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SOLUTION OF SYSTEM OF NONLINEAR EQUATIONS USING INTEGRATED RADM AND ADM

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Abstract: In this paper, we apply an incorporated restarted Adomian decomposition method (RADM) and Adomian decomposition method (ADM) for solving the system of nonlinear equations. Illustrative example has been presented to show the effectiveness of the proposed method. Obtained results show that the proposed method has been an efficient approach for solving the system of non linear equations.

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1. Introduction

Solutions system of nonlinear equations arises in many parts of the engineer-

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ing applications such as pressure driven analysis of water distribution network in earth and atmospheric science, heat and thermal conduction in mechanical engineering, power distribution system in electrical science, to name the few. The decomposition method was first introduced by Adomian [1, 2] at the beginning of the 1980's. This iterative scheme can be used to solve a wide range of problems whose mathematical models yield an equation or a system of algebraic, differential, integral and integro-differential equations. In this method the solution is considered as the sum of an infinite series, rapidly converging to an accurate solution.

Abboui and Cherrualt applied the Adomian decomposition method (ADM) to solve the equation $f(x) = 0$ where $f(x)$ is a nonlinear function [3]. Babolian et al. [4] applied the standard ADM to solve the system of nonlinear equations. Jafari and Daftardar [5] introduced Revised ADM for solving a system of nonlinear equations. By this modification the series solution converges faster than that of standard ADM.

Restarted Adomian decomposition method (RADM), based on standard ADM introduced by Babolian, et al. [6] for algebraic equations. H. Sadeghi et al. [7] applied the RADM for solving system of nonlinear Voltra integral equations.

2. ADM for System of Nonlinear Equations

Consider the following system of nonlinear equations

$$f_i(x_1, x_2, ..., x_n) = 0, \quad i = 1, 2, ..., n, \tag{1}$$

The equations (1) can be written in the following form

$$x_i = c_i + N_i(x_1, x_2, , x_n), \quad i = 1, 2, ..., n, \tag{2}$$

where c_i are constants and $N_i s$ are in general nonlinear functions on their arguments. The standard ADM yields the solution x_i in terms of the series

$$x_i = \sum_{j=0}^{\infty} x_{i,j}, \quad i = 1, 2, ..., n \tag{3}$$

and nonlinear functions N_i are expressed in terms of an infinite series of Adomian's polynomials

$$N_i(x_1, x_2, ..., x_n) = \sum_{j=0}^{\infty} A_{i,j}, \quad i = 1, 2, ..., n. \tag{4}$$

Here $A_(i,j)$ depend on $x_{1,0}, x_{1,1}, ..., x_{1,j}, x_{2,0}, x_{2,1}, ..., x_{2,j}, x_{n,0}, x_{n,1}, ..., x_{n,j}$.

In view of equations (3) and (4) we have

$$N_i\left(\sum_{j=0}^{\infty} x_{1,j}\lambda^j, \sum_{j=0}^{\infty} x_{2,j}\lambda^j, ..., \sum_{j=0}^{\infty} x_{n,j}\lambda^j\right) = \sum_{j=0}^{\infty} A_{i,j}, \quad i = 1, 2, ..., n, \tag{5}$$

which implies

$$A_{i,j} = \frac{1}{j!}\left[\frac{d^j}{d\lambda^j}\left(\sum_{j=0}^{\infty} x_{1,j}\lambda^j, \sum_{j=0}^{\infty} x_{2,j}\lambda^j, ..., \sum_{j=0}^{\infty} x_{n,j}\lambda^j\right)\right]_{\lambda=0}, \quad i = 1, 2, ..., n, \tag{6}$$

where λ is a parameter introduced for convenience. Hence equation (2) can be written as

$$\sum_{j=0}^{\infty} x_{i,j} = c_i + \sum_{j=0}^{\infty} A_{i,j}, \quad i = 1, 2, ..., n. \tag{7}$$

The ADM defines the components $x_{i,j}$, for $i = 1, 2, ..., n$ and $j \geq 0$, by the following recurrence relation

$$\begin{aligned} x_{i,0} =& c_i, \\ x_{i,j+1} =& A_{i,j}, \quad i = 0, 1, 2, \ldots. \end{aligned} \tag{8}$$

The solution x_i approximate by the truncated series

$$\Psi_{i,k} = \sum_{j=0}^{k-1} x_{i,j}, \text{ with } \lim \Psi_{i,k} = x_i, \ i = 1, 2, ... \tag{9}$$

Practically, in computing x_i, as n increases, the number of terms in the expression for $A_{i,n}$ increases and this causes propagation of round of errors, on the other hand the factor $\frac{1}{n!}$ in the formula of $A_{i,n}$, makes it very small. The Restarted Adomian Decomposition Method (RADM) resolves this problem and increases the accuracy of the solutions, dramatically.

3. Integrated RADM and ADM for System of Nonlinear Equations

Suppose the exact solution of nonlinear equation (1) is $X^* = (x_1^*, x_2^*, ...x_n^*)$.

Let s_i be a point close to x_i^* such that

$$|x_i^* - s_i| < |x_i^* - c_i|, \quad i = 1, 2, ..., n. \tag{10}$$

Adding s_i to both sides of (2), we obtain

$$x_i - N_i - s_i = c_i + s_i, \quad i = 1, 2, ..., n. \tag{11}$$

Equation (11) can be solved by the Adomian method through the following algorithm:

Choose small natural numbers m and k.

Step 1. Apply the Adomian method on equation (2) and calculate $x_{i,0}$, $x_{i,1}$, ..., $x_{i,n}$, $i = 1, 2, ..., n$.

Step 2: For $i = 1, 2, ..., n$, set $\Psi_i^(1) = x_{i,0} + x_{i,1} + ... + x_{i,k}$.

Step 3: For any $j = 2, ..., m$ do

$$\begin{aligned}
s_i =& \Psi_i^{j-1}, \quad i = 1, 2, ..., n;\\
x_{i,0} =& s_i, \quad i = 1, 2, ..., n;\\
x_{i,1} =& c_i - s_i + A_{i,1}(x_{1,0}, x_{2,0}, ..., x_{n,0}), \quad i = 1, 2, ..., n;\\
x_{i,2} =& A_{i,1}(x_{1,0}, x_{2,0}, ..., x_{n,0}, x_{2,1}, ..., x_{n,1}), \quad i = 1, 2, ..., n;\\
&\vdots\\
x_{i,k+1} =& A_{i,1}(x_{1,0}, x_{2,0}, ..., x_{n,0}, x_{2,1}, ..., x_{n,1}, x_{1,k}, x_{2,k}, ..., x_{n,k}), \quad i = 1, 2, ..., n;\\
\Psi_i^j =& x_{i,0} + x_{i,1} + ... + x_{i,k}, \quad i = 1, 2, ..., n.
\end{aligned}$$

In this algorithm $x_{i,0}$ (for $i = 1, 2, ..., n$) are updated in each step. Therefore we may choose m, k sufficiently small, for example: $2 \leq m$, $k \leq 5$.

4. Numerical Example

In this section an example is presented to consider the effectiveness of the integrated RADM and ADM.

Example. Consider the following system of nonlinear equations

$$\begin{aligned}
e^x + xy - 1 =& 0,\\
\sin(xy) + x + y - 1 =& 0.
\end{aligned}$$

The solutions are $X^* = (x^*, y^*) = (0.009, 0.997)$. For applying the ADM, we get

$$\begin{aligned} x =& \frac{1}{y}(1 - e^x), \\ y =& 1 - x - \sin(xy). \end{aligned}$$

By using equation (3), we receive

$$\begin{aligned} \sum x_j =& \sum (A_{x,j}\left(\frac{1}{y}\right) - \sum (A_{x,j}\left(\frac{1 - e^x}{y}\right), \\ \sum y_j =& 1 - \sum A_{y,j}(x) - \sum A_{y,j}(\sin(xy)). \end{aligned}$$

The Adomian polynomials of non linear terms x^n and y^n can be found easily by the equation (6) and the first few polynomials of complicated non linear function e^x are formulated as follows

$$\begin{aligned} A_0 =& e^{x_0}, \\ A_1 =& \frac{1}{1!}x_1 e^{x_0}, \\ A_2 =& \left(\frac{1}{2!}x_1^2 + x_2\right) e^{x_0}, \\ A_3 =& \left(\frac{1}{3!}x_1^3 + x_1 x_2 - x_3\right) e^{x_0}, \end{aligned}$$

and the first few Adomian polynomials for the non linear function can be formulated as follows

$$\begin{aligned} A_0 =& \sin(x_0 y_0), \\ A_1 =& \cos(x_0 y_0)(x_0 y_1 + y_0 x_1), \\ A_2 =& \cos(x_0 y_0)(x_1 y_1 + y_2 x_0 + x_2 y_3) \\ & - \sin(x_0 y_0)\left(\frac{1}{2!}x_1^2 y_0^2 + \frac{1}{2!}y_1^2 x_0^2 + x_0 y_0 y_1 x_1\right), \\ A_3 =& -\cos(x_0 y_0) X - \sin(x_0 y_0) Y, \end{aligned}$$

where

$$\begin{aligned} X = & \frac{1}{3!}y_0^2 x_1^3 + \frac{1}{3!}y_1^3 x_0^3 + \frac{1}{2!}y_0^2 x_1^2 x_0 + \frac{1}{2!}x_1^2 y_1 x_0 y_0^2 + \frac{1}{2!}x_1 y_1^2 x_0^2 y_0 \\ & + y_1 x_2 + x_1 y_2 + x_3 y_0 + y_3 x_0, \end{aligned}$$

$$Y = x_1^2 y_0 y_1 + x_1 y_1^2 x_0 + y_1 y_2 x_0^2 + y_1 x_2 y_0 x_0 + x_1 y_2 y_0 x_0.$$

By applying the procedure of ADM and calculating 10 terms of series, we have:

$$x = \phi_{x,10} = x_0 + x_1 + x_2 + ... + x_{10} = 0.00852,$$
$$y = \phi_{y,10} = y_0 + y_1 + y_2 + ... + y_{10} = 0.88364.$$

Now to apply RADM, we have chosen the values of $(x_0, y_0) = (0.00852, 0.88364)$. Therefore by (10), we have $s_1 = 0.00852$, $s_2 = 0.88364$ and $c_1 = 0, c_2 = 1$ and by calculating the three terms of the series, we have

$$\Psi_x^2 = x_0 + x_1 + x_2 + x_3 = 0.0090, \quad \Psi_y^2 = y_0 + y_1 + y_2 + y_3 = 0.9970.$$

Comparing the exact solution of this problem and the solution given by the RADM, we see that the solution produced by RADM exactly match with the exact solution and there is no significant error in the solution produced by RADM.

5. Conclusion

In this paper initial solution of RADM takes the solution values given by ADM in ten steps, with this initial solution, RADM finds the best solution within three steps. This incorporation of ADM and RADM makes RADM a best alternative approach for solving system of nonlinear equations in dynamic environments. Moreover the obtained results show that the proposed method has been an efficient approach for solving the system of non linear equations.

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