tilde{\subset}Cl(V,E)$
$(P,E)\tilde{\subset}Cl(\tau_\gamma - Int(P,E))\ \tilde{\subset}Cl(\tau_\gamma - Int(Cl(P,E)))$.
Hence (P,E) is β-γ-soft open set.

4. If (P,E) is α-γ-soft open set, then $(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(P,E)))$. We also know for any soft subset (T,E) of X τ_γ-Int(T,E) $\tilde{\subset}$ Int(T,E) $\tilde{\subset}$ (T,E)
$(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(P,E)))\tilde{\subset}\tau_\gamma - Int(Cl(P,E))$.
Hence (P,E) is pre-γ-soft open set.

5. If (P,E) is pre-γ-soft open set, then $(P,E)\tilde{\subset}\tau_\gamma - Int(Cl(P,E))$. We also know for any (V,E) $(V,E)\tilde{\subset}Cl(V,E)$
   $(P,E)\tilde{\subset}Cl(\tau_\gamma - Int(P,E))\ \tilde{\subset}Cl(\tau_\gamma - Int(Cl(P,E)))$.
   Hence (P,E) is β-γ-soft open set.

□

Theorem 26. *Every γ-soft open set (V,E) in a soft topological space (X,τ,E) is soft open set.*

Proof. Let (V,E) be any γ-soft open set of (X,τ,E).In order to prove, we just need to show that there exists a soft open set (P_x,E) for each $x\tilde{\in}(V,E)$ by definition of γ-soft open set, we have a soft open set (P_x,E) such that $x\tilde{\in}(P_x,E)$ and $\gamma(P_x,E)\tilde{\subset}(V,E)$.
Therefore $x\tilde{\in}(P_x,E)\tilde{\subset}\gamma(P_x,E)\tilde{\subset}(V,E)$
Hence $x\tilde{\in}(P_x,E)\tilde{\subset}(V,E)$
$\Rightarrow$ (V,E) is a soft open set. □

Remark 27. Converse of Theorem 26 need not be true as shown in the following example.

Example 28. Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ and $\tau=\{\tilde{\emptyset},\tilde{X},(F,E),(G,E),(H,E)\}$ where $F(e_1)=\{a\};F(e_2)=\{b\}$, $G(e_1)=\{b\};G(e_2)=X$, $H(e_1)=\emptyset;H(e_2)=\{b\}$.
Now γ operation defined as :

$$\gamma(P,E)=\begin{cases}\tilde{X}, & \text{if } (P,E)=(F,E);\\ (P,E), & \text{otherwise.}\end{cases} \quad \text{then } \tau_\gamma=\{\tilde{\emptyset},\tilde{X},(G,E),(H,E)\}$$

Hence (F,E) is soft open set but not γ-soft open set.

Corollary 29. *τ_γ-Int$(P,E)\tilde{\subset}$ Int(P,E).*

Corollary 30. *Cl(P,E) $\tilde{\subset}$ τ_γ-Cl(P,E).*

Theorem 31. *Every α-γ-soft open set is α-soft open set.*

Proof. Let (P,E) be α-γ-soft open set then by definition, we have, $(P,E)\tilde{\subset}$ τ_γ-Int(Cl(τ_γ-Int(P,E))) $\tilde{\subset}$ Int(Cl(τ_γ-Int(P,E))) $\tilde{\subset}$ Int(Cl(Int(P,E)))
$(P,E)\tilde{\subset}Int(Cl(Int(P,E)))$.
Hence (P,E) is α-soft open set. □

Remark 32. Converse of Theorem31 need not be true as shown in the following example.

Example 33. Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ and $\tau=\{\tilde{\emptyset},\tilde{X},(F,E),(G,E),(H,E)\}$ where $F(e_1)=\{a\};F(e_2)=\{b\}$, $G(e_1)=\{b\};G(e_2)=X$, $H(e_1)=\emptyset;H(e_2)=\{b\}$.
then $\tau_\gamma=\{\tilde{\emptyset},\tilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E)\}$ where $F_1(e_1)=\emptyset;F_1(e_2)=\{b\},F_2(e_1)=\emptyset;F_2(e_2)=X$, $F_3(e_1)=\{a\};F_3(e_2)=\{b\},F_4(e_1)=\{a\};F_4(e_2)=X$, $F_5(e_1)=\{b\};F_5(e_2)=\{b\},F_6(e_1)=\{b\};F_6(e_2)=X$, $F_7(e_1)=X;F_7(e_2)=\emptyset,F_8(e_1)=X;F_8(e_2)=\{b\}$

Now γ is defined by
$\gamma(P,E)=\begin{cases}\tilde{X}, & \text{if } (P,E)=(F_3,E);\\ (P,E), & \text{otherwise.}\end{cases}$

$$\Rightarrow \tau=\{\tilde{\emptyset},\tilde{X},(G,E),(H,E)\}$$

also $\tau_{\alpha-\gamma}=\{\tilde{\emptyset},\tilde{X},,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E)\}$
here (F_3,E) is α-soft open set but not α-γ-open set.
In fact (F_3,E) is soft open set but not α-γ-open set.

Remark 34. In fact (F,E) is soft open set.
$\Rightarrow$ soft open set doesn't imply α-γ-soft open set.

Remark 35. We observe that α-open set $\not\Rightarrow$ γ-open set.

3.2. Strong Form of γ-Soft Open Sets

Definition 36. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be strong α-γ-soft open set, if

$$(P,E)\tilde{\subset}Int(\tau_\gamma-Cl(Int(P,E))).$$

Definition 37. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be strong pre-γ-soft open set, if

$$(P,E)\tilde{\subset}Int(\tau_\gamma-Cl(P,E)).$$

Definition 38. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be strong semi-γ-soft open set, if

$$(P,E)\tilde{\subset}\tau_\gamma-Cl(Int(P,E)).$$

Definition 39. A soft subset (P,E) of a soft topological space (X,τ,E) is said to be strong β-γ-soft open set, if

$$(P,E)\tilde{\subset}\tau_\gamma-Cl(Int(\tau_\gamma-Cl(P,E))).$$

Theorem 40. *Let (X,τ,E) be a soft topological space, then the following properties holds for a soft subset (P,E) of a set X :*

1. *Every strong soft γ-α-soft open set is a strong soft γ-semi-soft open.*

2. *Every strong soft γ-semi-soft open set is a strong soft γ-β-soft open.*

3. *Every strong soft γ-α-soft open set is a strong soft γ-pre-soft open.*

4. *Every strong soft γ-pre-open set is a strong soft γ-β-soft open.*

Proof. 1. If (P,E) is strong soft γ-α-open set, then

$$(P,E)\tilde{\subset}Int(\tau_\gamma - Cl(Int(P,E)))$$

We also know for any soft subset (T,E) of X

$$(T,E)\tilde{\subset}Cl(T,E)\tilde{\subset}\tau_\gamma - Cl(T,E)$$

$\Rightarrow (P,E)\tilde{\subset}Int(\tau_\gamma - Cl(Int(P,E)))\ \tilde{\subset}\tau_\gamma - Cl(Int(P,E))$
$\Rightarrow (P,E)\tilde{\subset}\tau_\gamma - Cl(Int(P,E))$.
Hence (P,E) is strong semi-γ-soft open set.

2. If (P,E) is strong soft γ-semi-open set, then

$$(P,E)\tilde{\subset}\tau_\gamma - Cl(Int(P,E))$$

We also know for any (V,E)

$$(V,E)\tilde{\subset}Cl(V,E)$$

$\Rightarrow (P,E)\tilde{\subset}\tau_\gamma - Cl(Int(P,E))\ \tilde{\subset}\tau_\gamma - Cl(Int(\tau_\gamma - Cl(P,E)))$
$\Rightarrow (P,E)\tilde{\subset}\tau_\gamma - Cl(Int(\tau_\gamma - Cl(P,E)))$
Hence (P,E) is soft γ-β-open set.

3. If (P,E) is strong soft γ-α-open set, then

$$(P,E)\tilde{\subset}Int(\tau_\gamma - Cl(Int(P,E)))$$

We also know for any soft subset (P,E) of X

$$(P,E)\tilde{\subset}Int(\tau_\gamma - Cl(Int(P,E)))$$

$\Rightarrow (P,E)\tilde{\subset}Int(\tau_\gamma - Cl(Int(P,E)))\ \tilde{\subset}Int(\tau_\gamma - Cl(P,E)) \Rightarrow (P,E)\tilde{\subset}Int(\tau_\gamma - Cl(P,E))$.
Hence (P,E) is strong soft γ-pre-open set.

4. If (P,E) is strong soft γ-pre-open set, then

$$(P,E)\tilde{\subset}Int(\tau_\gamma - Cl(P,E))$$

We also know for any (V,E)

$$(V,E)\tilde{\subset}\tau_\gamma - Cl(V,E)$$

$\Rightarrow (P,E)\tilde{\subset}Int(\tau_\gamma - Cl(P,E))\ \tilde{\subset}\tau_\gamma - Cl(Int(\tau_\gamma - Cl(P,E))) \Rightarrow (P,E)\tilde{\subset}\tau_\gamma - Cl(Int(\tau_\gamma - Cl(P,E)))$ Hence (P,E) is strong soft γ-β-open set.

□

Theorem 41. *Every soft pre open set is strong soft γ-pre open set.*

Proof. Let (P,E) be soft pre-open set then by definition, we have,
$\Rightarrow (P,E)\tilde{\subset}Int(Cl(P,E))\tilde{\subset}Int(\tau_\gamma - Cl(P,E))$
$\Rightarrow (P,E)\tilde{\subset}Int(\tau_\gamma - Cl(P,E))$
$\Rightarrow (P,E)$ is strong soft γ pre-open set. □

Remark 42. Converse of Theorem need not be true as shown in the following example.

Example 43. Let $X=\{a,b\}$, $E=\{e_1,e_2\}$ and $\tau=\{\tilde{\emptyset},\tilde{X},(F,E),(G,E),(H,E)\}$, where $F(e_1)=\{a\};F(e_2)=\{b\}$, $G(e_1)=\{b\};G(e_2)=X$, $H(e_1)=\emptyset;H(e_2)=\{b\}$.

Then $\tau_{pre}=\{\tilde{\emptyset},\tilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E)\}$, where: $F_1(e_1)=\emptyset;F_1(e_2)=\{b\},F_2(e_1)=\emptyset;F_2(e_2)=X$, $F_3(e_1)=\{a\};F_3(e_2)=\{b\},F_4(e_1)=\{a\};F_4(e_2)=X$, $F_5(e_1)=\{b\};F_5(e_2)=\{b\},F_6(e_1)=\{b\};F_6(e_2)=X$, $F_7(e_1)=X;F_7(e_2)=\emptyset,F_8(e_1)=X;F_8(e_2)=\{b\}$
Hence, γ is defined by

$$\gamma(P,E)=\begin{cases}\tilde{X}, & \text{if } P(e_1)=\emptyset;\\ (P,E), & \text{otherwise,}\end{cases}$$

and strong $\tau_{pre-\gamma}=\{\tilde{\emptyset},\tilde{X},(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E),(F_7,E),(F_8,E),(G_1,E),(G_2,E)\}$, $G_1(e_1)=\{a\};G_1(e_2)=\{a\}$, $G_2(e_1)=X;G_2(e_2)=\{a\}$,
$\Rightarrow (G_1,E)$ is strong soft γ-pre open set which is not a pre soft open set.

Theorem 44. *Every soft α-open set is strong soft γ-α-open set.*

Proof. Let (P,E) be soft α-open set then by definition, we have

$$(P,E)\tilde{\subset}Int(Cl(Int(P,E)))\tilde{\subset}Int(\tau_\gamma - Cl(Int(P,E)))$$
$$\Rightarrow (P,E)\tilde{\subset}Int(\tau_\gamma - Cl(Int(P,E))).$$

Therefore (P,E) is γ-α-open set. □

Remark 45. Converse of the above statement may not be true.

Theorem 46. *Every soft β-open set is strong soft γ-β-open set.*

Proof. Let (P,E) be soft β-open set then by definition, we have

$$(P,E)\tilde{\subset}Cl(Int(Cl(P,E)))$$
$$\Rightarrow (P,E)\tilde{\subset}Cl(Int(\tau_\gamma - Cl(P,E)))\tilde{\subset}\tau_\gamma - Cl(Int(\tau_\gamma - Cl(P,E)))$$
$$\Rightarrow (P,E)\tilde{\subset}\tau_\gamma - Cl(Int(\tau_\gamma - Cl(P,E))),$$

i.e. (P,E) is strong soft γ-β-open set. □

Remark 47. Converse of the above statement may not be true.

Theorem 48. *Theorem 3.34. Every soft semi-open set is strong soft γ-semi-open set.*

Proof. Let (P, E) be soft semi-open set then by definition, we have,

$$(P, E)\tilde{\subset}Cl(Int(P, E))\tilde{\subset}\tau_\gamma - Cl(Int(P, E))\tilde{\subset}$$

Hence (P, E) is strong soft γ-semi-open set. □

Remark 49. Converse of the above statement may not be true.

Theorem 50. *If (P, E) is any soft open set and (V, E) any soft subset of X, then $Cl((P, E)\tilde{\cap}(V, E)) = Cl(P, E)\tilde{\cap}Cl(V, E)$*

Proof. Let (P, E) be any soft open set and (V, E) any soft subset of X,

$$(P, E)\tilde{\cap}(V, E)\tilde{\subseteq}(P, E)\tilde{\cap}Cl(V, E) \quad \Rightarrow \quad Cl((P, E)\tilde{\cap}(V, E))\tilde{\subset}Cl((P, E)\tilde{\cap}Cl(V, E)).$$

Conversely,

$$\begin{aligned}
&x\tilde{\in}Cl(P, E)\tilde{\cap}Cl(V, E)\\
&\Rightarrow \text{there exist a net } (x_\lambda) \text{ in } (P, E)\tilde{\cap}Cl(V, E)\\
&\Rightarrow (x_\lambda)\tilde{\in}(P, E) \text{ and } (x_\lambda)\tilde{\in}Cl(V, E)\\
&\Rightarrow \text{there exist a net } (x_{\lambda_k}) \text{ in } (V, E) \text{ such that } (x_{\lambda_k}) \to x\\
&\Rightarrow (x_{\lambda_k}) \text{ in } (P, E)
\end{aligned}$$

So, we choose

$$\begin{aligned}
&(x_{\lambda_k}) \to x\\
&\Rightarrow (x_{\lambda_k}) \text{ in } (P, E)\tilde{\cap}(V, E)\\
&\Rightarrow x\tilde{\in}Cl((P, E)\tilde{\cap}(V, E)).
\end{aligned}$$

□

Theorem 51. *The intersection of a α-γ-soft open set and soft open set is α-soft open set.*

Proof. Let (P, E) be α-γ-soft open set and (V, E) be soft open set.

$$\begin{aligned}
&\Rightarrow \ (J, E) = (P, E)\tilde{\cap}(V, E)\\
&\Rightarrow \ (P, E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(P, E))) \text{ and } (V, E)\tilde{\subset}Int(V, E)\\
&\Rightarrow \ (P, E)\tilde{\subset}Int(Cl(\tau_\gamma - Int(P, E)))\tilde{\subset}Int(Cl(Int(P, E)))\\
&\Rightarrow \ (J, E) = (P, E)\tilde{\cap}(V, E)\tilde{\subset}Int(Cl(Int(P, E)))\tilde{\cap}Int(V, E)\\
&\Rightarrow \ \tilde{\subset}Int(Cl(Int(P, E))\tilde{\cap}(V, E))\tilde{\subset}Int(Cl(Int(P, E)\tilde{\cap}(V, E)))\\
&\Rightarrow \ Int(Cl(Int(P, E)\tilde{\cap}(V, E))) = Int(Cl(Int(J, E)))\\
&\Rightarrow \ (J, E)\tilde{\subset}Int(Cl(Int(J, E))).
\end{aligned}$$

Hence (J, E) is α-soft open set. □

Theorem 52. *If $\{(A_k, E) : k \in \triangle\}$ are collection of α-γ-soft open set of a space X over same set of parameter E, then $\bigcup_{k\in\triangle}(A_k, E)$ is α-γ-soft open set.*

Proof. Since (A_k, E) are α-γ-soft open set,

$$(A_k, E)\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int(A_k, E)))$$
$$\Rightarrow \bigcup_{k\in\triangle}(A_k, E)\tilde{\subset}\bigcup_{k\in\triangle}\tau_\gamma - Int(Cl(\tau_\gamma - Int(A_k, E)))\tilde{\subset}\tau_\gamma - Int(\bigcup_{k\in\triangle}Cl(\tau_\gamma - Int(A_k, E)))$$
$$= \tau_\gamma - Int(Cl(\bigcup_{k\in\triangle}\tau_\gamma - Int(A_k, E)))\tilde{\subset}\tau_\gamma - Int(Cl(\tau_\gamma - Int\bigcup_{k\in\triangle}(A_k, E))).$$

Hence $\bigcup_{k\in\triangle}(A_k, E)$ is α-γ-soft open set. □

4. Seperation Axioms

4.1. γ-Soft Seperation Axioms

Definition 53. A soft topological space (X, τ_γ, E) with an operation γ on τ is said to be soft $\gamma - T_0$ if for each pair of distinct points x, y in X, if $\exists$ a soft γ-open set (P, E) such that $x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E)$ or $y\tilde{\in}(P, E)$ and $x\tilde{\notin}(P, E)$.

Definition 54. A soft topological space (X, τ_γ, E) with an operation γ on τ is said to be soft $\gamma - T_1$ if for each pair of distinct points x, y in X, if $\exists$ a soft γ-open sets (P, E) and (V, E) such that $x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E)$ and $y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E)$.

Definition 55. A soft topological space (X, τ_γ, E) with an operation γ on τ is said to be soft $\gamma - T_2$ if for each pair of distinct points x, y in X, if $\exists$ a soft γ-open sets (P, E) and (V, E) such that $x\tilde{\in}(P, E)$ and $y\tilde{\in}(V, E)$ and $(P, E)\bigcap(V, E) = \tilde{\phi}$

Theorem 56. *If (X, τ_γ, E) is soft $\gamma - T_1$ then (X, τ_γ, E) is soft $\gamma - T_0$.*

Proof. Let (X, τ_γ, E) be soft $\gamma - T_1$. $\Rightarrow \exists$ γ-soft open sets (P, E) and (V, E) s.t. $\Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E)$ $\bigwedge$ $y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E)$ $\Rightarrow$ $x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E)$ $\bigvee$ $y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E)$.
Thus (X, τ_γ, E) be soft $\gamma - T_0$. □

Remark 57. Converse may not be true.

Theorem 58. *If (X, τ_γ, E) is soft $\gamma - T_2$ then (X, τ_γ, E) is soft $\gamma - T_1$.*

Proof. Let (X, τ_γ, E) be soft $\gamma - T_2$. $\Rightarrow \exists$ γ-soft open sets (P, E) and (V, E) s.t. $\Rightarrow$ $x\tilde{\in}(P, E)$ and $y\tilde{\in}(V, E)$ and $(P, E)\tilde{\bigcap}(V, E) = \tilde{\phi}$. $\Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E)$ $\bigvee$ $y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E)$. Thus (X, τ_γ, E) be soft $\gamma - T_1$. □

Remark 59. Converse may not be true.

4.2. α-γ-Soft Seperation Axioms (Pre-γ, Semi-γ, β-γ Seperation Axioms)

Definition 60. A soft topological space (X, τ_γ, E) with an operation γ on τ is said to be soft $\alpha - \gamma - T_0$ if for each pair of distinct points x, y in X, if $\exists$ α-γ (pre-γ,semi-γ,β-γ) soft open set (P, E) such that $x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E)$ or $y\tilde{\in}(P, E)$ and $x\tilde{\notin}(P, E)$.

Definition 61. A soft topological space (X, τ_γ, E) with an operation γ on τ is said to be soft $\alpha - \gamma - T_1$ if for each pair of distinct points x, y in X, $\exists$ α-γ (pre-γ,semi-γ,β-γ) soft open set (P, E) and (V, E) such that $x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E)$ and $y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E)$.

Definition 62. A soft topological space (X, τ_γ, E) with an operation γ on τ is said to be soft $\alpha - \gamma - T_2$ if for each pair of distinct points x, y in X, if $\exists$ α-γ (pre-γ,semi-γ,β-γ) soft open set (P, E) and (V, E) such that $x\tilde{\in}(P, E)$ and $y\tilde{\in}(V, E)$ and $(P, E)\tilde{\bigcap}(V, E) = \tilde{\phi}$

Theorem 63. *If (X, τ_γ, E) is soft α-γ (pre-γ,semi-γ,β-γ)-T_1 then it is α-γ (pre-γ,semi-γ,β-γ)-T_0.*

Proof. Let (X, τ_γ, E) be soft $\alpha - \gamma - T_1$. $\Rightarrow \exists\ \alpha - \gamma$ -soft open sets (P, E) and (V, E) s.t. $\Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E) \bigwedge y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E) \Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E) \bigvee y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E)$.
Thus (X, τ_γ, E) be soft $\alpha - \gamma - T_0$. □

Remark 64. Converse may not be true.

Theorem 65. *If (X, τ_γ, E) is soft α-γ (pre-γ,semi-γ,β-γ)-T_2 then it is α-γ (pre-γ,semi-γ,β-γ)-T_1.*

Proof. Let (X, τ_γ, E) be soft $\alpha - \gamma - T_2$. $\Rightarrow \exists\ \alpha - \gamma$ -soft open sets (P, E) and (V, E) s.t. $\Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\in}(V, E)$ and $(P, E)\tilde{\bigcap}(V, E) = \tilde{\phi}$. $\Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\notin}(P, E) \bigvee y\tilde{\in}(V, E)$ and $x\tilde{\notin}(V, E)$.
Thus (X, τ_γ, E) be soft $\alpha - \gamma - T_1$. □

Remark 66. Converse may not be true.

Theorem 67. *If (X, τ_γ, E) is soft $\gamma - T_i$(α-γ-T_i, pre-γ-T_i)then it is soft α-$\gamma - T_i$(pre-γ-T_i, β-γ-T_i respectively) for $i = 0, 1, 2$.*

Proof. Let (X, τ_γ, E) be soft $\gamma - T_2$. $\Rightarrow \exists\ \gamma$ -soft open sets (P, E) and (V, E) s.t. $\Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\in}(V, E)$ and $(P, E)\tilde{\bigcap}(V, E) = \tilde{\phi}$. Every γ -soft open set is α-γ-soft open set. By theorem, (P, E) and (V, E) are α-γ-soft open sets s.t. $\Rightarrow x\tilde{\in}(P, E)$ and $y\tilde{\in}(V, E)$ and $(P, E)\tilde{\bigcap}(V, E) = \tilde{\phi}$. Thus (X, τ_γ, E) be soft $\alpha - \gamma - T_2$. □

Remark 68. Converse may not be true.

Theorem 69. *Let (X, τ_γ, E) be a soft topological space over X. If (x, E) is α-γ-soft closed set in (X, τ_γ, E) for each $x \in X$, then (X, τ_γ, E) is a soft α-$\gamma - T_1$ space.*

Proof. Let (x, E) be soft α-γ-soft closed set in X. $\Rightarrow (x, E)^c$ is α-γ -soft open in X and $x \tilde{\notin} (x, E)^c$. Similarly, it is true for each $x \in X$; let us take $y \in X$ s.t. $x \neq y \Rightarrow (y, X)^c$ is α-γ -soft open in X and $y \tilde{\notin} (y, E)^c$. But $y \tilde{\in} (x, E)^c$ and $x \tilde{\in} (y, E)^c$, which satisfy the definition of soft α-$\gamma - T_1$ space. □

Theorem 70. *Let (X, τ_γ, E) be a soft topological space over X. If (X, τ_γ, E) is a soft α-γ-T_2 space, then $(x, E) = \tilde{\bigcap} (P_j, E)$ for each $x \tilde{\in} (P_j, E)$.*

Proof. Let Let (X, τ_γ, E) be soft α-γ-T_2 space. For any $x, y \in X$ where $x \neq y, \exists$ α-γ -soft open sets (P, E) and (V, E) s.t. $x \tilde{\in} (P, E)$ and $y \tilde{\in} (V, E)$ and $(P, E) \tilde{\bigcap} (V, E) = \tilde{\phi}$.

$\Rightarrow \{x\} \subseteq P_j(e_i)$ for each $e_i \in E$.

$\Rightarrow \{x\} \subseteq \bigcap_j P_j(e_i)$ for each $e_i \in E$. $\Rightarrow (x, E) \subseteq \bigcap_j (P_j, E)$ for each $x \in (P_j, E)$. $\Rightarrow (x, E) \subseteq \bigcap_j (P_j, E)$. $(\Leftarrow)$ *i.e.* $\bigcap (P_j, E) \subseteq (x, E)$

Let if possible above is not true then $\exists$ $y \in X$ s.t. $x \neq y$ and $y \in \bigcap_j P_j(e_i)$ for some $e_i \in E$. For any $x, y \in X$ where $x \neq y$, $\exists$ α-γ -soft open sets (P, E) and (V, E) s.t. $x \tilde{\in} (P, E)$ and $y \tilde{\in} (V, E)$ and $(P, E) \tilde{\bigcap} (V, E) = \tilde{\phi}$.

$\Rightarrow (x, E) \subseteq (P, E) \Rightarrow (x, E) \subseteq (P, E) \tilde{\bigcap} (\tilde{\bigcap}_j (P_j, E))$. Since this is true for arbitrary y in X, $\Rightarrow (P, E) \tilde{\bigcap} (\tilde{\bigcap}_j (P_j, E)) \subseteq (x, E)$. So, $(x, E) = (\tilde{\bigcap} (P_j, E)$ for each $x \tilde{\in} (P_j, E)$. □

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