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ONE MODULO THREE ROOT SQUARE MEAN LABELING OF PATH RELATED GRAPHS

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Abstract: In this paper, we introduce a new labeling called one modulo three root square mean labeling. A graph $G$is said to be one modulo three root square mean graph if there is an injective function φ from the vertex set of $G$to the set $\{0, 1, 3, \ldots, 3q\text{-}2, 3q\}$where $q$is the number of edges of $G$and φ induces a bijection φ^* from the edge set of G to$\{1, 4, \ldots, 3q\text{-}2\}$given by $\varphi^*(\mathrm{uv}) = \left\lceil \sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}} \right\rfloor$ and the function φ is called one modulo three root square mean labeling of G. Furthermore, we prove that some path related graphs are one modulo three root square mean graphs.

Key Words: one modulo three root square mean labeling, one modulo three root square mean graphs

1. Introduction

We begin with simple, finite, connected and undirected graph. For standard terminology and notations we follow Harary [1]. A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s).

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If the domain of the mapping is the set of vertices (edges) then the labeling is called a *vertex labeling* (an *edge labeling*). Several types of graph labeling and a detailed survey is available in [2].

C. Jayasekaran and C. David Raj was introduced the concept of one modulo three harmonic mean labeling of graphs in [3]. Root square mean labeling was introduced by S. S. Sandhya, S. Somasundaram and S. Anusa in [4]. P. Jeyanthi and A. Maheswari was introduced the concept of one modulo three mean labeling of graphs in [5]. Also they proved one modulo three mean labeling of cycle related graphs in [6]. In this paper, we introduce a new labeling called one modulo three root square mean labeling and investigate one modulo three root square mean graphs.

We will provide a brief summary of definitions and other information's which are necessary for our present investigation.

Definition 1.1. A graph $G$is said to be one modulo three root square mean graph if there is an injective function φ from the vertex set of $G$to the set $\{0, 1, 3, \ldots, 3q\text{-}2, 3q\}$where $q$is the number of edges of $G$and φ induces a bijection φ^* from the edge set of G to$\{1, 4, \ldots, 3q\text{-}2\}$given by $\varphi^*(uv) = \left\lceil \sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{[\varphi(u)]^2+[\varphi(v)]^2}{2}} \right\rfloor$ and the function φ is called one modulo three root square mean labeling of G.

Definition 1.2. A walk of a graph G is an alternating sequence of points and lines $v_0, x_1, v_1, x_2, \ldots v_{n-1}, x_n, v$ beginning and ending with points such that each line x_i is incident with $v_{i-1}$and v_i.

Definition 1.3. A walk is called a path if all its points are distinct. A path on vertices is denoted by.

Definition 1.4. The corona of two graphs G_1 and G_2 is the graph $G = G_1 \odot G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where ith vertex of G_1 is adjacent to every vertices in the ith copy of G_2.

Definition 1.5. The graph $P_n \odot K_1$ is called a *comb*.

Definition 1.6. A *star* graph is a complete bigraph $K_{1,n}$.

Definition 1.7. A connected acyclic graph is called a *tree*.

Definition 1.8. Y− tree is a tree obtained by taking three paths of same length and identifying one end point of each path.

2. Main Results

Theorem 2.1. Any path P_n is a one modulo three root square mean graph.

Proof. Let $P_n$be the path $u_1u_2 \ldots u_n$. Then $V(P_n) = \{u_1, u_2, \ldots, u_n\}$and $E(P_n) = \{u_iu_{i+1} \ / \ 1\leq i\leq n\text{-}1\}$. Define a function $\phi : V(P_n) \rightarrow \{0, 1, 3,$

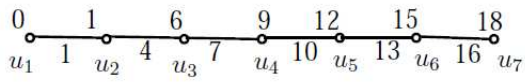

Figure 1: P_7

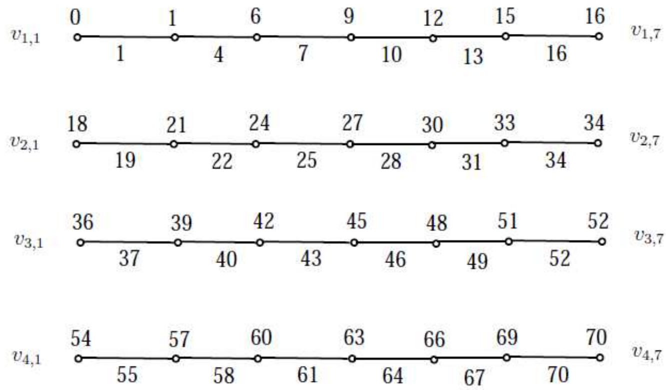

Figure 2: $4P_7$

$\ldots$, *3q-2, 3q*} by $\phi(u_1) = 0$, $\phi(u_2) = 1$, $\phi(u_i) = 3(i-1)$, $3\leq i\leq n$. Then ϕ induces a bijection $\phi* : E(P_n) \to \{1, 4, \ldots, 3q\text{-}2\}$, where $\phi^*(u_iu_{i+1}) = 3i-2$, $1\leq i \leq n-1$. Therefore, ϕ is a one modulo three root square mean labeling. Hence the path P_n is a one modulo three root square mean graph.

Example 2.2. One modulo three root square mean labeling of P_7 is given in Figure 1.

Theorem 2.3. *nP_m is a one modulo three root square mean graph.*

Proof. Let $v_{i,1}$ $v_{i,2}\ldots v_{i,m}$ be the i^{th} copy of P_m in nP_m, $1\leq i \leq n$. Then $V = \{v_{i,j} / 1\leq i \leq n, 1\leq j \leq m\}$is the vertex set and $E = \{v_{i,j}v_{i,j+1}/ 1\leq i \leq n, 1\leq j \leq m-1\}$is the edge set of nP_m. Define a function $\phi : V(nP_m) \to \{0, 1, 3, \ldots, 3q\text{-}2, 3q\}$ by $\phi(v_{1,1}) = 0$; $\phi(v_{1,2}) = 1$; $\phi(v_{1,j}) = 3(j-1)$, $3\leq j\leq m-1$; $\phi(v_{i,j}) = 3[m(i-1)+j-i]$, $2\leq i\leq n$, $1\leq j\leq m-1$; $\phi(v_{i,m}) = 3im-3i-2$, $1\leq i\leq n$. Then ϕ induces a bijection $\phi* : E(nP_m) \to\{1, 4, \ldots, 3q\text{-}2\}$, where $\phi^*(v_{i,j}v_{i,j+1}) = 3[m(i-1)+j-i]+1$, $1\leq i\leq n$, $1\leq j\leq m-1$. Therefore, ϕ is a one modulo three root square mean labeling. Hence nP_m is a one modulo three root square mean graph.

Example 2.4. One modulo three root square mean labeling of $4P_7$ is given in Figure 2.

Theorem 2.5. *Comb $P_n \odot K_1$ is a one modulo three root square mean*

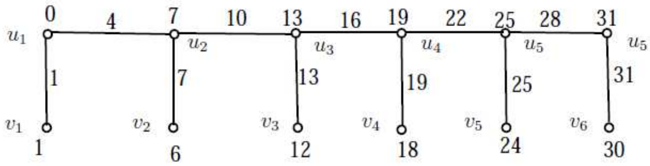

Figure 3: $P_6 \odot K_1$

graph.

Proof. Let P_n be the path $u_1u_2\ldots u_n$. Let v_i be the vertex adjacent to u_i, $1\leq i\leq n$. The resultant graph is $P_n \odot K_1$. Here $V(P_n \odot K_1) = \{u_i, v_i / 1\leq i\leq n\}$and $E(P_n \odot K_1) = \{u_iv_i, u_iu_{i+1}, u_nv_n / 1\leq i\leq n\text{-}1\}$. Then $P_n \odot K_1$ has 2n vertices and 2n-1 edges. Define a function $\phi : V(P_n \odot K_1) \to \{0, 1, 3, \ldots, 3q\text{-}2, 3q\}$ by $\phi(u_1) = 0$; $\phi(u_i) = 6i\text{-}5$, $2\leq i\leq n$; $\phi(v_1) = 1$, $\phi(v_i) = 6i\text{-}6$, $2\leq i\leq n$. Then ϕ induces a bijection $\phi* : E(P_n \odot K_1) \to \{1, 4, \ldots, 3q\text{-}2\}$, where $\phi^*(u_iu_{i+1}) = 6i\text{-}2$, $1\leq i \leq n\text{-}1$; $\phi^*(u_iv_i) = 6i\text{-}5$, $1\leq i \leq n$. Therefore, ϕ is a one modulo three root square mean labeling. Hence Comb $P_n \odot K_1$ is a one modulo three root square mean graph.

Example 2.6. One modulo three root square mean labeling of $P_6 \odot K_1$ is given in Figure 3.

Theorem 2.7. *$P_n \odot \bar{K}_2$ is a one modulo three root square mean graph.*

Proof. Let P_n be the path $u_1u_2\ldots u_n$. Let v_i, w_i be the vertices of i^{th} copy of $\bar{K}_2$. Join v_i and w_i with the vertex u_i, $1\leq i\leq n$. The resultant graph is $P_n \odot \bar{K}_2$ with $V(P_n \odot \bar{K}_2) = \{u_i, v_i, w_i / 1\leq i\leq n\}$and $E(P_n \odot \bar{K}_2) = \{u_iv_i, u_iw_i, u_ju_{j+1} / 1\leq i\leq n, 1\leq j\leq n\text{-}1\}$. Define a function $\phi : V(P_n \odot \bar{K}_2) \to \{0, 1, 3, \ldots, 3q\text{-}2, 3q\}$ by $\phi(u_1) = 1$; $\phi(u_2) = 9$; $\phi(u_i) = 9i\text{-}6$, $3\leq i\leq n$; $\phi(v_1) = 0$; $\phi(v_2) = 10$; $\phi(v_i) = 9(i\text{-}1)\text{-}2$, $3\leq i\leq n$; $\phi(w_1) = 6$; $\phi(w_2) = 15$; $\phi(w_i) = 9i\text{-}5$, $3\leq i\leq n$. Then ϕ induces a bijection $\phi* : E(P_n \odot \bar{K}_2) \to \{1, 4, \ldots, 3q\text{-}2\}$, where $\phi^*(u_iu_{i+1}) = 9i\text{-}2$, $1\leq i \leq n\text{-}1$; $\phi^*(u_iv_i) = 9i\text{-}8$, $1\leq i \leq n$; $\phi^*(u_iw_i) = 9i\text{-}5$, $1\leq i \leq n$. Therefore, ϕ is a one modulo three root square mean labeling. Hence $P_n \odot \bar{K}_2$ is a one modulo three root square mean graph.

Example 2.8. One modulo three root square mean labeling of $P_4 \odot \bar{K}_2$ is given in Figure 4.

Theorem 2.9. *A graph obtained by attaching P_3 at each vertex of P_n is one modulo three root square mean graph.*

Proof. Let P_n be the path $u_1u_2\ldots u_n$. Let $x_iv_iw_i$ be the i^{th} copy of P_3, $1\leq i\leq n$. Identify the vertex u_i with x_i, $1\leq i\leq n$. The resultant graph is G with $V(G) = \{u_i, v_i, w_i / 1\leq i\leq n\}$and $E(G) = \{u_iv_i, v_iw_i, v_nw_n, u_nv_n, u_iu_{i+1} / 1\leq$

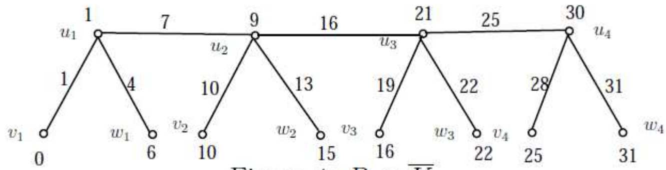

Figure 4: $P_4 \odot \overline{K}_2$

Figure 4: $\mathrm{P}_4 \odot \bar{K}_2$

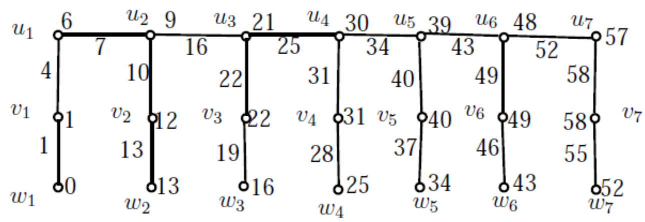

Figure 5

i≤n-1}. Then G has 3n vertices and 3n-1 edges. Define a function $\phi : V(G) \to \{0,\ 1,\ 3, \ldots,\ 3q\text{-}2,\ 3q\}$ by $\phi(u_i) = 3i+3$, $1 \le i \le 2$; $\phi(u_i) = 9i-6$, $3 \le i \le n$; $\phi(v_i) = 11i-10$, $1 \le i \le 2$; $\phi(v_i) = 9i-5$, $3 \le i \le n$; $\phi(w_i) = 13i-13$, $1 \le i \le 2$; $\phi(w_i) = 9i-11$, $3 \le i \le n$. Then ϕ induces a bijection $\phi* : E(G) \to \{1,\ 4,\ \ldots,\ 3q\text{-}2\}$, where $\phi^*(u_iu_{i+1}) = 9i-2$, $1 \le i \le n-1$; $\phi^*(u_1v_1) = 4$; $\phi^*(u_2v_2) = 10$; $\phi^*(u_iv_i) = 9i-5$, $3 \le i \le n$; $\phi^*(v_1w_1) = 1$; $\phi^*(v_2w_2) = 13$; $\phi^*(v_iw_i) = 9i-8$, $3 \le i \le n$. Therefore, ϕ is a one modulo three root square mean labeling. Hence G is a one modulo three root square mean graph.

Example 2.10. One modulo three root square mean labeling of G when n = 7 is given in Figure 5.

Theorem 2.11. *$P_n \odot \bar{K}_3$ is a one modulo three root square mean graph.*

Proof. Let $u_1u_2 \ldots u_n$ be the path P_n. Let v_i, x_i, y_i, z_i be the vertices of i^{th} copy of $K_{1,3}$ with central vertex v_i. Identify v_i with u_i, $1 \le i \le n$. The resultant graph is $G = P_n \odot \bar{K}_3$ with $V(G) = \{u_i, x_i, y_i, z_i / 1 \le i \le n\}$and $E(G) = \{u_ix_i, u_iy_i, u_iz_i, u_ju_{j+1} / 1 \le i \le n, 1 \le j \le n-1\}$. Then G has 4n vertices and 4n-1 edges. Define a function $\phi : V(P_n \odot \bar{K}_3) \to \{0,\ 1,\ 3, \ldots,\ 3q\text{-}2,\ 3q\}$ by $\phi(u_1) = 1$; $\phi(u_2) = 15$; $\phi(u_i) = 12i-8$, $3 \le i \le n$; $\phi(x_1) = 0$; $\phi(x_i) = 12i-14$, $2 \le i \le n$; $\phi(y_1) = 6$; $\phi(y_2) = 16$; $\phi(y_i) = 12i-9$, $3 \le i \le n$; $\phi(z_i) = 12i-3$, $1 \le i \le n$. Then ϕ induces a bijection $\phi* : E(P_n \odot \bar{K}_3) \to \{1,\ 4,\ \ldots,\ 3q\text{-}2\}$, where $\phi^*(u_iu_{i+1}) = 12i-2$, $1 \le i \le n-1$; $\phi^*(u_ix_i) = 12i-11$, $1 \le i \le n$; $\phi^*(u_iy_i) =$

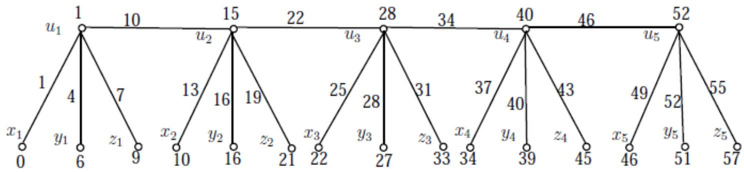

Figure 6: $P_5 \odot \bar{K}_3$

12i-8, 1≤ i ≤ n; $\phi^*(u_iz_i)$ =12i-5, 1≤ i ≤ n. Therefore, ϕ is a one modulo three root square mean labeling. Hence $P_n \odot \bar{K}_3$ is a one modulo three root square mean graph.

Example 2.12. One modulo three root square mean labeling of $P_5 \odot \bar{K}_3$ is given in Figure 6.

Theorem 2.13. *A graph obtained by attaching the central vertex of $K_{1,2}$ at each pendent vertex of a comb $P_n \odot K_1$ is a one modulo three root square mean graph.*

Proof. Let $u_1u_2 \ldots u_n$ be the path P_n and let v_i be a vertex adjacent to u_i, 1≤i≤n. The resultant graph is $P_n \odot K_1$. Let x_i, w_i, y_i be the vertices of i^{th} copy of $K_{1,2}$ with the central vertex w_i. Identify the vertex w_i with v_i, 1≤i≤n, we get the required graph G. Then $V(G)$ = {u_i, v_i, y_i, x_i / 1≤i≤n}and $E(G)$ = {u_iv_i, v_ix_i, v_iy_i, u_ju_{j+1} / 1≤i≤n, 1≤j≤n-1}and hence G has 4n vertices and 4n-1 edges. Define a function $\phi : V(G) \rightarrow \{0,\ 1,\ 3, \ldots,\ 3q\text{-}2,\ 3q\}$ by $\phi(u_1)$ = 6; $\phi(u_2)$ =13; $\phi(u_i)$ =12i-8, 3≤ i≤ n; $\phi(v_1)$ =1; $\phi(v_2)$ =19; $\phi(v_i)$ =12i-9, 3≤ i≤ n; $\phi(x_1)$ = 0; $\phi(x_2)$ = 4; $\phi(x_i)$ = 12i-14, 3≤ i≤ n; $\phi(y_i)$ = 9i, 1≤ i≤ 2; $\phi(y_i)$ = 12i-3, 3≤ i≤ n. Then ϕ induces a bijection $\phi* :\ E(G) \rightarrow \{1,\ 4,\ \ldots,\ 3q\text{-}2\}$, where $\phi^*(u_iu_{i+1})$ =12i-2, 1≤ i ≤ n-1; $\phi^*(u_iv_i)$ =12i-8, 1≤ i ≤ n; $\phi^*(v_iy_i)$ =12i-5, 1≤ i ≤ n; $\phi^*(v_ix_i)$ = 12i-11, 1≤ i ≤ n. Therefore, ϕ is a one modulo three root square mean labeling. Hence G is a one modulo three root square mean graph.

Example 2.14. One modulo three root square mean labeling of G when n = 5 is given in Figure 7.

Remark 2.15. If G is one modulo three root square mean graph, then the edge must get label with 1and so two adjacent vertices of G must get the label 0 and 1.

Theorem 2.16. *Star graph $K_{1,n}$is a one modulo three root square mean graph if and only if n ≤ 3.*

Proof. Let G be the star graph $K_{1,n}$. Let $V(G)$ = {u, u_i /1≤ i≤ n }and $E(G)$ = {uu_i /1≤ i≤ n}. $K_{1,1}$ is same as P_2 and $K_{1,2}$ is P_3. Hence by

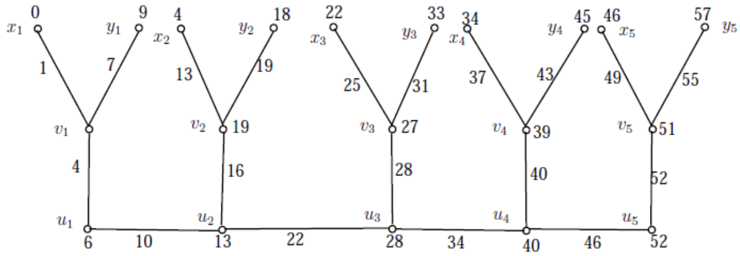

Figure 7

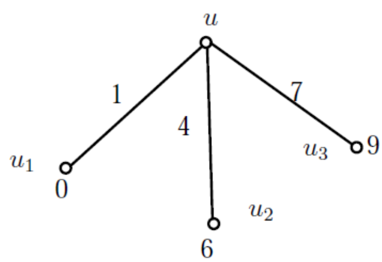

Figure 8: $K_{1,3}$

Theorem 2.1, $K_{1,1}$ and $K_{1,2}$ are one modulo three root square mean graph. One modulo three root square mean labeling for $K_{1,3}$ is shown in Figure 8.

Suppose $K_{1,n}$is a one modulo three root square mean labeling for n >3. Then there is a function $\phi : V(G) \rightarrow \{0, 1, 3, \ldots, 3q\text{-}2, 3q\}$. Let u be the central vertex of $K_{1,n}$. By remark 2.15, two adjacent vertices are labeled by 0 and 1. Then u must be labeled with either 0 or 1. The number of edges in $K_{1,n}$ is $q = n > 3$. Here we consider two cases.

Case 1. φ(u) = 0

Then clearly there is no edge with label *3q-2* $(q = n > 3)$, since the labels of the edges uu_i are less than or equal to $\left\lceil \sqrt{\frac{0^2+(3q)^2}{2}} \right\rceil$ *or* $\left\lfloor \sqrt{\frac{0^2+(3q)^2}{2}} \right\rfloor$. That is the labels of the edges uu_i are less than or equal to 2q. But 3q-2 is greater than or equal to 2q. Hence $K_{1,n}$ is not a one modulo three root square mean graph.

Case 2. φ(u) = 1

Then clearly there is no edge with label 3q-2 $(q = n > 3)$, since the labels of the edges uu_i are less than or equal to $\left\lceil \sqrt{\frac{1^2+(3q)^2}{2}} \right\rceil$ *or* $\left\lfloor \sqrt{\frac{1^2+(3q)^2}{2}} \right\rfloor$. Hence

$K_{1,n}$ is not a one modulo three root square mean graph.

Thus in the above two cases $K_{1,n}$ is not a one modulo three root square mean graph for n >3. Therefore star graph $K_{1,n}$is a one modulo three root square mean graph if and only if n $\leq$ 3.

Theorem 2.17. *Y- tree is a one modulo three root square mean graph.*

Proof. Let x_1, x_2, ..., x_n, y_1, y_2, ..., y_n and z_1, z_2, ..., z_n be the three paths of length n each. Identify x_1, y_1, and z_1 and label it as v. The resultant graph is G (Y- tree). Here $V(G)$ = {v, x_i, y_i, z_i / 2$\leq$ i$\leq$ n}and $E(G)$ ={vx_2, vy_2, vz_2, x_ix_{i+1}, y_iy_{i+1}, z_iz_{i+1}/ 2$\leq$i$\leq$n-1}. Then G has 3n-2 vertices and 3n-3 edges. Define a function $\phi : V(G) \rightarrow${*0, 1, 3, ..., 3q-2, 3q*} by ϕ(v)= 0, $\phi(x_2)$ = 1, $\phi(x_i)$ = 9i-14, 4$\leq$ i$\leq$ n and *i* is even, $\phi(x_i)$ = 9i-12, 3$\leq$ i$\leq$ n and *i* is odd; $\phi(y_2)$ = 10, $\phi(y_i)$ = 9i-6, 3$\leq$ i$\leq$ n; $\phi(z_2)$ = 6, $\phi(z_i)$ = 9i-9, 3$\leq$ i$\leq$ n. Then ϕ induces a bijection $\phi* :$ $E(G) \rightarrow$ {*1, 4, ..., 3q-2*}, where $\phi^*(vx_2)$ =1, $\phi^*(x_ix_{i+1})$ = 9i-8, 2$\leq$ i $\leq$ n-1; $\phi^*(vy_2)$ =7, $\phi^*(y_iy_{i+1})$ = 9i-2, 2$\leq$ i $\leq$ n-1; $\phi^*(vz_2)$ =4, $\phi^*(z_iz_{i+1})$ = 9i-5, 2$\leq$ i $\leq$ n-1. Therefore, ϕ is a one modulo three root square mean labeling. Hence Y- tree is a one modulo three root square mean graph.

Example 2.18. One modulo three root square mean labeling of G when n = 7 is given in Figure 9.

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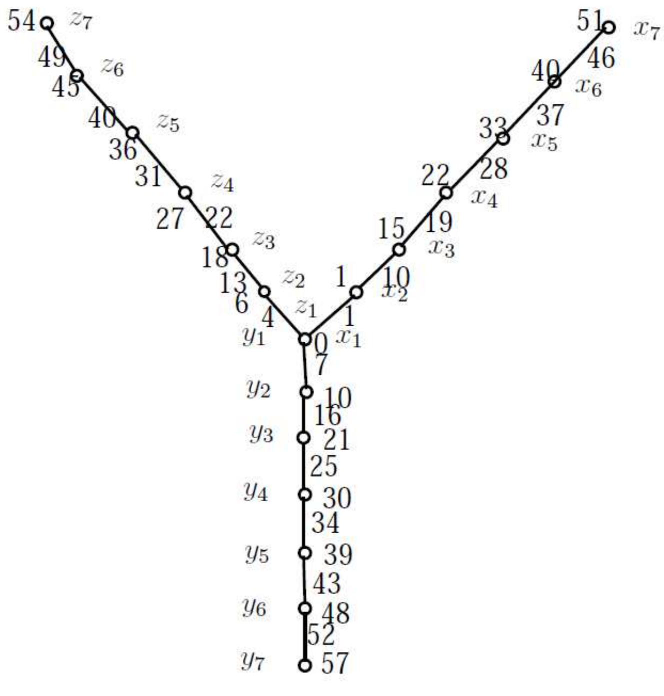

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Figure 9