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ECCENTRICITY BASED GEOMETRIC-ARITHMETIC AND ATOM-BOND CONNECTIVITY INDICES OF COPPER OXIDE CuO

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Abstract: Graph theory has much advancement in the field of mathematical chemistry. Recently, chemical graph theory has become very popular among researchers because of its wide applications in mathematical chemistry. The molecular topological descriptors are the numerical invariants of a molecular graph and are very useful for predicting their bioactivity.

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A great variety of such indices are studied and used in theoretical chemistry, pharmaceutical researchers, in drugs and in different other fields. Among topological descriptor, connectivity indices are very important and they have a prominent role in chemistry. In this article, we study the chemical graph of copper oxide and compute the eccentricity based atom-bond connectivity index and eccentricity based geometric-arithmetic index for copper oxide. Furthermore, we give analytically closeds formulas of these indices which are helpful in studying the underlying topologies.

AMS Subject Classification: 05C12, 05C90
Key Words: molecular graph, eccentricity based atom-bond connectivity index, eccentricity based geometric-arithmetic index, copper oxide

1. Introduction

There are a lot of chemical compounds, either organic or inorganic, which possess a level of commercial, industrial, pharmaceutical chemistry and laboratory importance. A relationship exists between chemical compounds and their molecular structures. Graph theory is a very powerful area of mathematics that has wide range of applications in many areas of science such as chemistry, biology, computer science, electrical, electronics and other fields. Chemical graph theory is a branch of mathematical chemistry in which we apply tools of graph theory to model the chemical phenomenon mathematically. This theory contributes a prominent role in the field of chemical sciences. Some references are given, which hopefully demonstrate the importance of this field [28, 29, 32, 33, 34, 36, 38, 39, 40].

Let $G = (V, E)$ be a graph, where V is a non-empty set of vertices and E is a set of edges. The chemical graph theory applies graph theory to mathematical modeling of molecular phenomena, which is helpful for the study of molecular structure. This theory contributes a prominent role in the field of chemical sciences. Chemical compounds have a variety of applications in chemical graph theory, drug design, etc. The manipulation and examination of chemical structural information is made conceivable by using molecular descriptors. A great variety of topological indices are studied and used in theoretical chemistry, pharmaceutical researchers. In chemical graph theory, there are many topological indices for a connected graph, which are helpful in study of chemical molecules. Development of chemical science had an important effect by this theory.

If $p, q \in V(G)$, then the distance $d(p, q)$ between p and q is defined as the length of any shortest path in G connecting p and q. Eccentricity is the distance of vertex u from the farthest vertex in G. In mathematical form,

$$\varepsilon(u) = max\{d(u,v)|\forall \ u \in V(G)\} \tag{1}$$

Recently, the eccentric fourth geometric-arithmetic index of NA_m^n nanotube is found by [41]. The eccentric fourth geometric-arithmetic index version of Polycyclic aromatic hydrocarbons ($PAHk$) is computed by [37]. The eccentricity based geometric-arithmetic index of a graph G is defined as[31],

$$GA_4(G) = \sum_{uv\epsilon E(G)} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)} \tag{2}$$

A new version of ABC index is introduced by Farahani [5] which is defined as

$$ABC_5(G) = \sum_{uv\in E(G)} \sqrt{\frac{\varepsilon(v)+\varepsilon(u)-2}{\varepsilon(v)\varepsilon(u)}} \tag{3}$$

Recently the eccentric atom-bond connectivity index of linear polycene parallelogram benzenoid is introduced by [27].

Form more information and history about eccentric connectivity indices, reader can see [4]-[26].

The copper oxide/cupric oxide form an inorganic chemical compound CuO. This is an essential mineral found in plants and animals. Copper has enormous applications in medical instruments, drugs, and as a heat conductor, among others. Some applications of copper and cupric oxide are given in [1, 35, 42].

In Fig. 1(a), the copper hydroxide is depicted and when hydrogen atoms are depleted from $Cu(OH)_2$ then the resultant graph is depicted in Fig. 1(b).

The $3D$ graph of copper oxide CuO is depicted in Fig. 2. Copper oxide is used as the source of copper in mineral and vitamin supplements and is considered safe. Its use in medical devices, and industrial and consumer products, is novel. The safety aspects of the use of copper oxide in products that come into contact with open and closed skin must be considered [2, 30].

2. Main Results and Discussion

In this section, we discussed the eccentricity based atom-bond connectivity index $ABC_5(G)$ and eccentricity based geometric-arithmetic index $GA_4(G)$ of copper oxide. Here we consider the copper oxide graph $CuO = G$. In this article, we consider the copper oxide molecular graph CuO, as depicted in Fig. 1(b). The construction of the CuO graph is such that the octagons are

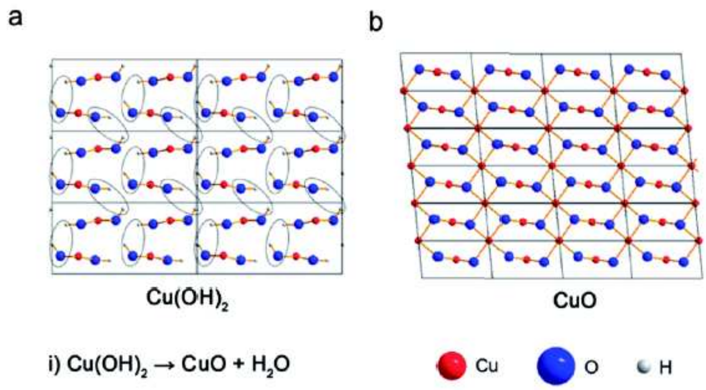

Figure 1: (a) $Cu(OH)_2$ (b) CuO

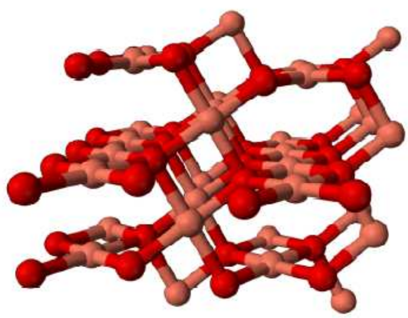

Figure 2: $3D$ Copper oxide CuO

connected to each other in columns and rows; the connection between two octagons is achieved by making one C_4 bond between two octagons. For our convenience, we take m and n as the number of octagons in rows and columns, respectively. The cardinality of vertices and edges in CuO are $4mn + 3n + m$ and $6mn + 2n$ respectively.

2.1. Geometric-Arithmetic Index of Copper Oxide ($CuO = G$)

In this section we find the eccentricity based geometric-arithmetic index of copper oxide, which is denoted as $GA_4(G(m, n))$.

Theorem 1. *Let $G(m, n)$, for all $m, n \in N$, where m and n both are even, be the copper oxide, then the eccentricity based geometric-arithmetic*

$(\varepsilon(u),\varepsilon(v))$	frequency	Range of m and n	Range of k
$(k,k+1)$	$4m$	$m\geq 2,\, n\geq 2m$	$k=2n$
$(4k+2m+1,4k+2m+2)$	$2(m+1)$	$m\geq 2,\, n\geq 2m$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-2}{2}$
$(4k+2m+2,4k+2m+3)$	$2(m+1)$	$m\geq 2,\, n\geq 2m$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-2}{2}$
$(4k+2m+3,4k+2m+4)$	$4m$	$m\geq 2,\, n\geq 2m$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-4}{2}$
$(4k+2m+4,4k+2m+5)$	$4m$	$m\geq 2,\, n\geq 2m$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-4}{2}$
$(k,k+1)$	$4m$	$m\geq 2,\, n\geq 2m$	$k=4n-1$

Table 1: Edge partition of Copper oxide for ((m,n)-levels) where m and n both are even, based on eccentricity of end vertices of each edge with existence of their frequencies.

$(\varepsilon(u),\varepsilon(v))$	frequency	Range of m and n	Range of k
$(k,k+1)$	$2(m+1)$	$m\geq 1,\, n\geq 2m+1$	$k=2n$
$(4k+2m+1,4k+2m+2)$	$4m$	$m\geq 1,\, n\geq 2m+1$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-3}{2}$
$(4k+2m+2,4k+2m+3)$	$4m$	$m\geq 1,\, n\geq 2m+1$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-3}{2}$
$(4k+2m+3,4k+2m+4)$	$2(m+1)$	$m\geq 1,\, n\geq 2m+1$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-3}{2}$
$(4k+2m+4,4k+2m+5)$	$2(m+1)$	$m\geq 1,\, n\geq 2m+1$	$\frac{n-m}{2}\leq k\leq \frac{2n-m-3}{2}$
$(k,k+1)$	$4m$	$m\geq 1,\, n\geq 2m+1$	$k=4n-1$

Table 2: Edge partition of Copper oxide for ((m,n)-levels) where m and n both are odd, based on eccentricity of end vertices of each edge with existence of their frequencies.

$(\varepsilon(u),\varepsilon(v))$	frequency	Range of m and n	Range of k
$(k,k+1)$	$4m$	$m\geq 1,\, n\geq 2m$	$k=2n$
$(4k+9,4k+10)$	$2(m+1)$	$m\geq 1,\, n\geq 2m$	$\frac{n-4}{2}\leq k\leq n-3$
$(4k+10,4k+11)$	$2(m+1)$	$m\geq 1,\, n\geq 2m$	$\frac{n-4}{2}\leq k\leq n-3$
$(4k+11,4k+12)$	$4m$	$m\geq 1,\, n\geq 2m$	$\frac{n-4}{2}\leq k\leq n-4$
$(4k+12,4k+13)$	$4m$	$m\geq 1,\, n\geq 2m$	$\frac{n-4}{2}\leq k\leq n-4$
$(k,k+1)$	$4m$	$m\geq 1,\, n\geq 2m$	$k=4n-1$

Table 3: Edge partition of Copper oxide for ((m,n)-levels) where m is odd and n is even, based on eccentricity of end vertices of each edge with existence of their frequencies.

index GA_4 of $G(m,n)$ is

$$GA_4(G(m,n)) = \sum_{m\geq 2}\{8m(\frac{\sqrt{2n(2n+1)}}{4n+1} + \frac{2\sqrt{n(4n-1)}}{8n-1})$$

$(\varepsilon(u), \varepsilon(v))$	frequency	Range of m and n	Range of k
$(k, k+1)$	$2(m+1)$	$m \geq 2$, $n > 2m$	$k = 2n$
$(4k+7, 4k+8)$	$4m$	$m \geq 2$, $n > 2m$	$\frac{n-3}{2} \leq k \leq n-3$
$(4k+8, 4k+9)$	$4m$	$m \geq 2$, $n > 2m$	$\frac{n-3}{2} \leq k \leq n-3$
$(4k+9, 4k+10)$	$2(m+1)$	$m \geq 2$, $n > 2m$	$\frac{n-3}{2} \leq k \leq n-3$
$(4k+10, 4k+11)$	$2(m+1)$	$m \geq 2$, $n > 2m$	$\frac{n-3}{2} \leq k \leq n-3$
$(k, k+1)$	$4m$	$m \geq 2$, $n > 2m$	$k = 4n-1$

Table 4: Edge partition of Copper oxide for ((m, n)-levels) where m is even and n is odd, based on eccentricity of end vertices of each edge with existence of their frequencies.

$$+4(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}\left(\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{8k+4m+3}+\frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{8k+4m+5}\right)$$
$$+8m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}\left(\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{8k+4m+7}+\frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{8k+4m+9}\right)\}.$$

Proof. Let $G(m, n)$, where m and n both are even, be the copper oxide contains $4mn+3n+m$ vertices and $6mn+2n$ edges.

The general formula of eccentricity based geometric arithmetic index is

$$GA_4(G) = \sum_{uv\epsilon E(G)} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}.$$

Using the edge partitioned from Table 1, we have the following computations

$$GA_4(G(m,n) = \sum_{m\geq 2}\{4m\sum_{k=2n}\frac{2\sqrt{k(k+1)}}{k+k+1}$$
$$+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(2(m+1)\frac{2\sqrt{(4k+2m+1)(4k+2m+2)}}{(4k+2m+1)+(4k+2m+2)}$$
$$+2(m+1)\frac{2\sqrt{(4k+2m+2)(4k+2m+3)}}{(4k+2m+2)+(4k+2m+3)})$$
$$+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(4m\frac{2\sqrt{(4k+2m+3)(4k+2m+4)}}{(4k+2m+3)+(4k+2m+4)}$$
$$+4m\frac{2\sqrt{(4k+2m+4)(4k+2m+5)}}{(4k+2m+4)+(4k+2m+5)})+4m\sum_{k=4n-1}\frac{2\sqrt{k(k+1)}}{k+k+1}\}$$

$$= \sum_{m\geq 2}\{8m\sum_{k=2n}\frac{\sqrt{k(k+1)}}{2k+1} + \sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(4(m+1)\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{4k+2m+1+4k+2m+2}$$
$$+ 4(m+1)\frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{4k+2m+2+4k+2m+3})$$
$$+ \sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(8m\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{4k+2m+3+4k+2m+4}$$
$$+ 8m\frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{4k+2m+4+4k+2m+5}) + 8m\sum_{k=4n-1}\frac{\sqrt{k(k+1)}}{2k+1}\}$$
$$= \sum_{m\geq 2}\{8m\frac{\sqrt{2n(2n+1)}}{4n+1} + 4(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{8k+4m+3}$$
$$+ \frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{8k+4m+5})$$
$$+ 8m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{8k+4m+7}$$
$$+ \frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{8k+4m+9}) + 8m\frac{\sqrt{4n(4n-1)}}{8n-1}\}.$$

Finally, for all $m, n \in N$, where m and n both are even, the eccentricity based geometric-arithmetic index of copper oxide $G(m,n)$ is

$$GA_4(G(m,n)) = \sum_{m\geq 2}\{8m(\frac{\sqrt{2n(2n+1)}}{4n+1} + \frac{2\sqrt{n(4n-1)}}{8n-1})$$
$$q + 4(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{8k+4m+3} + \frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{8k+4m+5})$$
$$+ 8m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{8k+4m+7} + \frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{8k+4m+9})\}. \quad \square$$

Theorem 2. *Let $G(m,n)$, for all $m, n \in N$, where m and n both are odd, be the copper oxide, then the eccentricity based geometric-arithmetic index GA_4 of $G(m,n)$ is*

$$GA_4(G(m,n)) = \sum_{m\geq 1}\{4(m+1)(\frac{\sqrt{2n(2n+1)}}{4n+1}) + 16m(\frac{\sqrt{n(4n-1)}}{8n-1})$$

$$+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(8m(\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{8k+4m+3}+\frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{8k+4m+5})$$

$$+4(m+1)(\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{8k+4m+7}+\frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{8k+4m+9}))\}.$$

Proof. Let $G(m,n)$, where m and n both are odd, be the copper oxide contains $4mn+3n+m$ vertices and $6mn+2n$ edges.

The general formula of eccentricity based geometric arithmetic index is

$$GA_4(G)=\sum_{uv\epsilon E(G)}\frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}.$$

Using the edge partitioned from Table 2, we have the following computations

$$GA_4(G(m,n)=\sum_{m\geq 1}\{2(m+1)\sum_{k=2n}\frac{2\sqrt{k(k+1)}}{k+k+1}$$

$$+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(4m\frac{2\sqrt{(4k+2m+1)(4k+2m+2)}}{(4k+2m+1)+(4k+2m+2)}$$

$$+4m\frac{2\sqrt{(4k+2m+2)(4k+2m+3)}}{(4k+2m+2)+(4k+2m+3)})+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(2(m+1)\frac{2\sqrt{(4k+2m+3)(4k+2m+4)}}{(4k+2m+3)+(4k+2m+4)}$$

$$+2(m+1)\frac{2\sqrt{(4k+2m+4)(4k+2m+5)}}{(4k+2m+4)+(4k+2m+5)})+4m\sum_{k=4n-1}\frac{2\sqrt{k(k+1)}}{k+k+1}\}$$

$$=\sum_{m\geq 1}\{4(m+1)\sum_{k=2n}\frac{\sqrt{k(k+1)}}{2k+1}+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(8m\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{4k+2m+1+4k+2m+2}$$

$$+8m\frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{4k+2m+2+4k+2m+3})+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(4(m+1)\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{4k+2m+3+4k+2m+4}$$

$$+4(m+1)\frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{4k+2m+4+4k+2m+5})+8m\sum_{k=4n-1}\frac{\sqrt{k(k+1)}}{2k+1}\}$$

$$=\sum_{m\geq 1}\{4(m+1)\frac{\sqrt{2n(2n+1)}}{4n+1}+8m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{8k+4m+3}$$

$$+\frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{8k+4m+5})+4(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{8k+4m+7}$$

$$+\frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{8k+4m+9})+8m\frac{\sqrt{4n(4n-1)}}{8n-1}\}.$$

Finally, for all $m, n \in N$, where m and n both are odd, the eccentricity based geometric-arithmetic index of copper oxide $G(m,n)$ is

$$GA_4(G(m,n)) = \sum_{m\geq 1}\{4(m+1)(\frac{\sqrt{2n(2n+1)}}{4n+1})+16m(\frac{\sqrt{n(4n-1)}}{8n-1})$$
$$+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(8m(\frac{\sqrt{(4k+2m+1)(4k+2m+2)}}{8k+4m+3}+\frac{\sqrt{(4k+2m+2)(4k+2m+3)}}{8k+4m+5})$$
$$+4(m+1)(\frac{\sqrt{(4k+2m+3)(4k+2m+4)}}{8k+4m+7}+\frac{\sqrt{(4k+2m+4)(4k+2m+5)}}{8k+4m+9}))\}.$$

□

Theorem 3. *Let $G(m,n)$, for all $m, n \in N$, where m is odd and n is even, be the copper oxide, then the eccentricity based geometric-arithmetic index GA_4 of $G(m,n)$ is*

$$GA_4(G(m,n)) = \sum_{m\geq 1}\{8m(\frac{\sqrt{2n(2n+1)}}{4n+1}+\frac{2\sqrt{n(4n-1)}}{8n-1})$$
$$+4(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\frac{\sqrt{(4k+9)(4k+10)}}{8k+19}+\frac{\sqrt{(4k+10)(4k+11)}}{8k+21})$$
$$+8m\sum_{k=\frac{n-4}{2}}^{n-4}(\frac{\sqrt{(4k+11)(4k+12)}}{8k+23}+\frac{\sqrt{(4k+12)(4k+13)}}{8k+25})\}.$$

Proof. Let $G(m,n)$, where m is odd and n is even, be the copper oxide contains $4mn+3n+m$ vertices and $6mn+2n$ edges.

The general formula of eccentricity based geometric arithmetic index is

$$GA_4(G) = \sum_{uv\epsilon E(G)}\frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}.$$

Using the edge partitioned from Table 3, we have the following computations

$$GA_4(G(m,n) = \sum_{m\geq 1}\{4m\sum_{k=2n}\frac{2\sqrt{k(k+1)}}{k+k+1}$$

$$+\sum_{k=\frac{n-4}{2}}^{n-3}(2(m+1)\frac{2\sqrt{(4k+9)(4k+10)}}{(4k+9)+(4k+10)}$$
$$+2(m+1)\frac{2\sqrt{(4k+10)(4k+11)}}{(4k+10)+(4k+11)})+\sum_{k=\frac{n-4}{2}}^{n-4}(4m\frac{2\sqrt{(4k+11)(4k+12)}}{(4k+11)+(4k+12)}$$
$$+4m\frac{2\sqrt{(4k+12)(4k+13)}}{(4k+12)+(4k+13)})+4m\sum_{k=4n-1}\frac{2\sqrt{k(k+1)}}{k+k+1}\}$$
$$=\sum_{m\geq 1}\{8m\sum_{k=2n}\frac{\sqrt{k(k+1)}}{2k+1}+\sum_{k=\frac{n-4}{2}}^{n-3}(4(m+1)\frac{\sqrt{(4k+9)(4k+10)}}{8k+19}$$
$$+4(m+1)\frac{\sqrt{(4k+10)(4k+11)}}{8k+21})+\sum_{k=\frac{n-4}{2}}^{n-4}(8m\frac{\sqrt{(4k+11)(4k+12)}}{8k+23}$$
$$+8m\frac{\sqrt{(4k+12)(4k+13)}}{8k+25})+8m\sum_{k=4n-1}\frac{\sqrt{k(k+1)}}{2k+1}\}$$
$$=\sum_{m\geq 1}\{8m\frac{\sqrt{2n(2n+1)}}{4n+1}+4(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\frac{\sqrt{(4k+9)(4k+10)}}{8k+19}$$
$$+\frac{\sqrt{(4k+10)(4k+11)}}{8k+21})+8m\sum_{k=\frac{n-4}{2}}^{n-4}(\frac{\sqrt{(4k+11)(4k+12)}}{8k+23}$$
$$+\frac{\sqrt{(4k+12)(4k+13)}}{8k+25})+8m\frac{\sqrt{4n(4n-1)}}{8n-1}\}.$$

Finally, for all $m, n \in N$, where m is odd and n is even, the eccentricity based geometric-arithmetic index of copper oxide $G(m,n)$ is

$$GA_4(G(m,n))=\sum_{m\geq 1}\{8m(\frac{\sqrt{2n(2n+1)}}{4n+1}+\frac{2\sqrt{n(4n-1)}}{8n-1})$$
$$+4(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\frac{\sqrt{(4k+9)(4k+10)}}{8k+19}+\frac{\sqrt{(4k+10)(4k+11)}}{8k+21})$$
$$+8m\sum_{k=\frac{n-4}{2}}^{n-4}(\frac{\sqrt{(4k+11)(4k+12)}}{8k+23}+\frac{\sqrt{(4k+12)(4k+13)}}{8k+25})\}. \quad \square$$

Theorem 4. *Let $G(m,n)$, for all $m,n \in N$, where m is even and n is odd, be the copper oxide, then the eccentricity based geometric-arithmetic index GA_4 of $G(m,n)$ is*

$$GA_4(G(m,n)) = \sum_{m\geq 2}\{4(m+1)(\frac{\sqrt{2n(2n+1)}}{4n+1}) + 16m(\frac{\sqrt{n(4n-1)}}{8n-1}) + \sum_{k=\frac{n-3}{2}}^{n-3}(8m(\frac{\sqrt{(4k+7)(4k+8)}}{8k+15} + \frac{\sqrt{(4k+8)(4k+9)}}{8k+17}) + 4(m+1)(\frac{\sqrt{(4k+9)(4k+10)}}{8k+19} + \frac{\sqrt{(4k+10)(4k+11)}}{8k+21}))\}.$$

Proof. Let $G(m,n)$, where m is odd and n is even, be the copper oxide contains $4mn+3n+m$ vertices and $6mn+2n$ edges.

The general formula of eccentricity based geometric arithmetic index is

$$GA_4(G) = \sum_{uv\epsilon E(G)} \frac{2\sqrt{\varepsilon(u)\cdot\varepsilon(v)}}{\varepsilon(u)+\varepsilon(v)}.$$

Using the edge partitioned from Table 4, we have the following computations

$$GA_4(G(m,n) = \sum_{m\geq 2}\{2(m+1)\sum_{k=2n}\frac{2\sqrt{k(k+1)}}{k+k+1} + \sum_{k=\frac{n-3}{2}}^{n-3}(4m\frac{2\sqrt{(4k+7)(4k+8)}}{(4k+7)+(4k+8)} + 4m\frac{2\sqrt{(4k+8)(4k+9)}}{(4k+8)+(4k+9)}) + \sum_{k=\frac{n-3}{2}}^{n-3}(2(m+1)\frac{2\sqrt{(4k+9)(4k+10)}}{(4k+9)+(4k+10)} + 2(m+1)\frac{2\sqrt{(4k+10)(4k+11)}}{(4k+10)+(4k+11)}) + 4m\sum_{k=4n-1}\frac{2\sqrt{k(k+1)}}{k+k+1}\}$$

$$= \sum_{m\geq 2}\{4(m+1)\sum_{k=2n}\frac{\sqrt{k(k+1)}}{2k+1} + \sum_{k=\frac{n-3}{2}}^{n-3}(8m\frac{\sqrt{(4k+7)(4k+8)}}{8k+15} + 8m\frac{\sqrt{(4k+8)(4k+9)}}{8k+17}) + \sum_{k=\frac{n-3}{2}}^{n-3}(4(m+1)\frac{\sqrt{(4k+9)(4k+10)}}{8k+19} + 4(m+1)\frac{\sqrt{(4k+10)(4k+11)}}{8k+21}) + 8m\sum_{k=4n-1}\frac{\sqrt{k(k+1)}}{2k+1}\}.$$

$$= \sum_{m\geq 2}\{4(m+1)\frac{\sqrt{2n(2n+1)}}{4n+1} + \sum_{k=\frac{n-3}{2}}^{n-3}(8m(\frac{\sqrt{(4k+7)(4k+8)}}{8k+15}$$

$$+ \frac{\sqrt{(4k+8)(4k+9)}}{8k+17})) + \sum_{k=\frac{n-3}{2}}^{n-3}(4(m+1)(\frac{\sqrt{(4k+9)(4k+10)}}{8k+19}$$

$$+ \frac{\sqrt{(4k+10)(4k+11)}}{8k+21})) + 8m\frac{\sqrt{4n(4n-1)}}{8n-1}\}.$$

Finally, for all $m, n \in N$, where m is even and n is odd, the eccentricity based geometric-arithmetic index of copper oxide $G(m,n)$ is

$$GA_4(G(m,n)) = \sum_{m\geq 2}\{4(m+1)(\frac{\sqrt{2n(2n+1)}}{4n+1}) + 16m(\frac{\sqrt{n(4n-1)}}{8n-1})$$

$$+ \sum_{k=\frac{n-3}{2}}^{n-3}(8m(\frac{\sqrt{(4k+7)(4k+8)}}{8k+15} + \frac{\sqrt{(4k+8)(4k+9)}}{8k+17})$$

$$+ 4(m+1)(\frac{\sqrt{(4k+9)(4k+10)}}{8k+19} + \frac{\sqrt{(4k+10)(4k+11)}}{8k+21}))\}. \quad \square$$

2.2. Atom-Bond Connectivity Index of Copper Oxide ($CuO = G$)

In this section we find the eccentricity based atom-bond connectivity index of copper oxide, which is denoted as $ABC_5(G(m,n))$.

Theorem 5. *Let $CG(m,n)$, for all $m, n \in N$, where m and n both are even, be the copper oxide, then the eccentricity based atom-bond connectivity index ABC_5 of $G(m,n)$ is*

$$ABC_5(G(m,n)) = \sum_{m\geq 2}\{4m(\sqrt{\frac{4n-1}{2n(2n+1)}} + \sqrt{\frac{8n-3}{4n(4n-1)}})$$

$$+ 2(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(\sqrt{\frac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}} + \sqrt{\frac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}})$$

$$+ 4m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(\sqrt{\frac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}} + \sqrt{\frac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}})\}.$$

Proof. Let $G(m,n)$, where m and n both are even, be the copper oxide contains $4mn + 3n + m$ vertices and $6mn + 2n$ edges.

The general formula of eccentricity based atom-bond connectivity index is

$$ABC_5(G) = \sum_{uv\epsilon E(G)} \sqrt{\frac{\varepsilon(u)+\varepsilon(v)-2}{\varepsilon(u)\cdot\varepsilon(v)}}.$$

Using the edge partitioned from Table 1, we have the following computations

$$\begin{aligned}
ABC_5(G(m,n)) = \sum_{m\geq 2}\{4m \sum_{k=2n} \sqrt{\frac{k+k+1-2}{k\cdot(k+1)}} \\
+ 2(m+1) \sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}} (\sqrt{\frac{4k+2m+1+4k+2m+2-2}{(4k+2m+1)(4k+2m+2)}} \\
+ \sqrt{\frac{4k+2m+2+4k+2m+3-2}{(4k+2m+2)(4k+2m+3)}}) \\
+ 4m \sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}} (\sqrt{\frac{4k+2m+3+4k+2m+4-2}{(4k+2m+3)(4k+2m+4)}} \\
+ \sqrt{\frac{4k+2m+4+4k+2m+5-2}{(4k+2m+4)(4k+2m+5)}}) \\
+ 4m \sum_{k=4n-1} \sqrt{\frac{k+k+1-2}{k\cdot(k+1)}}\} \\
= \sum_{m\geq 2}\{4m \sum_{k=2n} \sqrt{\frac{2k-1}{k\cdot(k+1)}} \\
+ 2(m+1) \sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}} (\sqrt{\frac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}} \\
+ \sqrt{\frac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}}) \\
+4m \sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}} (\sqrt{\frac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}} + \sqrt{\frac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}}) \\
+ 4m \sum_{k=4n-1} \sqrt{\frac{2k-1}{k\cdot(k+1)}}\}
\end{aligned}$$

$$= \sum_{m\geq 2}\{4m\sqrt{\frac{4n-1}{2n\cdot(2n+1)}} + 2(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(\sqrt{\frac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}}$$

$$+\sqrt{\frac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}}) + 4m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(\sqrt{\frac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}}$$

$$+\sqrt{\frac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}}) + 4m\sqrt{\frac{8n-3}{4n\cdot(4n-1)}}\}.$$

Finally, for all $m, n \in N$, where m and n both are even, the eccentricity based atom-bond connectivity index of copper oxide $G(m,n)$ is

$$ABC_5(G(m,n)) = \sum_{m\geq 2}\{4m(\sqrt{\tfrac{4n-1}{2n(2n+1)}} + \sqrt{\tfrac{8n-3}{4n(4n-1)}})$$
$$+ 2(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(\sqrt{\tfrac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}} + \sqrt{\tfrac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}})$$
$$+ 4m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(\sqrt{\tfrac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}} + \sqrt{\tfrac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}})\}. \qquad \square$$

Theorem 6. *Let $G(m,n)$, for all $m, n \in N$, where m and n both are odd, be the copper oxide, then the eccentricity based atom-bond connectivity index ABC_5 of $G(m,n)$ is*

$$ABC_5(G(m,n)) = \sum_{m\geq 1}\{2(m+1)\sqrt{\frac{4n-1}{2n(2n+1)}} + 4m\sqrt{\frac{8n-3}{4n(4n-1)}}$$

$$+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(4m(\sqrt{\frac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}}+\sqrt{\frac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}})$$

$$+2(m+1)(\sqrt{\frac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}}+\sqrt{\frac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}}))\}.$$

Proof. Let $G(m,n)$, where m and n both are odd, be the copper oxide contains $4mn + 3n + m$ vertices and $6mn + 2n$ edges.

The general formula of eccentricity based atom-bond connectivity index is

$$ABC_5(G) = \sum_{uv\epsilon E(G)}\sqrt{\frac{\varepsilon(u)+\varepsilon(v)-2}{\varepsilon(u)\cdot\varepsilon(v)}}.$$

Using the edge partitioned from Table 2, we have the following computations

$$ABC_5(G(m,n)) = \sum_{m\geq 1}\{2(m+1)\sum_{k=2n}\sqrt{\frac{k+k+1-2}{k\cdot(k+1)}}$$
$$+4m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(\sqrt{\frac{4k+2m+1+4k+2m+2-2}{(4k+2m+1)(4k+2m+2)}}$$
$$+\sqrt{\frac{4k+2m+2+4k+2m+3-2}{(4k+2m+2)(4k+2m+3)}})$$
$$+2(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(\sqrt{\frac{4k+2m+3+4k+2m+4-2}{(4k+2m+3)(4k+2m+4)}}$$
$$+\sqrt{\frac{4k+2m+4+4k+2m+5-2}{(4k+2m+4)(4k+2m+5)}})$$
$$+4m\sum_{k=4n-1}\sqrt{\frac{k+k+1-2}{k\cdot(k+1)}}\}$$
$$=\sum_{m\geq 2}\{2(m+1)\sum_{k=2n}\sqrt{\frac{2k-1}{k\cdot(k+1)}}+4m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(\sqrt{\frac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}}$$
$$+\sqrt{\frac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}})+2(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-4}{2}}(\sqrt{\frac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}}$$
$$+\sqrt{\frac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}})+4m\sum_{k=4n-1}\sqrt{\frac{2k-1}{k\cdot(k+1)}}\}$$
$$=\sum_{m\geq 2}\{2(m+1)\sqrt{\frac{4n-1}{2n\cdot(2n+1)}}+4m\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(\sqrt{\frac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}}$$
$$+\sqrt{\frac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}})+2(m+1)\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-3}{2}}(\sqrt{\frac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}}$$
$$+\sqrt{\frac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}})+4m\sqrt{\frac{8n-3}{4n\cdot(4n-1)}}\}.$$

Finally, for all $m,n\in N$, where m and n both are odd, the eccentricity based atom-bond connectivity index of copper oxide $G(m,n)$ is

$$ABC_5(G(m,n)) = \sum_{m\geq 1}\{2(m+1)\sqrt{\frac{4n-1}{2n(2n+1)}}+4m\sqrt{\frac{8n-3}{4n(4n-1)}}$$

$$+\sum_{k=\frac{n-m}{2}}^{\frac{2n-m-2}{2}}(4m(\sqrt{\frac{8k+4m+1}{(4k+2m+1)(4k+2m+2)}}+\sqrt{\frac{8k+4m+3}{(4k+2m+2)(4k+2m+3)}})$$

$$+2(m+1)(\sqrt{\frac{8k+4m+5}{(4k+2m+3)(4k+2m+4)}}+\sqrt{\frac{8k+4m+7}{(4k+2m+4)(4k+2m+5)}}))\}. \quad \square$$

Theorem 7. *Let $CG(m,n)$, for all $m,n \in N$, where m and n both are even, be the copper oxide, then the eccentricity based atom-bond connectivity index ABC_5 of $G(m,n)$ is*

$$ABC_5(G(m,n)) = \sum_{m\geq 1}\{4m(\sqrt{\frac{4n-1}{2n(2n+1)}}+\sqrt{\frac{8n-3}{4n(4n-1)}})$$

$$+2(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\sqrt{\frac{8k+17}{(4k+9)(4k+10)}}+\sqrt{\frac{8k+19}{(4k+10)(4k+11)}})$$

$$+4m\sum_{k=\frac{n-4}{2}}^{n-4}(\sqrt{\frac{8k+21}{(4k+11)(4k+12)}}+\sqrt{\frac{8k+23}{(4k+12)(4k+13)}})\}.$$

Proof. Let $G(m,n)$, where m is odd and n is even, be the copper oxide contains $4mn+3n+m$ vertices and $6mn+2n$ edges.
The general formula of eccentricity based atom-bond connectivity index is

$ABC_5(G) = \sum_{uv\epsilon E(G)}\sqrt{\frac{\varepsilon(u)+\varepsilon(v)-2}{\varepsilon(u)\cdot\varepsilon(v)}}$. Using the edge partitioned from Table 3, we have the following computations

$$ABC_5(G(m,n)) = \sum_{m\geq 1}\{4m\sum_{k=2n}\sqrt{\frac{k+k+1-2}{k\cdot(k+1)}}$$

$$+2(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\sqrt{\frac{4k+9+4k+10-2}{(4k+9)(4k+10)}}+\sqrt{\frac{4k+10+4k+11-2}{(4k+10)(4k+11)}})$$

$$+4m\sum_{k=\frac{n-4}{2}}^{n-4}(\sqrt{\frac{4k+11+4k+12-2}{(4k+11)(4k+12)}}$$

$$+\sqrt{\frac{4k+12+4k+13-2}{(4k+12)(4k+13)}})+4m\sum_{k=4n-1}\sqrt{\frac{k+k+1-2}{k\cdot(k+1)}}\}$$

$$= \sum_{m\geq 1}\{4m\sum_{k=2n}\sqrt{\frac{2k-1}{k\cdot(k+1)}}+2(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\sqrt{\frac{8k+17}{(4k+9)(4k+10)}}$$
$$+\sqrt{\frac{8k+19}{(4k+10)(4k+11)}})+4m\sum_{k=\frac{n-4}{2}}^{n-4}(\sqrt{\frac{8k+21}{(4k+11)(4k+12)}}$$
$$+\sqrt{\frac{8k+23}{(4k+12)(4k+13)}})+4m\sum_{k=4n-1}\sqrt{\frac{2k-1}{k\cdot(k+1)}}\}$$
$$= \sum_{m\geq 1}\{4m\sqrt{\frac{4n-1}{2n\cdot(2n+1)}}+2(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\sqrt{\frac{8k+17}{(4k+9)(4k+10)}}$$
$$+\sqrt{\frac{8k+19}{(4k+10)(4k+11)}})+4m\sum_{k=\frac{n-4}{2}}^{n-4}(\sqrt{\frac{8k+21}{(4k+11)(4k+12)}}$$
$$+\sqrt{\frac{8k+23}{(4k+12)(4k+13)}})+4m\sqrt{\frac{8n-3}{4n\cdot(4n-1)}}\}.$$

Finally, for all $m, n \in N$, where m is odd and n is even, the eccentricity based atom-bond connectivity index of copper oxide $G(m, n)$ is

$$ABC_5(G(m,n)) = \sum_{m\geq 1}\{4m(\sqrt{\frac{4n-1}{2n(2n+1)}}+\sqrt{\frac{8n-3}{4n(4n-1)}})$$
$$+2(m+1)\sum_{k=\frac{n-4}{2}}^{n-3}(\sqrt{\frac{8k+17}{(4k+9)(4k+10)}}+\sqrt{\frac{8k+19}{(4k+10)(4k+11)}})$$
$$+4m\sum_{k=\frac{n-4}{2}}^{n-4}(\sqrt{\frac{8k+21}{(4k+11)(4k+12)}}+\sqrt{\frac{8k+23}{(4k+12)(4k+13)}})\}. \quad \square$$

Theorem 8. *Let $CG(m, n)$, for all $m, n \in N$, where m is even and n is odd, be the copper oxide, then the eccentricity based atom-bond connectivity index ABC_5 of $G(m, n)$ is*

$$ABC_5(G(m,n)) = \sum_{m\geq 2}\{2(m+1)\sqrt{\frac{4n-1}{2n(2n+1)}}+4m\sqrt{\frac{8n-3}{4n(4n-1)}}$$

$$+\sum_{k=\frac{n-3}{2}}^{n-3}(4m(\sqrt{\frac{8k+13}{(4k+7)(4k+8)}}+\sqrt{\frac{8k+17}{(4k+8)(4k+9)}})$$
$$+2(m+1)(\sqrt{\frac{8k+17}{(4k+9)(4k+10)}}+\sqrt{\frac{8k+19}{(4k+10)(4k+11)}}))\}.$$

Proof. Let $G(m,n)$, where m is odd and n is even, be the copper oxide contains $4mn+3n+m$ vertices and $6mn+2n$ edges.

The general formula of eccentricity based atom-bond connectivity index is

$$ABC_5(G)=\sum_{uv\epsilon E(G)}\sqrt{\frac{\varepsilon(u)+\varepsilon(v)-2}{\varepsilon(u)\cdot\varepsilon(v)}}.$$

Using the edge partitioned from Table 4, we have the following computations

$$ABC_5(G(m,n))=\sum_{m\geq 2}\{2(m+1)\sum_{k=2n}\sqrt{\frac{k+k+1-2}{k\cdot(k+1)}}$$
$$+4m\sum_{k=\frac{n-3}{2}}^{n-3}(\sqrt{\frac{4k+7+4k+8-2}{(4k+7)(4k+8)}}+\sqrt{\frac{4k+8+4k+9-2}{(4k+8)(4k+9)}})$$
$$+2(m+1)\sum_{k=\frac{n-3}{2}}^{n-3}(\sqrt{\frac{4k+9+4k+10-2}{(4k+9)(4k+10)}}+\sqrt{\frac{4k+10+4k+11-2}{(4k+10)(4k+11)}})$$
$$+4m\sum_{k=4n-1}\sqrt{\frac{k+k+1-2}{k\cdot(k+1)}}\}$$
$$=\sum_{m\geq 2}\{2(m+1)\sqrt{\frac{4n-1}{2n\cdot(2n+1)}}$$
$$+4m\sum_{k=\frac{n-3}{2}}^{n-3}(\sqrt{\frac{8k+13}{(4k+7)(4k+8)}}+\sqrt{\frac{8k+15}{(4k+8)(4k+9)}})$$
$$+2(m+1)\sum_{k=\frac{n-3}{2}}^{n-3}(\sqrt{\frac{8k+17}{(4k+9)(4k+10)}}+\sqrt{\frac{8k+19}{(4k+10)(4k+11)}})$$
$$+4m\sqrt{\frac{8n-3}{4n\cdot(4n-1)}}\}.$$

Finally, for all $m, n \in N$, where m is even and n is odd, the eccentricity based atom-bond connectivity index of copper oxide $G(m,n)$ is

$$ABC_5(G(m,n)) = \sum_{m\geq 2}\{2(m+1)\sqrt{\frac{4n-1}{2n(2n+1)}} + 4m\sqrt{\frac{8n-3}{4n(4n-1)}}$$
$$+ \sum_{k=\frac{n-3}{2}}^{n-3}(4m(\sqrt{\frac{8k+13}{(4k+7)(4k+8)}} + \sqrt{\frac{8k+17}{(4k+8)(4k+9)}})$$
$$+ 2(m+1)(\sqrt{\frac{8k+17}{(4k+9)(4k+10)}} + \sqrt{\frac{8k+19}{(4k+10)(4k+11)}}))\}.\square$$

3. Conclusions

In this paper, we computed the atom-bond connectivity index ABC_5 and geometric-arithmetic index GA_4 of the copper oxide CuO.

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