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ON SUBSPACE-HYPERCYCLIC AND SUPERCYCLIC SEMIGROUP

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Abstract: A C_0-semigroup $\mathcal{T} = (T_t)_{t\geq 0}$ on a infinite-dimensional separable complex Banach space X, is called subspace-hypercyclic (resp. subspace-supercyclic) for a subspace M, if $Orb(\mathcal{T}, x) \bigcap M$ (resp. $\mathbb{C}Orb(\mathcal{T}, x) \bigcap M = \{\lambda T_t x : \lambda \in \mathbb{C}, t \geq 0\} \bigcap M$) is dense in M for a vector $x \in M$. In this paper we provide a Subspace-hypercyclicity Criterion and we show if $\mathcal{T} = (T_t)_{t\geq 0}$ and $\mathcal{S} = (S_t)_{t\geq 0}$ are C_0-semigroups and M_1, M_2 are nonzero closed subspaces of X and $(T_t \oplus S_t)_{t\geq 0}$ is $(M_1 \oplus M_2)$-hypercyclic C_0-semigroups, then $\mathcal{T}$ and $\mathcal{S}$ are M_1-hypercyclic and M_2-hypercyclic C_0-semigroups , respectively. At the same time, we also characterize other properties of subspace-hypercyclic (resp. subspace-supercyclic) C_0-semigroup.

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1. Introduction

Let X be a separable infinite dimensional Banach space over the scalar field $\mathbb{C}$ and let $\mathcal{B}(X)$ denote the set of all bounded linear operators on X and we will usually refer to elements of $\mathcal{B}(X)$ as just operators. A bounded linear operator $T : X \to X$ is called hypercyclic (respectively, supercyclic) if there is some

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vector $x \in X$ such that $Orb(T, x) = \{T^n x : n = 0, 1, 2, ...\}$ (respectively, the projective orbit $\mathbb{C}Orb(T, x) = \{\lambda T^n x : \lambda \in \mathbb{C}, n = 0, 1, 2, ...\}$) is dense in X. Such a vector x is said hypercyclic (respectively, supercyclic) for T. Refer to [3][12][22] for more informations about hypercyclicity and supercyclicity.

The definition and the properties of supercyclicity operators were introduced by Hilden and Wallen [10]. They proved that all unilateral backward weighted shifts on a Hilbert space are supercyclic. The study of supercyclic operators has experienced a great of development in recent years. Salas gave a characterization of supercyclic bilateral backward weighted shifts via the Supercyclicity Criterion in [20]. Montes and Salas [17] refined the Supercyclicity Criterion and proved that it is equivalent to the former given by Salas. Besides, T. Bermúdez, A. Bonilla and A. Peris [4] showed that the equivalence of two supercyclicity criteria given by N. Feldman, V. Miller and L. Miller in [8] to the Supercyclicity Criterion.

In 2011, B. F. Madore and R. A. Martnez-Avendano in [15] introduced and studied the concept of subspace-hypercyclicity for an operator. An operator T is subspace-hypercyclic or M-hypercyclic for a subspace M of X, if there exists $x \in X$ such that $Orb(T, x) \bigcap M$ is dense in M. Such a vector x is called a M-hypercyclic vector for T, they showed that there are operators which are M-hypercyclic but not hypercyclic. They introduced analogously the concept of subspace-transitivity. Let $T \in \mathcal{B}(X)$ and M be a closed subspace of X, we say that T is M-transitive, if for any non-empty open sets U, V in M, there exists $n \geq 0$ such that $T^{-n}(U) \bigcap V$ contain a non-empty open subset of M.The authors showed that M-transitivity implies M-hypercyclicity. Note that the converse is not true, this is proven recently by C. M. Le in [13]; for more informations see [11][18].

Similarly, for subspace-supercyclicity, Zhao, Y.L. Sun and Y.H. Zhou in [24] provided a Subspace-Supercyclicity Criterion and offered two necessary and sufficient conditions for a path of bounded linear operators to have a dense G_δ set of common subspace-hypercyclic vectors and common subspace-supercyclic vectors and they also constructed examples to show that subspace-supercyclic is not a strictly infinite dimensional phenomenon and that some subspace-supercyclic operators are not supercyclic.

In [2] Nareen Bamerni and Adem Kiliman studied the relation between subspace-hypercyclicity and the direct sum of two operators. In particular, they showed that if the direct sum of two operators is subspace-hypercyclic, then the both operators are subspace-hypercyclic; however, the converse is true for a stronger property than subspace-hypercyclicity. Moreover, they proved that if an operator T satisfies subspace-hypercyclic criterion, then $T \oplus T$ is subspace-hypercyclic. However, they showed that the converse is true under certain conditions.

Recall that a one-parameter family $(T_t)_{t\geq 0}$ of operators on X is called a strongly continuous semigroup (or C_0-semigroup) of operators, if $T_0 = I$, $T_{t+s} = T_tT_s$ for all $t, s \geq 0$ and $\lim_{t\longrightarrow s} T_t(x) = T_s(x)$ for all $s \geq 0$ and $x \in X$; see [7][12][16].

A C_0-semigroup $\mathcal{T} = (T_t)_{t\geq 0}$ of linear and continuous operators on X is said to be hypercyclic (respectively, supercyclic), if there is some vector $x \in X$ such that the set $Orb(\mathcal{T}, x) = \{T_tx : t \geq 0\}$ (respectively, the projective orbit $\mathbb{C}Orb(\mathcal{T}, x) = \{\lambda T_tx : \lambda \in \mathbb{C}, t \geq 0\}$) is dense in X. Such a vector x is said hypercyclic (respectively, supercyclic) for $\mathcal{T}$.

In [6] Jos A.Conjero, Alfredo Peris showed that different transitivity criteria for strongly continuous semigroups of operators are equivalent. They also gave new results concerning the equivalence of transitivity criteria in the case of iterations of a single operator. ; for more informations see [5].

Recently, in [21] Abdelaziz Tajmouati , Abdeslam El Bakkali and Ahmed Toukmati introduced and studied the M-Hypercyclicity of C_0-semigroup $\mathcal{T} = (T_t)_{t\geq 0}$ on an infinite-dimensional separable complex Banach space X, and gave sufficient conditions of being M-hypercyclic for this semigroup. Moreover, some proprieties and analogous results for the notion of M-Transitive.

In [23] we localized the notion of M-extended semigroup(resp. M-extended semigroup mixing) limit set of x under $\mathcal{T} = (T_t)_{t\geq 0}$ and we gave sufficient conditions of being M-hypercyclic for this semigroup. Then by this result, we proved that $(T_t^{-1})_{t\geq 0}$ is a M-hypercyclic.

In this present paper, we provide a Subspace-hypercyclicity Criterion and we show if $\mathcal{T} = (T_t)_{t\geq 0}$ and $\mathcal{S} = (S_t)_{t\geq 0}$ are C_0-semigroups and M_1, M_2 are nonzero closed subspaces of X and $(T_t \oplus S_t)_{t\geq 0}$ is $(M_1 \oplus M_2)$-hypercyclic C_0-semigroups, then $\mathcal{T}$ and $\mathcal{S}$ are M_1-hypercyclic and M_2-hypercyclic C_0-semigroups , respectively. At the same time, we also characterize other properties of subspace-hypercyclic (resp. subspace-supercyclic) C_0-semigroup.

2. Main results

We will assume that the subspace $M \subset X$ is topologically closed. We start with our main definitions.

Definition 2.1. [21] Let $\mathcal{T} = (T_t)_{t\geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. We say that $\mathcal{T}$ is $M-$hypercyclic if there exists a vector $x \in X$ such that $Orb(\mathcal{T}, x) \bigcap M$ is dense in M with $Orb(\mathcal{T}, x) = \{T_tx : t \geq 0\}$

Definition 2.2. Let $\mathcal{T} = (T_t)_{t\geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. We say that $\mathcal{T}$ is $M-$supercyclic if there exists a vector $x \in X$ such that $\mathbb{C}Orb(\mathcal{T}, x) \bigcap M = \{\lambda T_t x : \lambda \in \mathbb{C}, t \geq 0\} \bigcap M$ is dense in M. We call x a $M-$supercyclic vector.

Definition 2.3. Let $\mathcal{T} = (T_t)_{t\geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. We call that $\mathcal{T}$ is M-mixing if for every tow open, non-empty subsets U, V of M there is $t_0 \geq 0$ such that for all $t \geq t_0, T_t^{-1}(U) \bigcap V$ contains a non-empty open set of M

Example 1. [24] Let T be a supercyclic operator on X with supercyclic vector x and let I be the identity operator on X. Then the operator $T \oplus I : X \oplus X \to X \oplus X$ is subspace-supercyclic for the subspace $M := X \oplus \{0\}$ with the subspace-supercyclic vector $x \oplus \{0\}$, but $T \oplus I$ is not supercyclic on the space $X \oplus X$.

Definition 2.4. Let $\mathcal{T} = (T_t)_{t\geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X.

$J_s(\mathcal{T}, M, x) = \{y \in X$: for every relatively open neighborhoods U, V of x, y in M respectively, and every positive real numbers s, there exist $t > s$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $\lambda T_t(U) \cap V \neq \emptyset$ and $T_t(M) \subset M\}$ denotes M-extended C_0-semigroup limit set of x under $(T_t)_{t\geq 0}$.

Proposition 2.1. *An equivalent definition for the sets $J_S(\mathcal{T}, M, x)$ is the following:*

$J_s(\mathcal{T}, M, x) = \{y \in X$: *there exist a strictly increasing sequence of positive real numbers t_n with $t_n \to \infty$ and a sequence $(x_n)_n \subset X$ and $(\lambda_{t_n})_n \in \mathbb{C}\backslash\{0\}$ such that $x_n \to x$ and $\lambda_{t_n} T_{t_n} x_n \to y$ and for every n, $T_{t_n}(M) \subset M\}$*

Proof. Let $y \in J_s(T_t, M, x)$ and consider the open balls $U_n = B(x, \frac{1}{n}) \cap M, V_n = B(y, \frac{1}{n}) \cap M$ centered at $x, y \in X$ and radius $1/n$ for $n = 1, 2, ...$ and $s = t_{n-1}, t_0 = 1$. Then there exists $t_n > s = t_{n-1}$ and $(\lambda_{t_n})_n \in \mathbb{C}\backslash\{0\}$ such that

$$\lambda_{t_n} T_{t_n}(U_n) \cap V_n \neq \emptyset \text{ and } T_{t_n}(M) \subset M.$$

Hence there exists $x_n \in U_n$ such that $\lambda_{t_n} T_{t_n} x_n \in V_n$ and $T_{t_n}(M) \subseteq M$. Therefore, there exist a strictly increasing sequence of positive real numbers $t_n \geq 0$ and a sequence $(x_n)_n \subset X$ and $(\lambda_{t_n})_n \in \mathbb{C}\backslash\{0\}$ such that $x_n \to x$ and $\lambda_{t_n} T_{t_n} x_n \to y$ and for every n, $T_{t_n}(M) \subset M$. The converse is obvious. □

Theorem 2.1. *Let $\mathcal{T} = (T_t)_{t\geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. Then the following conditions are equivalent:*

1. *For every non-empty open U and V of M, there exists $t_0 \geq 0$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $(\lambda T_{t_0})^{-1}(U) \bigcap V$ contains a non-empty open subset of M.*

2. *For every non-empty open U and V of M, there exists $t_0 \geq 0$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $(\lambda T_{t_0})^{-1}(U) \bigcap V$ is non-empty and $T_t(M) \subset M$.*

3. *For every non-empty open U and V of M, there exists $t_0 \geq 0$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $(\lambda T_{t_0})^{-1}(U) \bigcap V$ is non-empty open subset of M.*

Proof. (1) $\Rightarrow$ (2). Let U and V be tow nonempty open subsets of M. By (1) there exists $t_0 \geq 0$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $(\lambda T_{t_0})^{-1}(U) \bigcap V$ contains a non-empty open W of M, it follows that $W \subset (\lambda T_{t_0})^{-1}(U) \cap V$ and $(\lambda T_{t_0})^{-1}(U) \cap V \neq \emptyset$.

Next, We prove that $T_{t_0}(M) \subset M$.

Let $x \in M$, we have $W \subset (\lambda T_{t_0})^{-1}(U) \cap V$, this implies that $T_{t_0}(W) \subset U \subset M$. Let $x_0 \in W$, since W is open of M then for all r enough small we have $x_0 + rx \in W$, therefore $\lambda T_{t_0}(x_0 + rx) = (\lambda T_{t_0} x_0 + \lambda r T_{t_0} x) \in M$. Since $\lambda T_{t_0} x_0 \in M$, it follows that $\lambda r T_{t_0} x \in M$. We then conclude that $T_{t_0}(M) \subset M$.

(2) $\Rightarrow$ (3). Let U and V be nonempty open subsets of M, by (2) there exists $t_0 \geq 0$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $(\lambda T_{t_0})^{-1}(U) \bigcap V$ is non-empty and $T_{t_0}(M) \subset M$.

Since $(\lambda T_{t_0})|M : M \longrightarrow M$ is continuous, then $(\lambda T_{t_0})^{-1}(U)$ is open in M, therefore $(\lambda T_{t_0})^{-1}(U) \cap V$ is nonempty open of M.

At last, we see that the implication (3) $\Rightarrow$ (1) is obvious and this completes the whole proof of the theorem. □

Corollairy 2.1. *Let $\mathcal{T} = (T_t)_{t \geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. Then the following conditions are equivalent*

1. *For every non-empty open U and V of M, there exists $t_0 \geq 0$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $(\lambda T_{t_0})^{-1}(U) \bigcap V$ contains a non-empty open subset of M.*

2. *For every $x \in M, J_s(\mathcal{T}, M, x) = M$.*

Proof. we use a proof method of the [23, Theorem 2.3] □

Theorem 2.2. *Let $\mathcal{T} = (T_t)_{t \geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. For every $x \in M, J_s(\mathcal{T}, M, x) = M$. Then $\mathcal{T}$ is subspace-supercyclic semigroup for M.*

Proof. For any nonempty sets $U \subseteq M$ and $V \subseteq M$, both relatively open, consider $x \in U, y \in V$. Since $J_s(\mathcal{T}, M, x) = M$, there exist $t > s = 1$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $\lambda T_t(U) \cap V \neq \emptyset$ and $T_t(M) \subset M$. By Theorem 2.1, $\mathcal{T}$ is subspace-supercyclic semigroup for M. □

Theorem 2.3. *Let $\mathcal{T} = (T_t)_{t\geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. If for every $x \in M, J_s(\mathcal{T}, M, x) = M$ and T_t, $t \geq 0$ be an invertible operators. Then $\mathcal{T}^{-1}$ is subspace-supercyclic semigroup for M.*

Proof. First, by Theorem 2.2, $\mathcal{T}$ is subspace-supercyclic semigroup for M. For any $x, y \in M$, by assumption, $J_s(\mathcal{T}, M, x) = M$. For any nonempty sets $U \subseteq M$ and $V \subseteq M$, both relatively open such that contain x, y respectively, then there exist $t > 1$ and $\lambda \in \mathbb{C}\backslash\{0\}$ such that $\lambda T_t(U) \cap V \neq \emptyset$ and $T_t(M) \subset M$. Hence for every $y \in M$, $J_s(\mathcal{T}^{-1}, M, y) = M$. By Theorem 2.2, $\mathcal{T}^{-1}$ is also subspace-supercyclic semigroup for M. □

Let M_1 and M_2 be subspaces of a Banach space X, then the direct sum of M_1 and M_2 is defined as follows

$$M_1 \oplus M_2 = \{(x, y) : x \in M_1, y \in M_2\}$$

Theorem 2.4. *Let $\mathcal{T} = (T_t)_{t\geq 0}$ and $\mathcal{S} = (S_t)_{t\geq 0}$ be a C_0-semigroups and M_1, M_2 be a nonzero closed subspaces of X and $(T_t \oplus S_t)_{t\geq 0}$ is $(M_1 \oplus M_2)$-hypercyclic C_0-semigroup, then $\mathcal{T}$ and $\mathcal{S}$ are M_1-hypercyclic and M_2-hypercyclic C_0-semigroups, respectively.*

Proof. Let $x_1 \in M_1$ and $x_2 \in M_2$, and let $(x, y) \in HC(T_t \oplus S_t, M_1 \oplus M_2)$, then there exist an $\varepsilon > 0$ and an increasing sequence of positive real numbers $(t_n)_{n\in\mathbb{N}}$ such that

$$||(T_{t_n} \oplus S_{t_n})(x, y) - (x_1, x_2)||_{M_1\oplus M_2} \leq \varepsilon.$$

It follows

$$||T_{t_n}x - x_1||_{M_1} + ||S_{t_n}y - x_2||_{M_2} \leq \varepsilon.$$

Then

$$||T_{t_n}x - x_1||_{M_1} \leq \varepsilon \text{ and } ||S_{t_n}y - x_2||_{M_2} \leq \varepsilon.$$

Thus, there exists an increasing sequence of positive real numbers $(t_n)_{n\in\mathbb{N}}$ such that $\{T_{t_n} : t_n \text{ in } \mathbb{R}_+\}$ and $\{S_{t_n} : t_n \text{ in } \mathbb{R}_+\}$ are dense in M_1 and M_2, respectively. Therefore $Orb(\mathcal{T}, x)$ and $Orb(\mathcal{S}, y)$ are dense in M_1 and M_2, respectively. □

Theorem 2.5. *Let $\mathcal{T} = (T_t)_{t\geq 0}$ and $\mathcal{S} = (S_t)_{t\geq 0}$ be a C_0-semigroups and M_1, M_2 be a nonzero closed subspaces of X and $(T_t \oplus S_t)_{t\geq 0}$ is $(M_1 \oplus M_2)$-mixing C_0-semigroups, then $\mathcal{T}$ and $\mathcal{S}$ are M_1-mixing and M_2-mixing C_0-semigroups , respectively.*

Proof. let U_1 and U_2 be open sets in M_1, and V_1 and V_2 be open sets in M_2, then $U_1 \oplus V_1$ and $U_2 \oplus V_2$ are open in $M_1 \oplus M_2$. So there exists an $t_0 \geq 0$ such that

$$(T_t \oplus S_t)^{-1}(U_1 \oplus V_1) \cap (U_2 \oplus V_2) \neq \emptyset,$$

and

$$(T_t \oplus S_t)(M_1 \oplus M_2) \subseteq (M_1 \oplus M_2)$$

for all $t \geq t_0$. Then

$$T_t^{-1}(U_1) \bigcap U_2 \neq \emptyset,\ S_t^{-1}(V_1) \bigcap V_2 \neq \emptyset,\ T_t(M_1) \subset M_1 \text{ and } S_t(M_2) \subset M_2.$$

Therefore, $\mathcal{T}$ and $\mathcal{S}$ are M_1-mixing and M_2-mixing C_0-semigroups , respectively. □

Next we get the following theorem, which is the Subspace-hypercyclicity Criterion C_0-semigroup, which is similar to the hypercyclictiy Criterion that was stated in [4]; see also [12, Theorem 7.27].

Theorem 2.6. *(Subspace-hypercyclicity Criterion C_0-semigroup) Let $\mathcal{T} = (T_t)_{t\geq 0}$ be a C_0-semigroup and M be a nonzero subspace of X. Assume that there exist M_0 and M_1, dense subsets of M, an increasing sequence $(t_n)_{n\geq 0}$ in $\mathbb{R}_+$ with $t_n \to \infty$, and a sequence of mappings $S_{t_n} : M_1 \to M$, $n \in \mathbb{N}$ such that*

1. *For each $x \in M_0$, $T_{t_n}x \to 0$,*
2. *For each $y \in M_1$, $S_{t_n}y \to 0$,*
3. *For each $y \in M_1$, $(T_{t_n} \circ S_{t_n})y \to y$,*
4. *M is an invariant subspace for T_{t_n} for all $n \geq 0$.*

Then $\mathcal{T} = (T_t)_{t\geq 0}$ is subspace-hypercyclic C_0-semigroup for M.

Proof. Let U and V be non-empty open subsets of M. By [21, Theorem 2.1], it is enough to prove that there exist $t_0 \geq 0$ such that

$$T_{t_0}^{-1}(U) \bigcap V \text{ is non-empty and } T_{t_0}(M) \subset M.$$

Since M_0 and M_1 are dense in M, there exist $x \in M_0 \cap V, y \in M_1 \cap U$. And since U and V are nonempty open subsets, there exists $\varepsilon > 0$ such that $B_M(x, \varepsilon) \subseteq V$ and $B_M(y, \varepsilon) \subseteq U$. By assumption, there exist $(t_n)_{n\in\mathbb{N}}$ such that

$$||T_{t_n}x|| < \frac{\varepsilon}{2}, ||S_{t_n}y|| < \frac{\varepsilon}{2} \text{ and } ||T_{t_n}S_{t_n}y - y|| < \frac{\varepsilon}{2}.$$

Define $u = x + S_{t_n}y$. We know that $u \in M$ and $u \in V$, since $||u - x|| = ||S_{t_n}y|| \leq \frac{\varepsilon}{2}$. Observe that $T_{t_n}u = T_{t_n}x + T_{t_n}S_{t_n}y$, so $T_{t_n}u \in M$. Since

$$||T_{t_n}u - y|| \leq ||T_{t_n}x|| + ||T_{t_n}S_{t_n}y - y|| < \varepsilon,$$

we have that $T_{t_n}u \in U$. Then $T_{t_n}^{-1}(U) \bigcap V \neq \emptyset$ and $\mathcal{T}$ is subspace-hypercyclic C_0-semigroup for M. □

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