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# SOME PROPERTIES OF CONTRA $\breve{g}$ CONTINUOUS FUNCTIONS

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**Abstract:** In [4], Dontchev introduced and investigated a new notion of continuity called contra continuity. Later, Jafari and Noiri [3] introduced and investigated the concept of contra $\alpha$ continuous. The aim of this paper is to introduce and study the concept of contra $\breve{g}$ continuous and the relationship with other contra functions and their characteristics are obtained.

**AMS Subject Classification:** $\breve{g}$ closed, $\breve{g}$ - continuity and Contra $\breve{g}$ continuous
**Key Words:** 54C08, 54C10

## 1. Introduction

In 1996, Dontchev [4] introduced the notion of contra continuous functions. In 2007, Caldas, Jafari, Noiri and Simoes [2] introduced a new class of functions called generalized contra continuous (contra $g$ - continuous) functions. They defined a function $f : X \longrightarrow Y$ to be contra $g$ continuous if pre image every open subsets of y is $g$ closed in X. New types of contra generalized continuity such as contra $g^{\#}$p continuous function by Alli [7] and contra $gs$ continuous [5] and contra $\alpha^*$ continuous functions, almost contra $\alpha^*$ continuous function

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[12], contra $\pi gr$ continuous, almost contra $\pi gr$ continuous [6]. The purpose of this paper is to introduce some properties of notion of contra $\breve{g}$ continuity via the concept $\breve{g}$ closed set, $\breve{g}$ continuity.

## 2. Preliminaries

Throughout this paper,$(X,\tau),(Y,\sigma)$ or X,Y represent non empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. Let A be a subset of a space X. The closure and interior of A are denoted by Cl(A) and int(A), respectively.

Definition: 2.1
1. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra continuous [4] if $f^{-1}(v)$ is closed set in $(X,\tau)$ for every open set $v$ in $(Y,\sigma)$

2. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra $g$ continuous [2] if $f^{-1}(v)$ is $g$-closed set in $(X,\tau)$ for every open set $v$ in $(Y,\sigma)$

3. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra $\alpha$ continuous [3] if $f^{-1}(v)$ is $\alpha$-closed set in $(X,\tau)$ for every open set $v$ in $(Y,\sigma)$

4. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra semi-continuous [5] if $f^{-1}(v)$ is semi-closed set in $(X,\tau)$ for every open set $v$ in $(Y,\sigma)$

5. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra $gs$ continuous (resp.contra $sg$-continuous) if the preimage of every open subset of $Y$ is $gs$ -closed set (resp.$sg$ closed)[5] in $X$

6. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra $g\alpha$ continuous [7] if $f^{-1}(v)$ is $g\alpha$ -closed set in $(X,\tau)$ for every open set $v$ in $(Y,\sigma)$

7. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra-pre-continuous [8] if $f^{-1}(v)$ is pre-closed set in $(X,\tau)$ for every open set $v$ in $(Y,\sigma)$

8. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra $g^{\#}p$ - continuous [7] if $f^{-1}(v)$ is $g^{\#}p$ - closed set in $(X,\tau)$ for every open set $v$ in $(Y,\sigma)$

9. A function $f:(X,\tau)\longrightarrow(y,\sigma)$ is called contra $gb$ - continuous [9] if $f^{-1}(v)$

is $gb$ - closed set in $(X, \tau)$ for every open set $v$ in $(Y, \sigma)$

10. A function $f : (X, \tau) \longrightarrow (y, \sigma)$ is called contra $\omega$ - continuous [14] if $f^{-1}(v)$ is $\omega$ - closed set in $(X, \tau)$ for every open set $v$ in $(Y, \sigma)$

11. A function $f : (X, \tau) \longrightarrow (y, \sigma)$ is called contra $g\alpha$ - continuous [7] if $f^{-1}(v)$ is $g\alpha$ - closed set in $(X, \tau)$ for every open set $v$ in $(Y, \sigma)$

Definition: 2.2
Let A be a subset of a topological space $(X, \tau)$. The set $\cap\{U \in \tau / A \subset U\}$ is called the kernel of A

Lemma 2.3 [3]
The following properties hold for subsets A,B of a space X:
(1)$x \in ker(A)$ if and only if $A \cap F \neq \emptyset$, for any $F \in C(X, x)$
(2)A $\subset ker(A)$ and $A = ker(A)$ if A is open in X
(3)if A $\subset$ B, then $ker(A) \subset ker(B)$.

Theorem 2.4 [15]
Let $(X, \tau)$ be a topological spaces.Then
(1) Every closed set is $\breve{g}$ closed.

(2) Every $\breve{g}$-closed set is $\breve{g}_\alpha$ closed.

(3) Every $\breve{g}$-closed set is $sg$ closed.

(4) Every $\breve{g}$-closed set $g$ closed.

(5) Every $\breve{g}$-closed set is $\alpha g$ closed.

(6) Every $\breve{g}$-closed set is $gs$ closed.

(7) Every $\breve{g}$-closed set is $gsp$ closed.

## 3. Contra $\breve{g}$-Continuous Functions

Definition: 3.1
A functions $f : (X, \tau) \to (y, \sigma)$ is called contra $\breve{g}$ continuous function if $f^{-1}(O)$ is $\breve{g}$ closed in $(X, \tau)$ for every open set O in $(Y, \sigma)$.

Example: 3.2
Let X = Y = $\{a, b, c\}, \tau = \{\phi, \{b\}, X\}$ and $\sigma = \{\phi, \{a, c\}, X\}$ we have, $\breve{G}(C(X)) = \{\phi, \{a, c\}, X\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the identity map. Then $f$ is called contra $\breve{g}$ continuous.

Theorem: 3.3
Every contra continuous is a contra $\breve{g}$ continuous function.
Proof:
Let $f : (X, \tau) \to (y, \sigma)$ be a function. Let O be an open set in $(Y, \sigma)$. Since, $f$ is contra continuous, then, $f^{-1}(O)$ is closed in $(X, \tau)$. [Since, every closed set is $\breve{g}$ closed], then, $f^{-1}(O)$ is $\breve{g}$-closed in $(X, \tau)$.
Therefore, $f$ is contra $\breve{g}$ continuous.

Examples: 3.4
Let X = Y = $\{a, b, c\}, \tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a, \}, Y\}$ we have, $\breve{G}(C(X)) = \{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the identity map. Since $f^{-1}\{b, c, \} = \{b, c\}$ is contra $\breve{g}$ continuous but not continuous.

REMARKS: 3.5

(i)Every contra $\breve{g}$ continuous is contra $\breve{g}_\alpha$ continuous. But converse of the above need not be true.
Examples: 3.5.1
Let X = Y = $\{a, b, c\}, \tau = \{\phi, \{a, \}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$ we have, $\breve{G}(C(X)) = \{\phi, \{b, c\}, X\}$ and $\breve{G}_\alpha C(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. Let $f : (X, \tau) \longrightarrow (Y, \sigma)$ be the identity map. Then $f$ is called contra $\breve{g}_\alpha$ continuous but not contra $\breve{g}$ continuous, since $f^{-1}\{c\} = \{c\}$ is not contra $\breve{g}$ continuous.

(ii)Every contra $\breve{g}$ continuous is contra $sg$ continuous. But converse of the above need not be true.

Examples: 3.5.2
Let X = Y = $\{a, b, c\}, \tau = \{\phi, \{a, \}, X\}$ and $\sigma = \{\phi, \{a, b\}, Y\}$ we have,

$\breve{G}(C(X)) = \{\phi, \{b,c\}, X\}$ and $SGC(X) = \{\phi, \{b\}, \{c\}, \{b,c\}, X\}$. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be the identity map. Then $f$ is called contra $sg$ continuous but not contra $\breve{g}$ continuous, since $f^{-1}\{c\} = \{c\}$ is not contra $\breve{g}$ continuous

(iii)Every contra $\breve{g}$ continuous is contra $g$ continuous. But converse of the above need not be true.
Examples: 3.5.3
Let X = Y = $\{a,b,c\}, \tau = \{\phi, \{a,\}, X\}$ and $\sigma = \{\phi, \{a,b\}, Y\}$ we have, $\breve{G}(C(X)) = \{\phi, \{b,c\}, X\}$ and $GC(X) = \{\phi, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be the identity map. Then $f$ is called contra $g$ continuous but not contra $\breve{g}$ continuous, since $f^{-1}\{c\} = \{c\}$ is not contra $\breve{g}$ continuous

(iv)Every contra $\breve{g}$ continuous is contra $\alpha g$ continuous. But converse of the above need not be true.
Examples: 3.5.4
Let X = Y = $\{a,b,c\}, \tau = \{\phi, \{a,\}, X\}$ and $\sigma = \{\phi, \{a,b\}, Y\}$ we have, $\breve{G}(C(X)) = \{\phi, \{b,c\}, X\}$ and ${}_{\alpha}GC(X) = \{\phi, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be the identity map. Then $f$ is called contra $\alpha g$ continuous but not contra $\breve{g}$ continuous, since $f^{-1}\{c\} = \{c\}$ is not contra $\breve{g}$ continuous

(v)Every contra $\breve{g}$ continuous is contra $gs$ continuous. But converse of the above need not be true.
Example: 3.5.5
Let X = Y = $\{a,b,c\}, \tau = \{\phi, \{a,\}, X\}$ and $\sigma = \{\phi, \{a,b\}, Y\}$ we have, $\breve{G}(C(X)) = \{\phi, \{b,c\}, X\}$ and $GSC(X) = \{\phi, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be the identity map. Then $f$ is called contra $gs$ continuous but not contra $\breve{g}$ continuous, since $f^{-1}\{c\} = \{c\}$ is not contra $\breve{g}$ continuous

(vi)Every contra $\breve{g}$ continuous is contra $gsp$ continuous. But converse of the above need not be true.
Examples: 3.5.6
Let X = Y = $\{a,b,c\}, \tau = \{\phi, \{b,\}, X\}$ and $\sigma = \{\phi, \{a,b\}, Y\}$ we have, $\breve{G}(C(X)) = \{\phi, \{a,c\}, X\}$ and $GSPC(X) = \{\phi, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, X\}$. Let $f : (X,\tau) \longrightarrow (Y,\sigma)$ be the identity map. Then $f$ is called contra $gsp$ continuous but not contra $\breve{g}$ continuous, since $f^{-1}\{c\} = \{c\}$ is not contra $\breve{g}$ continuous.

Theorem: 3.6
For a function $f : (X,\tau) \longrightarrow (y,\sigma)$ the following continuous are equivalent
(1) $f$ is a contra $\breve{g}$ continuous.

(2) for every closed subset F of Y,$f^{-1}(F) \in \breve{g}\ O(X,\tau)$.
(3) for each x $\in$ X, and each F $\in$ C(Y,$f(x)$) there exists U $\in$ $\breve{g}O(X,x)$ such that $f(U) \subseteq$ F.
(4) $f(cl_{\breve{g}(A)}) \subseteq ker(f(A))$ for every subset A of X.
(5) cl$_{\breve{g}}(f^{-1}(B)) \subseteq f^{-1}(ker(B))$ for every subset B of Y.

Proof:
(1) $\Longleftrightarrow$ (2) and (2) $\Longrightarrow$ (3) are obvious.

(3) $\Longrightarrow$ (2): Let F be any closed set of Y and x $\in$ $f^{-1}(F)$.Then $f(x) \in$ F and there exists U$_x$ $\in$ $\breve{g}O(X,x)$, such that $f(U_x) \subseteq$ F.Therefore we obtain $f^{-1}(F) = U\{U_x : x \in f^{-1}(F)\}$, which is $\breve{g}$ open in X.

(2) $\Longrightarrow$ (4): Let A be any subset of X. Suppose that Y $\notin ker(f(A))$. Then by lemma (2.3), there exist F$\in C(Y, f(x))$ such that $f(A) \cap F = \phi$. Thus,we have A $\cap f^{-1}(F) = \phi$ and since $f^{-1}(F)$ is $\breve{g}$ - open.then we have $\breve{g}cl(A) \cap f^{-1}(f) = \phi$. Therefore we obtain $f(\breve{g}cl(A) \cap f^{-1}(F) = \phi\ )$ and $y \notin \breve{g}cl(A))$. This implies that $f[\breve{g}cl(A)] \subseteq ker(f(A))$.

(4) $\Longrightarrow$ (5): Let B be any subset of Y. By(4) and lemma (2.3), we have
$f[\breve{g}(clf^{-1}(B))] \subseteq ker(f(f^{-1}B)) \subseteq ker(B)$. Thus $\breve{g}cl(f^{-1}(B)) \subseteq f^{-1}(ker(B))$

(5) $\Longrightarrow$ (1): Let V be any open set of Y .Then by lemma (2.3), We have $\breve{g}(cl(f^{-1}(V))) \subseteq f^{-1}(ker(V))$=$f^{-1}(V)$ and $\breve{g}cl(f^{-1}V)) = f^{-1}(V)$. This show that $f^{-1}(V)$ is $\breve{g}$ - closed in X.

Theorem: 3.7
If a function $f : X \longrightarrow Y$ is contra $\breve{g}$ continuous and Y is regular ,Then $f$ is $\breve{g}$ continuous.
Proof:
Let $x$ be an arbitrary point of X and let V be an open set of Y containing $f(x)$, since Y is regular, there exists an open set G in Y containing $f(x)$ such that cl(G) $\subset$ V. Since $f$ is contra $\breve{g}$ continuous, by theorem: 3.6(3) there exists U$\in$ $\breve{g}$ O(X,$x$) such that $f(U) \subseteq$ cl(G). Then $f(U) \subseteq cl(G) \subset V$. hence $f$ is $\breve{g}$ continuous.

Definition: 3.8
A space $(X,\tau)$ is said to be
(a) $\breve{g}$- space if every $\breve{g}$-open set of X is open in X.

(b) locally $\breve{g}$- indiscrete if every $\breve{g}$- open set of X is closed in X

The following two results follows immediately.
Theorem: 3.9
If a function $f : X \longrightarrow Y$ is contra $\breve{g}$ continuous and X is $\breve{g}$ space,then $f$ is contra continuous.
Proof:
Let V $\in$ O(Y). Then $f^{-1}(V)$ is $\breve{g}$- closed in X. Since X is $\breve{g}$- space, $f^{-1}(V)$ is closed in X. Thus, $f$ is contra continuous.

Theorem: 3.10
Let X be locally $\breve{g}$- indiscrete. If a function $f : X \longrightarrow Y$ is contra $\breve{g}$ continuous, then it is continuous.
Proof:
Let V $\in$ O(Y).Then $f^{-1}(V)$ is $\breve{g}$-closed in X.Since X is locally $\breve{g}$-indiscrete space, $f^{-1}(v)$ is open in X. Thus $f$ is continuous.

## 4. CLOSED GRAPHS

Recall that for a function $f : X \longrightarrow Y$,the subset $\{(x, f(x)) : x \in X\} \subset$ X x Y is called the graph of $f$ and is denoted by G(f).

Definition: 4.1
The graph G(f) of a function $f : (x, \tau) \longrightarrow (y, \sigma)$ is said to be contra $\breve{g}$-closed graph in X x Y,if for each $(x, y) \in (X, Y)$- G(f). There exist $U \in \breve{g}O(X, x)$ and $V \in C(Y, y)$ such that (U x V) $\cap$ G(f) = $\emptyset$.

Lemma: 4.2
The graph G(f) of a function $f : (x, \tau) \longrightarrow (y, \sigma)$ is contra $\breve{g}$ closed in X x Y if and only if for each $(x, y) \in (X, Y) - G(f)$ there exist $U \in \breve{g}O(X, x)$ and $V \in C(Y, y)$ such that $f(U) \cap V = \emptyset$
Proof:
We shall prove that $f(U) \cap V = \emptyset \iff (UxV) \cap G(f) = \emptyset$. Let (U x V) $\cap G(f) \neq \emptyset$. Then there exist $(x, y) \in (X, Y)$ and $(x, y) \in G(f)$. This implies that $x \in U, y \in V$ and $y = f(x) \in V$. Therefore,$f(U) \cap V \neq \emptyset$. Hence the result follows.

Theorem : 4.3
Let $f : X \longrightarrow Y$ be a function and let $g : X \longrightarrow$ X x Y be the graph function

of $f$ defined by $g(x) = (x, f(x))$ for every $x \in X$. If g is contra $\breve{g}$ -continuous, then $f$ is contra $\breve{g}$ continuous.
Proof:
Let U be an open set in Y, then X x U is an open set in X x Y. since g is contra $\breve{g}$ continuous.It follows that $f^{-1}(U) = g^{-1}$(X x U) is an $\breve{g}$ - closed in X. Thus $f$ is contra $\breve{g}$ continuous.

Theorem: 4.4
If $f : X \longrightarrow Y$ is contra $\breve{g}$ continuous and Y is Urysohn, then G(f) is contra $\breve{g}$-closed in X x Y
Proof:
Let $(x, y) \in (X, Y)$- G(f).Then $y \neq f(x)$ and there exists open sets V,W such that $f(x) \in V, y \in W$ and $cl(V) \cap cl(W) = \emptyset$. Since $f$ is contra $\breve{g}$ continuous, there exists $U \in \breve{g}\ O(X, x)$ such that $f(U) \subseteq cl(V)$.Therefore we obtain $f(U) \cap cl(W) = \emptyset$. This shows that G(f) is contra $\breve{g}$- closed.

Theorem: 4.5
If $f : X \longrightarrow Y$ is contra $\breve{g}$ continuous, $g : X \longrightarrow Y$ contra continuous and Y is Urysohn, then $E = \{x \in X : f(x) = g(x)\}$ is $\breve{g}$-closed in X.

Proof:
Let $x \in X - E$. Then $f(x) \neq g(x)$. Since Y is Urysohn, there exists open sets V and W such that $f(x) \in V, g(x) \in W$ and $cl(V) \cap cl(W) = \emptyset$. Since $f$ is contra $\breve{g}$ continuous, then $f^{-1}(cl(V))$ is $\breve{g}$ - open in X and g is contra continuous, then $g^{-1}(cl(W))$ is open in X. Let $U = f^{-1}(cl(V))$ and $G = g^{-1}(cl(W))$. Then U and G contain $x$. Set $A = U \cap G$ is $\breve{g}$ - open in X. And $f(A) \cap g(A) \subset f(U) \cap g(G) \subset cl(V) \cap cl(W) = \emptyset$. Hence $f(A) \cap g(A) = \emptyset$ and $A \cap E = \emptyset$ where A is $\breve{g}$ - open therefore $x \notin \breve{g}$ cl(E). Thus E is $\breve{g}$ - closed in X

Theorem: 4.6
If $f : X \longrightarrow Y$ is $\breve{g}$ - continuous and Y is $T_1$, then G(f) is contra $\breve{g}$ - closed in X x Y.
proof:
Let $(x, y) \in (XxY) - G(f)$.Then $Y \neq f(x)$ and there exist open set V of Y, such that $f(x) \in V$ and $y \notin V$. Since $f$ is $\breve{g}$ continuous, there exists $U \in \breve{g}O(X, x)$ such that $f(U) \subseteq V$. Therefore, we obtain $f(U) \cap (Y - V) = \emptyset$ and $(Y - V) \in C(Y, y)$. This shows that G(f) is contra $\breve{g}$ - closed in $X$ x $Y$

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