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NUMERICAL MODELING OF A SYSTEM OF INTERRELATED CONSOLIDATION AND MECHANICAL-CHEMICAL SUFFUSION PROCESSES IN HETEROGENIC POROUS MEDIA

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Abstract: The process of the soil consolidation with consideration of mechanical and chemical suffusions has been considered. An improved mathematical model has been explored numerically. A solution of the boundary problem has been programmed with the finite elements method. The influence of mechanical and chemical suffusions phenomena on the soil surface subsidence has been analyzed based on a model problem.

AMS Subject Classification: 65C20
Key Words: heterogenic porous media, soil consolidation, mechanical suffusion, chemical suffusion, finite elements method

1. Introduction

The article considers the system of interrelated physical-chemical processes in a completely saturated porous media, exemplified by soils. Soil is a complicated system, which in this case is composed from porous liquid and solid particles that form the soil's skeleton. Here we ignore the gaseous soil components. Concerning the solubility criterion, solid particles may be divided into two types – solid non-soluble components and solid water-soluble components (salts' minerals). Because of the presence of water-soluble particles in soil, the porous liquid is a certain chemical solution. This factor substantially influences the

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properties of soil permeability for the porous liquid [1] (see also the survey of dependences presented there). Moreover, with the change of the temperature of the soil surface (e.g. during the construction of a building) or under the necessity of pumping hot liquid into soil [2] it is also necessary to take into consideration the changed temperature conditions of the porous medium. That is, a soil – is a system where complicated interrelated processes take place, and the necessity of taking those into account is dictated by practical tasks.

Equations of the filtration consolidation of soils in the cases, when unspecified number of physical-chemical factors influences them, are generalized in the research [3]. The beginning of such researches had been initiated in works [1, 4, 5]. However, in [3] only a mathematical model was built, and other factors, such as the quantitative effect on soils consolidation or joint processes of mechanical-chemical suffusion were not studied yet. This is the aim of the present article.

The consolidation of a certain soil mass is connected with its settling ("settling" – is a conditional term, which may include physical processes both of settling and of swelling). In an article [6] the kinematic boundary condition is generalized on a mobile surface of soil mass, which is being consolidated. Therefore, another aim of the article is the disclosure of the influence degree of interrelated consolidation processes and of mechanical-chemical suffusion on the settlement (or swelling) of soil surface.

There is a number of researches concerning the interrelated filtration-suffusion processes in soils. In [7] experimental researches into the processes of mechanical and chemical suffusion in soils are provided. Besides, a mathematical model of these processes was built based on the research data. In [8] mathematical models of mechanical suffusion in soils were built and researched. In the research [9] the interrelated processes of mechanical and chemical suffusion were studied experimentally. The case of radionuclides migration – the process of colloidal migrating particles adsorbing radioactive pollutions and becoming carriers of radiation, is also described in the mentioned researches. Besides, a case of interrelation and dependence of processes, when the concentration of salts in the solid phase falls lower a certain level, which results in the activation of mechanical suffusion, and as a consequence – in mudding. In work [10] the migration of colloidal particles in soils was studied theoretically and experimentally. In [11] a research into mechanical suffusion was carried out: a mathematical model was built, numerical solutions were found, the verification of mathematical model was implemented based on their own experiments. In work [12] processes of filtration, suffusion and mud grouting injection were researched in saturated porous media experimentally. The above given concise survey of scientific works shows a wide circle of practical tasks and the topicality of researches on the direction of interrelated non-linear multi-factorial processes in porous media.

2. Statement of Problem in Physical Domain

The process of the filtration consolidation of soil mass is investigated in one-dimensional case. Let us make the following assumptions. Soil is salinized – salts' minerals are present in its (soil's) solid component. As a result, porous liquid is a one-component chemical solution. The dissolution of salts of the solid component leads to the development of dangerous processes from the building practice point of view – the processes of the chemical suffusion of soils. Freely mobile non-soluble solid particles are present in soil. Their availability may cause the so-called phenomena of mechanical suffusion, which are not less dangerous than the phenomena of chemical suffusion. Besides, if a load in the form of a foundation of some structure is present, it can cause a changing heat state of the porous medium.

Because of the applied load, soil mass may settle. In addition, such factors as mechanical suffusion, chemical suffusion, change of soil filtration properties, temperature and chemical fields will influence the dynamics of settlement processes. The system analysis and the research into the degree of the effect of the said factors on the settlement (swelling) of soil surface is one of the aims of this article.

3. Mathematical Model of the Problem

Let us assume that the process is studied in a one-dimensional domain $\Omega(t) = (l(t),\ L)$. The upper mobile boundary of soil mass is described here with the equation $x = l(t),\ l(t)|_{t=0} = l_0 \geq 0$. The lower soil mass boundary is non-mobile: $x = L > 0$. This results in the following mathematical model [3, 6]:

$$\gamma a\frac{\partial h}{\partial t} - (1+e)^2\frac{\partial s}{\partial t} - (1+e)^2\frac{1}{\rho_N}\frac{\partial N}{\partial t} -$$
$$-e\left(\frac{1}{\rho_p}\left(\frac{\partial \rho_p}{\partial T}\frac{\partial T}{\partial t} + \frac{\partial \rho_p}{\partial c}\frac{\partial c}{\partial t}\right) - \frac{1}{\rho_m}\left(\frac{\partial \rho_m}{\partial T}\frac{\partial T}{\partial t} + \frac{\partial \rho_m}{\partial N}\frac{\partial N}{\partial t}\right)\right) =$$
$$= (1+e)\left(\frac{\partial}{\partial x}\left(k_h\left(\Phi\right)\frac{\partial h}{\partial x}\right) - \frac{\partial}{\partial x}\left(k_c\frac{\partial c}{\partial x}\right) - \frac{\partial}{\partial x}\left(k_T\frac{\partial T}{\partial x}\right)\right),$$

$$x \in \Omega(t),\ t > 0, \tag{1}$$

$$\sigma\left(1-\frac{c}{\rho_p}\frac{\partial \rho_p}{\partial c}\right)\frac{\partial c}{\partial t}=\frac{\partial}{\partial x}\left(D_c\frac{\partial c}{\partial x}\right)-u\left(1-\frac{c}{\rho_p}\frac{\partial \rho_p}{\partial c}\right)\frac{\partial c}{\partial x}+$$
$$+\frac{c}{\rho_p}\frac{\partial \rho_p}{\partial T}\left(\sigma\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right)-\frac{\partial N}{\partial t},\quad x \in \Omega(t),\ t > 0, \tag{2}$$

$$c_T\frac{\partial T}{\partial t}=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)-\rho_p c_p u\frac{\partial T}{\partial x}-\left(\rho_s c_s\frac{\partial s}{\partial t}+\rho_N c_N\frac{\partial N}{\partial t}\right)T,$$
$$x \in \Omega(t), t > 0, \tag{3}$$

$$\frac{\partial s}{\partial t}=\frac{\partial}{\partial x}\left(D_s\frac{\partial s}{\partial x}\right)-\alpha_{er}u\frac{\partial s}{\partial x}, x \in \Omega(t),\ t > 0, \tag{4}$$

$$\frac{\partial N}{\partial t}=-\gamma_1(C_m-c), x \in \overline{\Omega}(t), t > 0, \tag{5}$$

$$u=-k_h\ (\Phi)\frac{\partial h}{\partial x}+k_c\frac{\partial c}{\partial x}+k_T\frac{\partial T}{\partial x}, x \in \overline{\Omega}(t), t \geq 0, \tag{6}$$

$$h(x,t)|_{x=l(t)}=0,\ u(x,t)|_{x=L}=0, \tag{7}$$

$$c(x,t)|_{x=l(t)}=C_l(t),\ q_c(x,t)|_{x=L}=0, \tag{8}$$

$$q_c(x,t)=-D_c\frac{\partial c}{\partial x}+uc, x \in \overline{\Omega}(t), t \geq 0, \tag{9}$$

$$T(x,t)|_{x=l(t)}=T_l(t),\ q_T(x,t)|_{x=L}=0, \tag{10}$$

$$q_T(x,t)=-\lambda\frac{\partial T}{\partial x}+\rho_p c_p uT, x \in \overline{\Omega}(t), t \geq 0, \tag{11}$$

$$s(x,t)|_{x=l(t)}=S_l(t),\ q_s(x,t)|_{x=L}=0, \tag{12}$$

$$q_s(x,t)=-D_s\frac{\partial s}{\partial x}+\alpha_{er}us, x \in \overline{\Omega}(t), t \geq 0, \tag{13}$$

$$\frac{dl(t)}{dt} = -\int_{l(t)}^{L} \left(\frac{e}{1+e} \left(\frac{1}{\rho_p} \left(\frac{\partial \rho_p}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial \rho_p}{\partial c} \frac{\partial c}{\partial t} \right) - \right.\right.$$
$$\left.\left. - \frac{1}{\rho_m} \left(\frac{\partial \rho_m}{\partial T} \frac{\partial T}{\partial t} + \frac{\partial \rho_m}{\partial N} \frac{\partial N}{\partial t} + \frac{\partial \rho_m}{\partial s} \frac{\partial s}{\partial t} \right) \right) + \frac{\partial u}{\partial x} \right) dz, \tag{14}$$

$$l(t)|_{t=0} = l_0 \geq 0, \tag{15}$$

$$h(x,0) = h_0(x), c(x,0) = C_0(x), T(x,0) = T_0(x),$$
$$s(x,0) = S_0(x), N(x,0) = N_0(x), x \in \overline{\Omega}, \tag{16}$$

where γ is the specific weight of the salt solution; a is the soil compression coefficient; $h(x,t)$ is the unknown function of excessive pressures in a porous liquid; t is the time; e is the soil void ratio; $s(x,t)$ is the relative voluminous concentration of suffusion particles; ρ_p is the density of the porous liquid; $T(x,t)$ is the temperature of the porous medium; $c(x,t)$ is the concentration of one-component porous salt solution; ρ_m is the density of solid soil particles (including water-soluble and non-soluble components); $k_h\,(\Phi)$is the filtration coefficient, which depends on the factors of influence vector $\Phi = (h, c, T, s, N)$; k_c, k_T are the coefficients of correspondingly chemical and thermal osmosis; σ is the soil porosity (do not confuse with e. These coefficients are connected by the dependence $\sigma = \frac{e}{1+e}$); D_c is the coefficient of convective diffusion of chemical solution in a porous liquid; u is the filtration rate of salt solution vector; c_T is the soil's voluminous heat capacity; λ is the effective heat conductivity of humid soil coefficient; c_p, c_N, c_s are the specific heat capacities of a porous solution; of a solid component of salts and of suffusion particles correspondingly; D_s is the suffusion particles dispersion coefficient; $\alpha_{er}$is the soil washout coefficient; γ_1 is the mass exchange rate constant; C_m is the maximum saturation concentration; q_c, q_T, q_s are the vectors of soluble salts flows, of the heat, and also of suffusion particles correspondingly; $C_l(t)$, $T_l(t)$, $S_l(t)$, $h_0(x)$, $C_0(x)$, $T_0(x)$, $S_0(x)$, $N_0(x)$ are the specified functions; and l_0 is the known constant, which specifies the initial position of the upper mobile boundary of the soil mass.

Equation (1) – is the equation of filtration consolidation [3], where the effect of physical-chemical processes in soil is considered in relation to the unknown function of excess pressures $h(x,t)$. Equations (2)-(5) describe the change of salts' concentration in a liquid and solid component, the change of soil temperature and the change of suffusion particles. In (5) the presence of salts in the form of dispersed particles in a solid component of soil is taken into account.

It is important to point out, that the functions $\frac{\partial\rho_p}{\partial T}$, $\frac{\partial\rho_p}{\partial c}$, $\frac{\partial\rho_m}{\partial T}$, $\frac{\partial\rho_m}{\partial N}$, $\frac{\partial\rho_m}{\partial s}$ in (1), (2), (14) are to be known. In particular, in [3] it was shown that

$$\frac{\partial\rho_m}{\partial T} = -\alpha_m\rho_m, \frac{\partial\rho_m}{\partial N} = 1, \frac{\partial\rho_m}{\partial s} = \rho_s,$$

where α_m is the voluminous thermal extension of solid particles coefficient. In [3, 6] a survey of relationships $\rho_p = \rho_p(c,T)$is also presented. A survey of dependences of filtration coefficient k_h on the concentration of salts and temperature is provided in work [1]; on the concentration of suffusion particles and porosity coefficient e - in [13].

4. Numerical Solution of the Boundary Problem by Finite Element Method

Suppose H_0 is the space of vector-functions $\varphi(x) = \left\{\varphi^{(k)}(x)\right\}_{k=1}^{5}$, each component $\varphi^{(k)}(x)$, $k = \overline{1,5}$ of which in domain $\Omega(t)$ belongs to Sobolev space $W_2^1(\Omega)$, whereas $\varphi^{(1)}(x)$, $\varphi^{(2)}(x)$, $\varphi^{(3)}(x)$ and $\varphi^{(4)}(x)$ acquire zero values on those particles of domain boundary $\Omega(t)$, where, for functions $h(x,t)$, $c(x,t)$, $T(x,t)$and $s(x,t)$ specified conditions of the first row are set correspondingly.

Let us use the standard procedure: we multiply equations (1)-(5) and each of initial conditions (16), by arbitrary functions $\varphi^{(k)}(x)$, $k = \overline{1,5}$ such that $\left\{\varphi^{(k)}(x)\right\}_{k=1}^{5} \in H_0$, integrate them in the domain $\Omega(t)$ and use the formula of integrating by part. We get:

$$\int_{l(t)}^{L} \frac{\partial h}{\partial t}\varphi^{(1)}(x)dx - \int_{l(t)}^{L} \frac{(1+e)^2}{\gamma a}\frac{\partial s}{\partial t}\varphi^{(1)}(x)dx - \\ - \int_{l(t)}^{L} \left(\frac{(1+e)^2}{\gamma a\rho_N} - \frac{e}{\gamma a\rho_m}\right)\frac{\partial N}{\partial t}\varphi^{(1)}(x)dx - \\ - \int_{l(t)}^{L} \left(\frac{e}{\gamma a\rho_p}\frac{\partial\rho_p}{\partial T} + \frac{e\alpha_m}{\gamma a\rho_m}\right)\frac{\partial T}{\partial t}\varphi^{(1)}(x)dx - \\ - \int_{l(t)}^{L} \frac{e}{\gamma a\rho_p}\frac{\partial\rho_p}{\partial c}\frac{\partial c}{\partial t}\varphi^{(1)}(x)dx + \\ + \int_{l(t)}^{L} \frac{(1+e)}{\gamma a}\left(k_h(\Phi)\frac{\partial h}{\partial x} - k_c\frac{\partial c}{\partial x} - k_T\frac{\partial T}{\partial x}\right)\frac{d\varphi^{(1)}}{dx}dx = 0, \quad (17)$$

$$\int_{l(t)}^{L} \sigma\left(1 - \frac{c}{\rho_p}\frac{\partial\rho_p}{\partial c}\right)\frac{\partial c}{\partial t}\varphi^{(2)}(x)dx + \int_{l(t)}^{L} D_c\frac{\partial c}{\partial x}\frac{d\varphi^{(2)}}{dx}dx +$$

$$+\int_{l(t)}^{L} u\left(1-\frac{c}{\rho_p}\frac{\partial \rho_p}{\partial c}\right)\frac{\partial c}{\partial x}\varphi^{(2)}(x)dx-$$
$$-\int_{l(t)}^{L}\frac{c}{\rho_p}\frac{\partial \rho_p}{\partial T}\left(\sigma\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}\right)\varphi^{(2)}(x)dx+\int_{l(t)}^{L}\frac{\partial N}{\partial t}\varphi^{(2)}(x)dx=0, \quad (18)$$

$$\int_{l(t)}^{L} c_T\frac{\partial T}{\partial t}\varphi^{(3)}(x)dx+\int_{l(t)}^{L}\lambda\frac{\partial T}{\partial x}\frac{d\varphi^{(3)}}{dx}dx+$$
$$+\int_{l(t)}^{L}\left(\rho_p c_p u\frac{\partial T}{\partial x}+\left(\rho_s c_s\frac{\partial s}{\partial t}+\rho_N c_N\frac{\partial N}{\partial t}\right)T\right)\varphi^{(3)}(x)dx=0, \quad (19)$$

$$\int_{l(t)}^{L}\frac{\partial s}{\partial t}\varphi^{(4)}(x)dx+\int_{l(t)}^{L}\left(D_s\frac{\partial s}{\partial x}\frac{d\varphi^{(4)}}{dx}+\alpha_{er}u\frac{\partial s}{\partial x}\varphi^{(4)}(x)\right)dx=0, \quad (20)$$

$$\int_{l(t)}^{L}\frac{\partial N}{\partial t}\varphi^{(5)}(x)dx+\int_{l(t)}^{L}\gamma_1(C_m-c)\varphi^{(5)}(x)dx=0, \quad (21)$$

$$\int_{l_0}^{L} h(x,0)\varphi^{(1)}(x)dx=\int_{l_0}^{L} h_0(x)\varphi^{(1)}(x)dx,$$
$$\int_{l_0}^{L} c(x,0)\varphi^{(2)}(x)dx=\int_{l_0}^{L} C_0(x)\varphi^{(2)}(x)dx,$$
$$\int_{l_0}^{L} T(x,0)\varphi^{(3)}(x)dx=\int_{l_0}^{L} T_0(x)\varphi^{(3)}(x)dx,$$
$$\int_{l_0}^{L} s(x,0)\varphi^{(4)}(x)dx=\int_{l_0}^{L} S_0(x)\varphi^{(4)}(x)dx,$$
$$\int_{l_0}^{L} N(x,0)\varphi^{(5)}(x)dx=\int_{l_0}^{L} N_0(x)\varphi^{(5)}(x)dx. \quad (22)$$

The approximate generalized solution of problem (1)-(16) can be found the following way:

$$(h(x,t);c(x,t);T(x,t);s(x,t);N(x,t))\approx\left\{\sum_{j=1}^{n}a_j^{(k)}(t)\varphi_j(x)\right\}_{k=1}^{5}, \quad (23)$$

Where n is the number of knots in the finite element coverage of the closing $\overline{\Omega}(t)$ of the domain$\Omega(t)$; $a_j^{(k)}(t)$, $j=\overline{1,n}$, $k=\overline{1,5}$, are the unknown coefficients dependent only on time; $\varphi_j(x)$ are the polynomial basic functions of the finite element method (FEM) of a certain natural degree..

From equations (17)-(22) applying in turns functions $\varphi^{(k)}(x)$, $k=\overline{1,5}$, equal to each basic function $\varphi_j(x)$, $j=\overline{1,n}$, considering (23) we get Cauchy problem for the system of non-linear differential equations concerning the vector $A(t)=\left\{A^{(k)}(t)\right\}_{k=1}^{5}$:

$$\sum_{k=1}^{5} R^{(pk)}\cdot\frac{dA^{(k)}}{dt}+\sum_{k=1}^{5} G^{(pk)}\left(A\right)\cdot A^{(k)}=F^{(i)},\ p=\overline{1,5}, \tag{24}$$

$$\overline{R}^{(p)}\cdot A^{(p)(0)}=\overline{F}^{(p)},\ p=\overline{1,5}, \tag{25}$$

where

$$A^{(k)}(t)=\left\{a_i^{(k)}(t)\right\}_{i=1}^{n}; A^{(k)(0)}=A^{(k)}(t)\Big|_{t=0}; R^{(pk)}=\left\{r_{ij}^{(pk)}(t)\right\}_{i,j=1}^{n};$$

$$G^{(pk)}=\left\{g_{ij}^{(pk)}(t)\right\}_{i,j=1}^{n}; F^{(k)}=\left\{f_i^{(k)}\right\}_{i=1}^{n}; \overline{R}^{(k)}=\left\{\overline{r}_{ij}^{(k)}(t)\right\}_{i,j=1}^{n};$$

$$\overline{F}^{(k)}=\left\{\overline{f}_i^{(k)}\right\}_{i=1}^{n};$$

$$r_{ij}^{(11)}=\int_{l(t)}^{L}\varphi_i(x)\varphi_j(x)dx; r_{ij}^{(12)}=-\int_{l(t)}^{L}\frac{e}{\gamma a\rho_p}\frac{\partial\rho_p}{\partial c}\varphi_i(x)\varphi_j(x)dx;$$

$$r_{ij}^{(13)}=-\int_{l(t)}^{L}\left(\frac{e}{\gamma a\rho_p}\frac{\partial\rho_p}{\partial T}+\frac{e\alpha_m}{\gamma a}\right)\varphi_i(x)\varphi_j(x)dx,$$

$$r_{ij}^{(14)}=-\int_{l(t)}^{L}\frac{(1+e)^2}{\gamma a}\varphi_i(x)\varphi_j(x)dx,$$

$$r_{ij}^{(15)}=-\int_{l(t)}^{L}\left(\frac{(1+e)^2}{\gamma a\rho_N}-\frac{e}{\gamma a\rho_m}\right)\varphi_j(x)\varphi_j(x)dx,$$

$$g_{ij}^{(11)}=\int_{l(t)}^{L}\frac{(1+e)}{\gamma a}k_h\left(\Phi\right)\frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx}dx,$$

$$g_{ij}^{(12)} = -\int_{l(t)}^{L} \frac{(1+e)}{\gamma a} k_c \frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx}dx, g_{ij}^{(13)} = -\int_{l(t)}^{L} \frac{(1+e)}{\gamma a} k_T \frac{d\varphi_i}{dx}\frac{d\varphi_j}{dx}dx,$$

$$\overline{r}_{ij}^{(1)} = \int_{l_0}^{L} \varphi_i(x)\varphi_j(x)dx; \overline{f}_i^{(1)} = \int_{l_0}^{L} h_0(x)\varphi_i(x)dx.$$

To find the numerical solution to Cauchy problem (24), (25) we divide the time interval $[0; t_{\max}]$ by M equal parts with the step $\tau = \frac{t_{\max}}{M}$. We designate approximate solutions for Cauchy problem (24), (25) through $A^{(m)} = \left\{A^{(m)(k)}\right\}_{k=1}^{5}$, under $t = m\tau$. We introduce the following designation as well: $A^{(m+1/2)} = \frac{1}{2}(A^{(m+1)} + A^{(m)})$. The approximate solution to the system of non-linear differential equations (24) can be received with the help of Krank-Nikolson scheme [14]:

$$\sum_{k=1}^{5} R^{(pk)} \cdot \frac{A^{(m+1)(k)} - A^{(m)(k)}}{\tau} +$$

$$+\sum_{k=1}^{5} G^{(pk)}\left(A^{\left(m+1/2\right)}\right) \cdot A^{\left(m+1/2\right)(k)} = F^{\left(m+1/2\right)(p)},\ p = \overline{1,5},$$

$m = 0, 1, 2, ..., M-1$. As Krank-Nikolson scheme requires the solution of a system of non-linear equasions at each time layer, then, in order to avoid these difficulties, approximate solutions of Cauchy problem may be found using the predictor-corrector approximation [15].

Equations (2)-(4) contain members of convective transfer. Therefore, the application of FEM based on Galerkin classical scheme may lead to the loss of the stability of approximate solution (the solution is obtained in the form of a "saw"). For such type of problems we can use SUPG scheme. For example, for equation (3) it appears in the form:

$$\int_{l(t)}^{L} c_T \frac{\partial T}{\partial t}\varphi_3(x)dx + \int_{l(t)}^{L} \lambda \frac{\partial T}{\partial x}\frac{d\varphi_3}{dx}dx + \int_{l(t)}^{L} \rho_p c_p u \frac{\partial T}{\partial x}\varphi_3(x)dx +$$

$$+\sum_{\Omega_e} \tau_e^{(T)} \int_{l(t)}^{L} \left(c_T \frac{\partial T}{\partial t} - \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \rho_p c_p u \frac{\partial T}{\partial x} + \right.$$

$$\left. + \left(\rho_s c_s \frac{\partial s}{\partial t} + \rho_N c_N \frac{\partial N}{\partial t}\right) T \right) \rho_p c_p u \frac{d\varphi_3}{dx}dx +$$

$$+\int_{l(t)}^{L}\left(\rho_s c_s \frac{\partial s}{\partial t} + \rho_N c_N \frac{\partial N}{\partial t}\right) T\varphi_3(x)dx = 0,$$

where $\Omega = \bigcup \Omega_e$, $\Omega_e$are the finite elements; $\tau_e^{(T)}$ are the stabilization parameters. The rules of the regularization parameters selection are described, for example, in works [16, 17]. Analogous schemes may be provided for equations (2), (4). Only as to avoid extra awkwardness no expositions of these schemes are presented in this article.

As a result of discretization of kinematic boundary condition (14) we get:

$$\frac{l^{(m+1)} - l^{(m)}}{\tau} = -\int_{l^{(m)}}^{L} \frac{e}{\rho_p(1+e)}\left(\frac{\partial \rho_p}{\partial T}\frac{T^{(m+1)} - T^{(m)}}{\tau} + \right.$$
$$\left. + \frac{\partial \rho_p}{\partial c}\frac{c^{(m+1)} - c^{(m)}}{\tau}\right) dz +$$
$$+\int_{l^{(m)}}^{L}\left(\frac{e}{\rho_m(1+e)}\left(-\alpha_m\rho_m\frac{T^{(m+1)} - T^{(m)}}{\tau} + \right.\right.$$
$$\left.\left. + \frac{N^{(m+1)} - N^{(m)}}{\tau} + \rho_s\frac{s^{(m+1)} - s^{(m)}}{\tau}\right) - \frac{\partial u^{(m+1)}}{\partial x}\right) dz. \quad (26)$$

From the equation (26) the position of the upper mobile soil boundary is determined at time layer $(m+1)$. The integral in the right part of (26) may be calculated using quadrature formulas.

5. Results of Numerical Experiments

Let us study boundary value problem (1)-(16) with the following numeric entry data:
$a = 5.12\frac{m^2}{H}$; $\sigma_0 = 0.375$; $e_0 = \frac{\sigma_0}{1-\sigma_0} = 0.6$; $c_p = 4.128\frac{kJ}{kg\cdot {}^0C}$; $c_s = 0.92\frac{kJ}{kg\cdot {}^0C}$; $c_N = 0.8709\frac{kJ}{kg\cdot {}^0C}$; $c_m = 0.8\frac{kJ}{kg\cdot {}^0C}$; $\rho_p = 1100\frac{kg}{m^3}$; $\rho_s = 2200\frac{kg}{m^3}$; $\rho_N = 2170\frac{kg}{m^3}$; $\rho_m = 1500\frac{kg}{m^3}$; $k_c = 2.8\cdot 10^{-6}\frac{m^5}{kg\cdot day}$; $k_T = 2.8\cdot 10^{-7}\frac{m^2}{{}^0C\cdot day}$; $D_c = 9.51\cdot 10^{-5}\frac{m^2}{day}$; $\gamma_1 = 10^{-6}day^{-1}$; $\lambda = 108\frac{kJ}{m\cdot {}^0C\cdot day}$; $l_0 = 20m$; $L = 20m$; $h_0(x) = 20m$; $C_l(t) = C_m = 350\frac{kg}{m^3}$; $C_0(x) = C_{\min} = 8\frac{kg}{m^3}$; $T_l(t) = T_{\max} = 40^0C$; $T_0(x) = T_{\min} = 4^0C$; $\sigma_s = 0.5$; $s_{\max} = (1-\sigma_s)\,\sigma_0 = 0.1875$; $S_0(x) = s_{\max}$; $S_l = 0$; $N_{\max} = 120\frac{kg}{m^3}$; $N_{\min} = 4\frac{kg}{m^3}$; $\alpha_{er} = 0.01$;

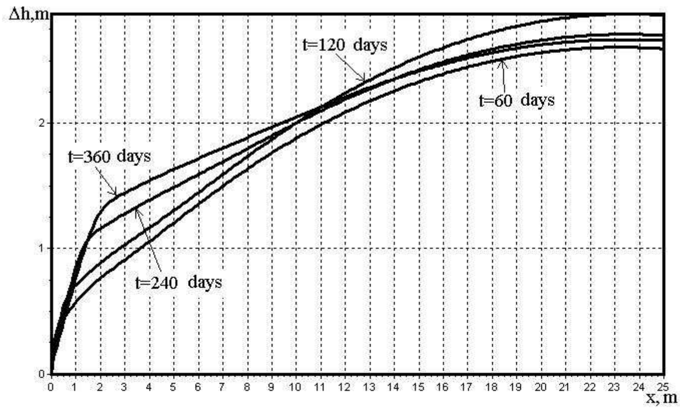

Figure 1: Graphs of differences of excess pressures

$$N_0(x, y) = 4\left(N_{\max} - N_{\min}\right)\left(-\left(\frac{x}{L}\right)^2 + \frac{x}{L}\right) + N_{\min}.$$

The coefficient of clean water filtration is $k_0 = 0.01\ m/day$. The voluminous heat capacity of a porous medium is:

$$c_T = \rho_p c_p \sigma + \rho_s c_s s + \rho_N c_N N + \rho_m c_m \left(1 - \sigma_0\right).$$

The suffusion particles dispersion coefficient is $D_s = \alpha\left|u\right|,\ \ \alpha = 0.1(m)$. Also,

$$\sigma(x, t) = \sigma_0 - \frac{N(x, t)}{\rho_N} - \frac{s(x, t)}{\rho_s}, e = \frac{\sigma}{1 - \sigma},$$

where σ_0 is the porosity of the "skeleton" of the porous medium (porosity in the case of complete absence of suffusion and water soluble particles); $\rho_N = const$ and $\rho_s = const$ are the density of water- soluble formations and the material of suffusion particles respectively. The dependence of filtration coefficient on salts concentration and temperature was determined in accordance with [1]. The dependence $\rho_p = \rho_p\left(c, T\right)$was obtained from [6].

We developed a program implementing the algorithms of the numerical solution of the problem through the finite element method. The created program allows to build graphs of all unknown functions, including when these or other factors were or were not taken into consideration. Also, there is a possibility of deriving numerical values of unknown functions in any point of the boundary problem domain, tracing the displacement of each point of the domain in

Time moment, day	Coordinates of soil mass' upper knot (not taking into account influence of mechanical-chemical suffusion) (m)	Coordinates of upper node of soil mass (in accordance with mathematical model (1)-(16))	Relative deviations (%)
60	0,34349	0,43189	25,70
120	0,45390	0,55884	23,12
180	0,50529	0,63994	26,65
240	0,54532	0,70547	29,37
300	0,59506	0,76382	28,36
360	0,61921	0,81757	32,03

Table 1: Settling of upper boundary of soil mass in different time moments

time. In particular, as shown in the table, the settling of soil surface in the case when the effect of mechanical–chemical suffusion was taken into consideration increases. It is important to note that the range of the relative increased settling of soil surface during a year lies within 23-32 percent.

On Fig. 1 graphs of the difference in pressures are provided with and without taking the influence of mechanical-chemical suffusion into account. As it is seen from graphs in Fig.1, pressures in the case, where the influence of suffusion processes was taken into account, disperse quicker than without considering it. Qualitatively this correlates with the physical essence of the process – the carrying away of suffusion particles and the dissolution of salts in the solid component causes the increased volume of pores, and, hence, the acceleration of pressures decrease in a porous liquid. Besides, the built model also permits the quantitative evaluation of discussed effects.

The effect of the influence of suffusion processes on soils' consolidation is studied in the article through the example of a one-dimensional problem. Therefore, next researches may concern two- and three-dimensional cases. Also, the questions of the qualitative investigation of the said boundary problems remain open. The authors' aim for the future is to both establish conditions concerning the existence and the unity of boundary problems solution and to research into the qualitative characteristics of approximate solutions.

6. Summary

The article considers the system of interrelated physical-chemical processes in completely saturated porous media exemplified by soils. In particular, interactions of processes of soil compaction, of mechanical and chemical suffusion, and also of heat field were studied. A mathematical model was built in relation to the physical statement of problem. A numerical solution of corresponding non-linear boundary problem in the domain with a mobile boundary was found using the finite element method. It was established, that the activation of processes of chemical and mechanical suffusion leads to increased settling of a soil mass surface. The tendency was observed for speedier dispersion of excess pressures in the case of the influence of mechanical-chemical suffusion. A program was created, which permits the evaluation of said effects. Guidelines for further studies were determined.

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