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ON TOPOLOGICAL INDICES OF LINE GRAPH OF SUBDIVISION OF STAR AND FRIENDSHIP GRAPHS

Waqas Nazeer[1], Muhammad Akmal[2],
Muhammad Imran[3], Mohammad Reza Farahani[4 §]

[1]Division of Science and Technology
University of Education
Lahore, PAKISTAN

[2]Divisional Public School and College Sahiwal
PAKISTAN

[3]Department of Mathematical Sciences
United Arab Emirates University
P.O. Box 15551, Al Ain, UNITED ARAB EMIRATES

[3] Department of Mathematics, School of Natural Sciences (SNS)
National University of Sciences and Technology (NUST)
Sector H-12, Islamabad, PAKISTAN

[4]Department of Applied Mathematics
Iran University of Science and Technology (IUST)
Narmak, Tehran 16844, IRAN

Abstract: In this report, we compute newly defined topological indices, namely, Arithmetic-Geometric index (AG_1 index), SK index, SK_1 index, and SK_2 index of the Line Graph of subdivision of star and friendship graphs. We also compute sum connectivity index and modified *Randić* index of underling graphs.

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§Correspondence author

1. Introduction

Chem-informatics is an emerging field in which quantitative structure-activity (QSAR) and Structure-property (QSPR) relationships predict the biological activities and properties of nano-material see [1]. In these studies, some physcio-chemical properties and topological indices are used to predict bioactivity of the chemical compounds see [2].

The branch of chemistry which deals with the chemical structures with the help of mathematical tools is called mathematical chemistry. The chemical graph theory is that branch of mathematical chemistry which applies graph theory to mathematical modeling of chemical phenomena. In chemical graph theory a molecular graph is a simple graph (having no loops and multiple edges) in which atoms and chemical bonds between them are represented by vertices and edges respectively. A graph G with vertex set $V(G)$ and edge set $E(G)$ is connected, if there exists a connection between any pair of vertices in G. The degree of a vertex is the number of vertices which are connected to that fixed vertex by the edges. In a chemical graph the degree of any vertex is at most 4. The number of vertices of G, adjacent to a given vertex v, is the "degree" of this vertex, and will be denoted by d_v. The concept of degree in graph theory is closely related (but not identical) to the concept of valence in chemistry.

The first Zagreb index and the second Zagreb index, introduced by Gutman and Trinajstic [3] and are defined as:

$$M_1(G) = \sum_{v \in V(G)} (d_v)^2 \text{ and } M_2(G) = \sum_{uv \in V(G)} d_u d_v.$$

We details about chemical graph theory please see [4, 5, 6, 7, 8, 9].

V. S. Shigehalli and Rachanna Kanabur [11] introduced following new degree-based topological indices: Arithmetic-Geometric (AG1) index

$$AG_1(G) = \sum_{v \in V(G)} \frac{d_u + d_v}{2\sqrt{d_u d_v}},$$

$$SK(G) = \sum_{v \in V(G)} \left(\frac{d_u + d_v}{2} \right),$$

$$SK_1(G) = \sum_{v \in V(G)} \frac{d_u + d_v}{2\sqrt{d_u d_v}},$$

$$SK_2(G) = \sum_{v \in V(G)} \left(\frac{d_u + d_v}{2} \right)^2.$$

In this article, we compute some newly defined degree-based topological indices of the line graph of the subdivision graph of Star and friendship groups.

Lemma 1. *Let G be a graph with $u, v \in V(G)$ and $e = uv \in E(G)$. Then $de = du + dv - 2$*

Lemma 2. *Let G be a graph of order p and size q, then the line graph $L(G)$ of G is a graph of order p and size $\frac{1}{2}M_1(G) - q$.*

2. Main Results

In this section, we give our computational results.

Theorem 3. *Let G be the line graph of subdivision of the Star graph with $n \geq 2$. Then:*

1. $\chi(G) = \frac{n-1}{\sqrt{n}} + \frac{n^2-3n+2}{2\sqrt{2n-2}}$;
2. $R(G) = \frac{n}{2}$;
3. $AG_1 = \frac{n\sqrt{n-2}}{2} + \frac{n^2}{2} - \frac{3n}{2} + 1$;
4. $SK = \frac{1}{2}n(n-1) + \frac{1}{2}(n^2 - 3n + 2)(n-1)$;
5. $SK_1 = \frac{1}{2}(n-1)^2 + \frac{1}{2}\left(\frac{1}{2}n^2 - \frac{3}{2}n + 1\right)(n-1)^2$;
6. $SK_2 = \frac{1}{2}n^2(n-1) + \frac{1}{2}(n^2 - 3n + 2)(n-1)^2$.

Proof. Star graph S_n is the complete bipartite graph $K_{1,n}$, a tree with one internal node and n leaves with $n+1$ vertices and n edges. The subdivision graph $S(S_n)$ is the graph obtained from S_n by replacing each of its edge by a path of length 2 or equivalently by inserting an additional vertex into each edge of S_n. The line graph $G = L[S(S_n)]$ of a subdivision graph $S(S_n)$ is the graph whose vertices are the edges of $S(S_n)$.

There are two types of vertices with respect to degree in line graph of subdivision graph of star graph $G = L[S(S_n)]$ their degree are $n-1$ and 1. $n-1$ vertices have degree $n-1$ and $n-1$ vertices have degree 1. $G = L[S(Sn)]$ contains $2(n-1)$ vertices and $\frac{n^2-n}{2}$ edges. There are two types of edges in based on degrees of end vertices of each edge. The first edge partition $E_1(G)$ contains $n-1$ edges uv, where $d_u = 1, d_v = n-1$. The second edge partition $E_2(G)$ contains $\frac{n^2-3n+2}{2}$ edges uv, where $d_u = 1, d_v = n-1$. From the definitions we get our results. □

Theorem 4. *Let G be the line graph of subdivision of the Star graph with $n \geq 2$. Then:*

1. $\chi(G) = \frac{3}{2}n + \frac{2n-1}{2} + \sqrt{n}$;
2. $R(G) = \frac{5}{2}n + \frac{n}{2}$;
3. $AG_1 = n^{3/2} - 2n^2 + \sqrt{n} + 2n$;
4. $SK = 4n^3 + 8n$;
5. $SK_1 = 4n^4 - 2n^3 + 4n^2 + 6n$;
6. $SK_2 = 8n^4 - 2n^3 + 4n^2 + 14n$.

Proof. Friendship graph F_n is a planner undirected graph with $2n+1$ vertices and $3n$ edges, can be constructed by joining n copies of cycle graph C_n with a common vertex. The subdivision graph $S(F_n)$ is the graph obtained from F_n by replacing each of its edge by a path of length 2 or equivalently by inserting an additional vertex into each edge of F_n. The line graph $L[S(F_n)]$ of a subdivision graph $S(F_n)$ is the graph whose vertices are the edges of $S(F_n)$, two vertices e and f are incident if and only if they have common end vertex in $S(F_n)$. There are two types of vertices with respect to degree in line graph of subdivision graph of friendship graph $G = L[S(F_n)]$, their degrees are $2n$ and 2. $2n$ vertices have degree $2n$ and $4n$ vertices have degree 2. $G = L[S(F_n)]$ contains $6n$ vertices and $2n(n+2)$ edges. In G, there are three types of edges based on degrees of end vertices of each edge. The first edge partition $E_1(G)$ contains $3n$ edges uv, where $d_u = d_v = 2$. The second edge partition $E_2(G)$ contains $n(2n-1)$ edges uv, where $d_u = d_v = 2n$. The third edge partition $E_3(G)$ contains $2n$ edges uv, where $d_u = 2n, d_v = 2$ From the definitions we get our results. □

3. Conclusion

In this article we computed Arithmetic-Geometric index (AG_1 index), SK index, SK_1 index, and SK_2 index, sum connectivity index and modified *Randić* index of the line graph of subdivision of Star and friendship graphs. These results can play a vital role in determining properties of this network and its uses in industry, electronics, and pharmacy.

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