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NUMERICAL STUDY OF MHD FLOW OF A THIRD GRADE FLUID IN A NON-DARCIAN POROUS CHANNEL

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Abstract: This paper aims to study, numerically, the flow and heat transfer of a magnetohydrodynamic third grade fluid in a horizontal porous channel. The flow is induced by a constant pressure gradient along the flow direction and a uniform magnetic field applied in the transverse direction. A modified Darcy resistance term for a third grade fluid is taken in the momentum equation. The governing equations are modelled, discretised by suitable finite difference technique, and solved by damped Newton's method. The effect of various flow parameters on the velocity and temperature profiles is presented in form of graphs and discussed.

AMS Subject Classification: 76Z05, 76W05, 76S05, 80A20
Key Words: third-grade fluid, modified Darcy's resistance, Hartmann number, Heat transfer, Wall shear stress

1. Introduction

The study of non-Newtonian flow in a porous medium has gained the attention of researchers in various branches of sciences, engineering and technology, ow-

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ing to its wide range of applications in surface and colloid physical chemistry, physiology, rheology, geophysics, soil physics, hydrology, petroleum engineering, metal casting, and in the technologies of wood, paper, textiles, leather, cement and solid fuels [1]. In order to describe the properties of non-Newtonian fluids several constitutive equations have been proposed to model fluid flow. The class of visco-elastic fluids amongst non-Newtonian fluids are the most widely accepted models. For applying our study to the real world problem we have considered the fluid to be human blood which is a biological fluid. The behavior of blood is Newtonian in large arteries whereas, in small vessels like arterioles and capillaries its behavior is mostly non-Newtonian. Due to the shear thinning property possesed by blood it falls in the category of visco-elastic third grade fluids. It is a magnetohydrodynamic fluid as the presence of haemoglobin in the RBCs of human blood makes it susceptible to magnetisation, and the presence of salts and other impurities makes it electrically conducting in nature. The study analyses the flow of the third grade fluid in a porous medium. For being the simplest model, Darcy's law is mostly used to describe the flow in porous media. Darcy's law in its original form is a linear model that relates the local pressure gradient in the flow direction to the superficial velocity of the fluid through the viscosity of the fluid and permeability of the medium. The validity of the Darcy's law is restricted to laminar, isothermal, purely viscous, incompressible Newtonian flows. To accomodate the third grade phenomenon of our problem, modified Darcy's law is used.

Ayub et.al. [2] presented solutions for steady third grade rheological fluid flow past an infinite surface using homotopy method and the perturbation technique. The steady laminar flow of a third grade fluid through porous channel was studied by Ariel [3]. Hayat et.al. [4] used homotopy analysis method to obtain the solution of a pressure driven third grade fluid flow past a porous channel with suction/injection at the walls. Aksoy and Pakdemirli [5] used perturbation method to study analytically, the flow of a third grade fluid in a parallel plate channel filled with porous medium and the plates are at different temperatures. The exact solution for the unsteady flow of an incompressible fluid along an infinite plane porous plate by using translational type of symmetries combined with finite difference method was studied by Fakhar et. al. [6]. Danish and Kumar [7] analysed the steady Poiseuille and Couette-Poiseuille flow of a third grade fluid between parallel plates by similarity transformation. Homotopy perturbation technique was applied to the parallel plate flow and heat transfer of a third grade fluid with constant viscosity by Siddiqui et.al. [8].

In the present study, we extend the work conducted by Beg et. al. [9], where we have changed the geometry of the problem from Darcian permeable half space to a porous channel obeying modified Darcian law for a third grade fluid. Moreover, we consider the fluid to be magnetohydrodynamic, thereby depicting the effect of applied magnetic field on the flow, with the aim of applying the study in biofluid dynamics. Further we perform a heat transfer analysis of the problem. We obtain the governing equations from the Cauchy's momentum equation, the energy equation, the Ohm's law, and the modified Darcy's law for third grade fluid. We implement a suitable finite technique to discretise the governing ordinary differential equations and solve the resulting non-linear algebraic equations by damped Newton's method. We illustrate our obtained results in form of graphs and discuss the effects of the governing flow parameters on the velocity distribution and temperature distribution.

2. Basic Equations

The fundamental equations governing the MHD flow of an incompressible electrically conducting fluid are

$$\nabla \cdot \vec{V} = 0, \tag{1}$$

$$\rho \frac{D\vec{V}}{Dt} = \nabla \cdot \vec{\tau} + \vec{J} \times \vec{B}, \quad \vec{J} = \sigma(\vec{E} + \vec{V} \times \vec{B}), \tag{2}$$

$$\rho C_p \frac{DT}{Dt} = \kappa \nabla^2 T + \vec{\tau} \cdot \nabla \vec{V}, \tag{3}$$

where $\vec{V}$ is the fluid velocity, ρ is the fluid density, σ is the electrical conductivity of the fluid, $\vec{J}$ is the current density, $\vec{E}$ is the total electric field, $\frac{D}{DT}$ is the material time derivative, $\vec{B}$ is the magnetic induction such that $\vec{B} = \vec{B_0} + \vec{b}$ ($\vec{B_0}$ and $\vec{b}$ are applied and induced magnetic fields, respectively), κ is the thermal conductivity, C_p is the specific heat, T is the fluid temperature, and $\vec{\tau}$ is the Cauchy stress tensor.

The constitutive equation for Cauchy stress tensor of a third grade fluid is

$$\begin{aligned}\vec{\tau} = & - pI + \mu A_1 + \alpha_1 A_2 + \alpha_2 A_1^2 + \beta_1 A_3 + \beta_2 (A_1 A_2 + A_2 A_1) \\ & + \beta_3 (trA_1^2) A_1,\end{aligned} \tag{4}$$

where p is the pressure, μ is the dynamic viscosity, α_i's ($i = 1, 2$) and β_j's ($j = 1, 2, 3$) are material constants, and A_i's ($i = 1, 2, 3$) are the Rivlin-Ericksen tensors given by

$$A_1 = \nabla \vec{V} + (\nabla \vec{V})^T,$$

$$A_n = \frac{DA_{n-1}}{Dt} + A_{n-1}(\nabla\vec{V}) + (\nabla\vec{V})^T A_{n-1} \quad \text{for} \quad n \geq 2.$$

The Clausius-Duhem inequality and the result that specific Helmholtz free energy is minimum when the fluid is at rest provide the following constraints [10]

$$\mu \geq 0, \ \ \alpha_1 \geq 0, \ \ |\, \alpha_1 + \alpha_2 \,| \leq \sqrt{24\mu\beta_3}, \ \ \beta_1 = \beta_2 = 0, \ \ \beta_3 \geq 0. \tag{5}$$

The modified Darcian law for flow of a third grade fluid in a porous medium [11] adds the following term in the Cauchy's momentum equation

$$\vec{R} = -\left[\mu + 2\beta_3\left(\frac{du}{dy}\right)^2\right]\frac{\phi\vec{V}}{k}, \tag{6}$$

where ϕ and k are the porosity and intrinsic permeability of the porous medium.

3. Problem Formulation

We consider the two-dimensional steady laminar flow of an electrically conducting non-Newtonian fluid, that obeys third grade fluid model, in a porous channel. We choose the cartesian coordinate system to model the problem with x-axis parallel to the direction of fluid flow, and y-axis being in the transverse direction of the flow. The channel walls are represented by equations $y = \pm h$. We confine our study to the region $0 \leq y \leq h$ by considering the flow to be symmetric about the centreline ($y = 0$) of the channel. The flow is driven by a constant pressure gradient in the flow direction and by the imposition of a uniform magnetic field of strength B_0 along the transverse direction. There is cross flow due to an uniform injection of the fluid at one wall with velocity v_0 and an equal suction at the other wall. Both the walls are maintained at a constant temperature T_w. We neglect the effect of gravity, induced magnetic strength, and buoyant forces in this study.

The schematic configuration of the physical problem is shown in Figure 1. With the above assumptions, the continuity equation and momentum equations those govern the flow are

$$\frac{\partial u}{\partial x} = 0 \qquad \text{or} \qquad u = u(y), \tag{7}$$

$$\mu\frac{d^2u}{dy^2} + \alpha_1 v_0\frac{d^3u}{dy^3} - \rho v_0\frac{du}{dy} + 6\beta_3\left(\frac{du}{dy}\right)^2\frac{d^2u}{dy^2} - \left[\mu + 2\beta_3\left(\frac{du}{dy}\right)^2\right]\frac{\phi u}{k}$$

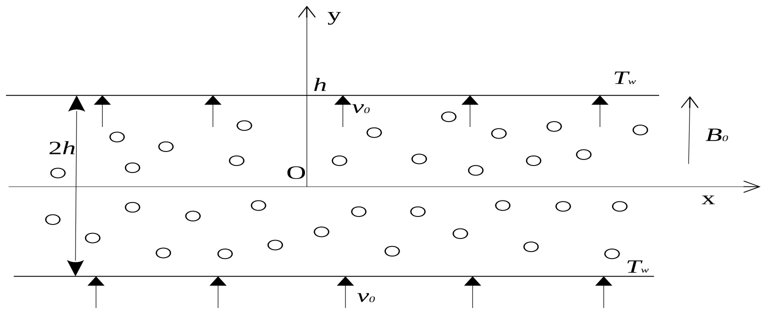

Figure 1: Sketch of the problem

$$-\sigma B_0^2 u - \frac{\partial \hat{p}}{\partial x} = 0, \tag{8}$$

$$\frac{\partial \hat{p}}{\partial y} = 0, \tag{9}$$

where $$\hat{p} = p - (2\alpha_1 + \alpha_2)\left(\frac{du}{dy}\right)^2. \tag{10}$$

The energy equation for the problem under consideration is

$$\rho C_p v_0 \frac{dT}{dy} - \mu\left(\frac{du}{dy}\right)^2 - \alpha_1 v_0 \frac{du}{dy}\frac{d^2u}{dy^2} - 2\beta_3\left(\frac{du}{dy}\right)^4$$
$$-\left[\mu + 2\beta_3\left(\frac{du}{dy}\right)^2\right]\frac{\phi u^2}{k} - \kappa\frac{d^2T}{dy^2} - \sigma B_0^2 u^2 = 0. \tag{11}$$

The boundary conditions for velocity and temperature are prescribed as

$$\begin{aligned} u &= 0,\ T = T_w \text{ at } y = h, \\ u &= U_0(\text{Free stream velocity}),\ \frac{du}{dy} = 0,\ \frac{dT}{dy} = 0 \text{ at } y = 0. \end{aligned} \tag{12}$$

Introducing the following non-dimensional variables

$$\eta = \frac{y}{h}, \quad U = \frac{u}{U_0}, \quad \theta = \frac{T}{T_w}, \tag{13}$$

and substituting in the momentum and energy equations, we obtain

$$\frac{d^2U}{d\eta^2} + \alpha\frac{d^3U}{d\eta^3} - Re_C\frac{dU}{d\eta} + 6\beta\left(\frac{dU}{d\eta}\right)^2\frac{d^2U}{d\eta^2}$$

$$-\left[\frac{\phi}{Da}\left(1+2\beta\left(\frac{dU}{d\eta}\right)^2\right)+Ha^2\right]U+P=0, \tag{14}$$

$$\frac{d^2\theta}{d\eta^2}-Pr.Re_C\frac{d\theta}{d\eta}+Br\left[\left(\frac{dU}{d\eta}\right)^2+\alpha\frac{dU}{d\eta}\frac{d^2U}{d\eta^2}+2\beta\left(\frac{dU}{d\eta}\right)^4\right.$$
$$\left.+\left\{\frac{\phi}{Da}\left(1+2\beta\left(\frac{dU}{d\eta}\right)^2\right)+Ha^2\right\}U^2\right]=0, \tag{15}$$

where $Re_C = \frac{\rho h v_0}{\mu}$ is the cross flow Reynold's number, $\alpha = \frac{\alpha_1 v_0}{\mu h}$ is the visco-elastic parameter, $\beta = \frac{6\beta_3 U_0^2}{\mu h^2}$ is the third grade fluid material parameter, $Da = \frac{k}{h^2}$ is the Darcy's number, $Ha = B_0 h\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann number, $P = -\frac{h^2}{\mu U_0}\frac{d\hat{p}}{dx}$ is the dimensionless pressure gradient, $Pr = \frac{\mu C_p}{\kappa}$ is the Prandtl number, and $Br = \frac{\mu U_0^2}{\kappa T_w}$ is the Brinkman number.
The corresponding boundary conditions in non-dimensional form become

$$\begin{aligned} &U=0, \quad \theta=1 \quad \text{at} \quad \eta=1, \\ &U=1, \quad \frac{dU}{d\eta}=0, \quad \frac{d\theta}{d\eta}=0 \quad \text{at} \quad \eta=0. \end{aligned} \tag{16}$$

4. Numerical Methods

The resulting momentum equation arising from the problem of magnetohydrodynamic flow of a third grade fluid past a porous channel subjected to an external magnetic field is non-linear in nature and does not admit an exact analytical solution. We, therefore, look for a numerical solution of the problem under consideration. For simplicity, better stability, accuracy and efficiency, we use the finite difference technique for discretization. In order to discretize the differential quotients, we opt the following suitable finite difference strategy

$$\begin{aligned} &\eta_i = i\Delta\eta, && i=1,2,...,N \\ &\eta_0 = 0, \qquad \eta_{N+1}=0, \\ &U_i = U(\eta_i), \\ &U_0 = 1, \qquad U_{N+1}=0. \end{aligned}$$

Using Euler's forward finite difference scheme in the boundary condition at the initial point $(i=0)$, we get $U_1 = 1$.

The following central difference scheme is used for discretisation of $\frac{dU}{d\eta}$ and $\frac{d^2U}{d\eta^2}$ at the nodes $i = 1, 2, ..., N$

$$\begin{aligned} \frac{dU}{d\eta} &= \frac{U_{i+1} - U_{i-1}}{2\Delta\eta}, \\ \frac{d^2U}{d\eta^2} &= \frac{U_{i+1} - 2U_i + U_{i-1}}{(\Delta\eta)^2}. \end{aligned}$$

We use the following finite difference technique to discretize $\frac{d^3U}{d\eta^3}$: At $i = 1$,

$$\frac{d^3U}{d\eta^3} = \frac{U_{i+2} - 3U_{i+1} + 3U_i - U_{i-1}}{(\Delta\eta)^3}, \tag{17}$$

At $i = 2, 3, ..., N - 1$,

$$\frac{d^3U}{d\eta^3} = \frac{U_{i+2} - 3U_{i+1} + 3U_i - U_{i-1}}{(\Delta\eta)^3}, \tag{18}$$

At $i = N$,

$$\frac{d^3U}{d\eta^3} = \frac{U_{i+1} - 3U_i + 3U_{i-1} - U_{i-2}}{(\Delta\eta)^3}. \tag{19}$$

The resulting algebraic system of equations is then expressed in the residual form as

$$\mathcal{R}_i = 0 \qquad\qquad i = 1, 2, ..., N. \tag{20}$$

where the residuals are given as follows

$$\begin{aligned} \mathcal{R}_1 &= \frac{\alpha}{(\Delta\eta)^3}(U_3 - 3U_2 + 2) + \frac{(U_2 - 1)}{2(\Delta\eta)^2}(2 + Re_C\Delta\eta) + \frac{6\beta}{4(\Delta\eta)^4}(U_2 - 1)^3 \\ &- \left[\frac{\phi}{2Da(\Delta\eta)^2}\left\{2(\Delta\eta)^2 + \beta(U_2 - 1)^2\right\} + Ha^2\right] + P, \\ \mathcal{R}_m &= \frac{\alpha}{(2\Delta\eta)^3}\left(U_{i+2} - 2U_{i+1} + 2U_{i-1} - U_{i-2}\right) + \frac{1}{(\Delta\eta)^2}\left(U_{i+1} - 2U_i + U_{i-1}\right) \\ &+ \frac{Re_C}{2\Delta\eta}\left(U_{i+1} - U_{i-1}\right) + \frac{6\beta}{4(\Delta\eta)^4}\left(U_{i+1} - U_{i-1}\right)^2\left(U_{i+1} - 2U_i + U_{i-1}\right) \\ &- \left[\frac{\phi}{2Da(\Delta\eta)^2}\left\{2(\Delta\eta)^2 + \beta\left(U_{i+1} - U_{i-1}\right)^2\right\} + Ha^2\right]U_i + P, \\ &\qquad\qquad m = 2, 3, ..., N - 1, \\ \mathcal{R}_N &= \frac{\alpha}{(\Delta\eta)^3}(-3U_N + 3U_{N-1} - U_{N-2}) + \frac{(-2U_N + U_{N-1})}{(\Delta\eta)^2} - \frac{Re_C U_{N-1}}{2\Delta\eta} \end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{6\beta}{4(\Delta\eta)^4}(U_{N-1})^2(-2U_N + U_{N-1}) \\
&- \quad \left[\frac{\phi}{2Da(\Delta\eta)^2}\left\{2(\Delta\eta)^2 + \beta(U_{N-1})^2\right\} + Ha^2\right]U_N + P.
\end{aligned}
$$

In order to obtain the solution of non-linear system of algebraic equations 20, to stabilize the convergence at an early stage of the iteration, and to save computing time, we use the damped Newton's method [12] which is given by

$$
\mathbf{U}^{k+1} = \mathbf{U}^k - \lambda^k J^{-1}(\mathbf{U}^k)\mathcal{R}(\mathbf{U}^k), \qquad k = 0, 1, ...
$$

where $\mathbf{U} = [U_1, U_2, ..., U_N]^T$ represents the column vector of unknowns, $0 < \lambda^k < 1$ is the k^{th} damping parameter which fulfills the criteria $\|\mathcal{R}(\mathbf{U^{k+1}})\| < \|\mathcal{R}(\mathbf{U^k})\|$, and $J(\mathbf{U}^k)$ is the Jacobian matrix evaluated at the k^{th} iterate whose element is given by

$$
J_{ij} = \frac{\partial \mathcal{R}_i^k}{\partial U_j^k}, \quad i,\ j = 1,\ 2,\ ...,\ N. \tag{21}
$$

A good initial guess is necessary for the convergence of the damped Newton's method and very important for the fast solution of the iterative process. The initial solution guess for the axial velocity is approximated according to the boundary conditions. For this problem the convergence of the damped Newton method is achieved when

$$
\|J^{-1}(\mathbf{U}^{k+1})\|_2 - \|J^{-1}(\mathbf{U}^k)\|_2 < 10^{-4}, \qquad k = 0, 1, ... \tag{22}
$$

After $U(\eta)$ is determined, we solve the energy equation subject to the boundary conditions using finite diffference technique. Using the central difference scheme, the heat transfer equation becomes

$$
\begin{aligned}
& (2 - \Delta\eta Pr Re_C)\theta_{i+1} - 4\theta_i + (2 - \Delta\eta Pr Re_C)\theta_{i-1} \\
= \quad & -2(\Delta\eta)^2 Br\left[\left(\frac{dU_i}{d\eta}\right)^2 + \alpha\frac{dU_i}{d\eta}\frac{d^2U_i}{d\eta^2} + 2\beta\left(\frac{dU_i}{d\eta}\right)^4\right. \\
& \left. + \left\{\frac{\phi}{Da}\left(1 + 2\beta\left(\frac{dU_i}{d\eta}\right)^2\right) + Ha^2\right\}U_i^2\right],
\end{aligned} \tag{23}
$$

which produces a tridiagonal system of equations and is solved using the Thomas algorithm. The temperature distribution $\theta(\eta)$ is characterised by the dimensionless parameters α, β, Da, Ha, Pr, Re_C, and Br.

5. Results and Discussion

In this section, we discuss the results of our investigation of the magnetohydrodynamic flow of a third grade fluid in a porous channel under the influence of an externally applied magnetic field and a constant pressure gradient. We produce the computed results in form of graphs of U and θ, which correspond to the axial velocity and temperature, for various governing flow parameters. The effect of the visco-elastic parameter α, the third grade material parameter β, the cross flow Reynold's number Re_C, the Hartmann number Ha, the Darcy's number Da, the Prandtl number Pr, and the Brinkman number Br on the flow is investigated. With a view of illustrating the applicability of this study in the real world, we have modelled blood as the magnetohydrodynamic third grade fluid and the arteries to be the porous channel. For numerical computation it is necessary to assign the values of the dimensionless parameters those govern the blood flow. We are providing a list of standard values of the governing flow parameters that suits the chosen realistic case in Table 1.

Table 1: List of standard values of the parameters.

α	Visco-elastic parameter	0.01
β	Shear thinning parameter	0.005
Da	Darcy's number	0.05
Ha	Hartmann number	2.0
Re_C	Cross flow Reynold's number	0.5
Pr	Prandtl number	7.0
Br	Brinkman number	0.001
ϕ	Porosity of medium	0.2
P	Dimensionless pressure gradient	1.0

For the sake of comparison we have also examined the results for different values of the parameters and these deviations are mentioned in the figures. Although computation for several combinations of flow parameters were performed, only some key graphs are provided in this paper.

Figures 2 and 3 depict the effect of the visco-elastic parameter α and the shear thinning parameter β on the blood flow, respectively. These figures show that the velocity increases with the increase in the values of these non-Newtonian parameters, which is physically relevant. Red blood cells (RBCs), the major constituent of blood, have a tendency to deform and aggregate, and this is the contributing factor to the visco-elastic behavior of blood. As the visco-elasticity parameter increases, the shear rate increases leading to the increase in deformation, rearrangement and orientation of red blood cells, thereby, increasing

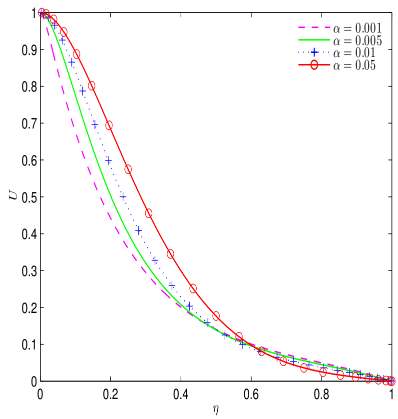

Figure 2: Axial velocity distribution U for different values of α.

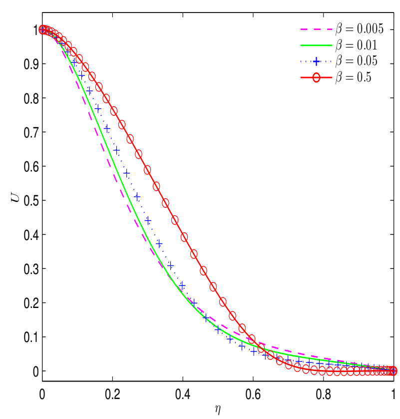

Figure 3: Axial velocity distribution U for different values of β.

the blood velocity in the channel. Due to the shear thinning of blood, viscosity reduces and hence the velocity increases. The nature of the graph for blood velocity with varying cross flow Reynold's number or the suction parameter is shown in Fig. 4. Figure 5 illustrates that the fluid velocity decreases with an increase in the magnetic parameter Ha. This result can be implemented as a control measure in handling blood flow during surgeries and other cases where the biological fluid flow needs to be checked. There is a corresponding rise in the fluid velocity as the porous permeability parameter, Da rises as observed in Fig. 6. This reveals the fact that homogeneity of the channel can be achieved by increasing the porous permeability of the medium.

Further, it is observed from Fig. 7 that there is an increase in the fluid temperature with increasing Da, which is a consequence of the increased diffusion of heat within the channel. The variation in the temperature with varying visco-elastic parameter, α and third grade fluid material parameter, β is shown in Fig. 8 and Fig. 9, respectively. The curves in these figures convey that there is a rise in temperature when the values of these parameters are raised. As the suction parameter, Re_C increases, temperature of the fluid increases which is given in Fig. 10. The response of fluid temperature to the magnetic field parameter, Ha is represented in Fig. 11. It is observed that an increase in the magnetic field intensity results in the increase in temperature of the fluid which is due to Ohmic heating phenomenon. This fact can be used in the clinical cases where the temperature of the body fluid needs to be raised, such as

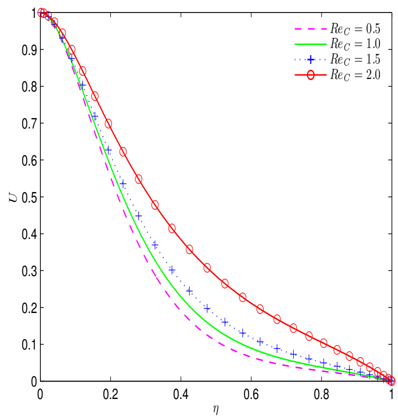

Figure 4: Axial velocity distribution U for different values of Re_C.

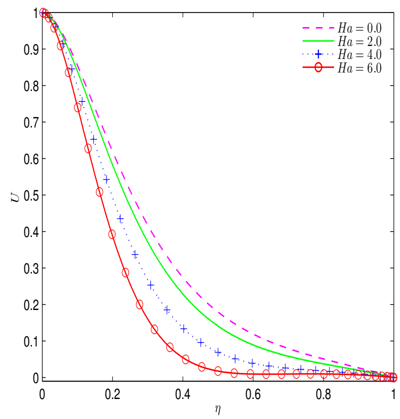

Figure 5: Axial velocity distribution U for different values of Ha.

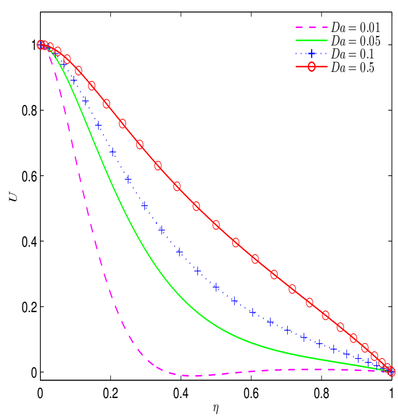

Figure 6: Axial velocity distribution U for different values of Da.

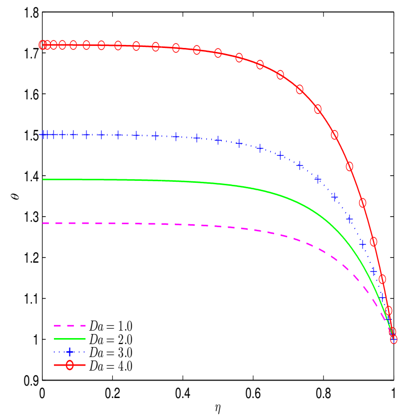

Figure 7: Temperature distibution θ for different values of Da.

hyperthermia for the treatment of cancer/ malignant tumours. The effect of Prandtl number and Brinkman number on the temperature profile are presented in Fig.12 and Fig.13, respectively. The case of blood flow is dominated by momentum diffusivity. It is evident from Fig.12 that as the viscous diffusion rate increases, the temperature increases. An increase in the Brinkman number leads to an increase in the temperature distribution within the flow channel

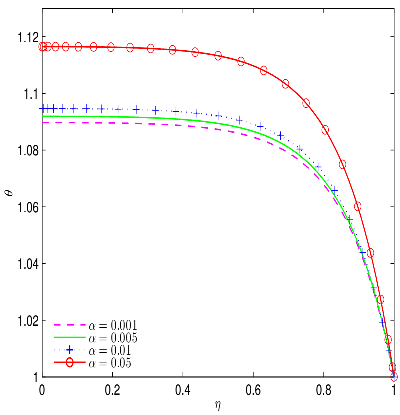

Figure 8: Temperature distibution θ for different values of α.

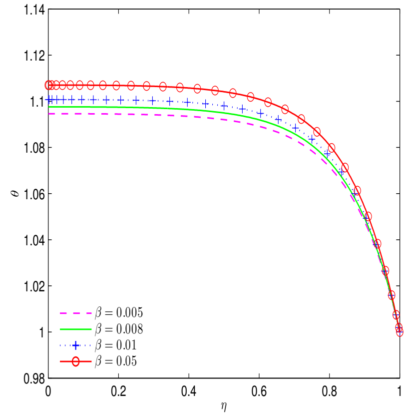

Figure 9: Temperature distibution θ for different values of β.

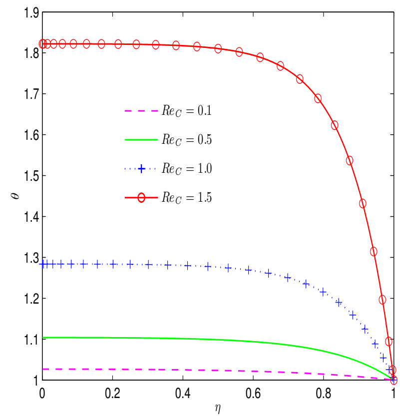

Figure 10: Temperature distibution θ for different values of Re_C.

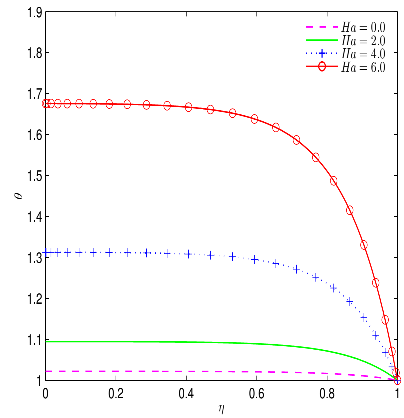

Figure 11: Temperature distibution θ for different values of Ha.

due to the fact that viscous heating is the source of heat that is stored in the channel and as this source of heat increases it enhances the temperature in the fluid. The volumetric rate of flow is given by

$$Q = 2\int_0^1 U d\eta. \tag{24}$$

The volumetric rate of flow increases with increasing α, β and Da and decreases

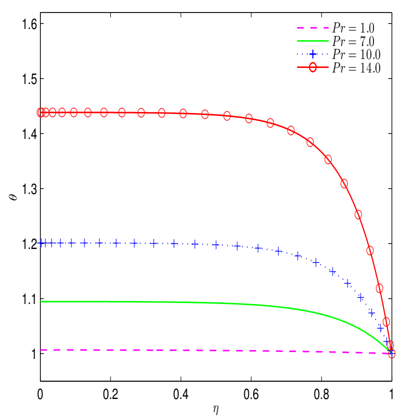

Figure 12: Temperature distibution θ for different values of Pr.

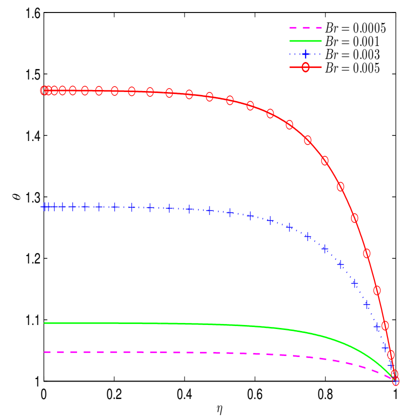

Figure 13: Temperature distibution θ for different values of Br.

with increasing magnetic parameter Ha as shown in Fig. 14.

The dimensionless wall shear stress is calculated as

$$\tau^\star = \frac{\tau_{xy}\mid_{y=h}}{\frac{\mu U_0}{h}} = \frac{dU}{d\eta} + \alpha \frac{d^2U}{d\eta^2} + 2\beta \left(\frac{dU}{d\eta}\right)^3 \mid_{\eta=1}. \tag{25}$$

The wall shear stress (WSS) is the shearing stress generated on the wall of the channel. There is a possibility of rupturing of the arterial wall or damage to the constituents of blood when the wall shear stress exceeds a certain limit. Therefore the investigation of WSS is important in the study of blood flow in arteries. Figure 15 depicts the influence of various flow parameters on the WSS. The negative sign of the WSS signifies that the wall exerts stress on the fluid. The magnitude of WSS decreases for increasing values of α, β and Ha whereas it increases for increasing porous permeability parameter Da.

6. Conclusion

The objective of the present paper is to study the effect of externally applied magnetic field on the magnetohydrodynamic flow of blood, modelled as a third grade fluid, in a porous channel obeying modified Darcy's resistance law, and analyse the heat transfer phenomenon associated with the problem. This study is motivated towards the human blood flow dynamics in thin arteries where

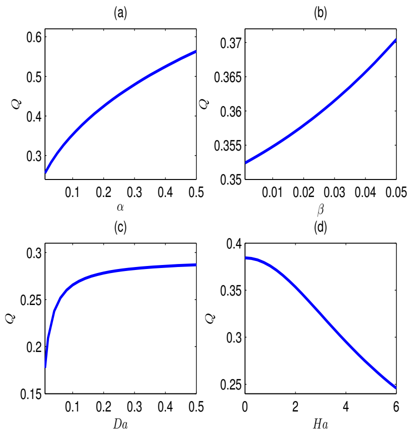

Figure 14: Volume rate of flow for different flow parameters.

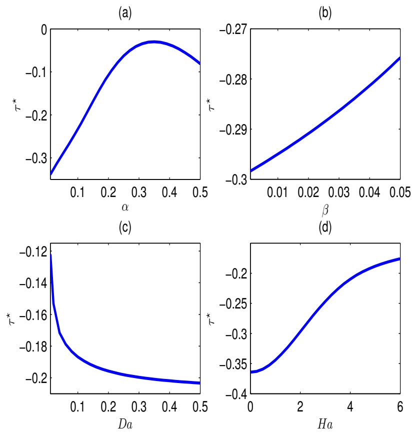

Figure 15: Wall shear stress for different flow parameters.

behavior of blood is non-Newtonian and mostly shear thinning. A suitable finite difference technique followed by the damped Newton's method is used to solve the non-linear Cauchy's momentum equation, and the energy equation is solved by Thomas algorithm. The effect of dimensionless governing flow parameters on the axial velocity distribution and temperature distribution are presented in form of graphs. The results obtained in the present study are listed below

1. Axial velocity and temperature of fluid in the non-Darcian porous channel is maximum at the centerline and decreases gradually while approaching the wall.

2. Velocity of the flow and temperature within the channel increases with increasing visco-elasticity and shear-thinning behavior of the fluid.

3. As the suction velocity and permeability of the wall increases it raises the fluid velocity as well as the temperature.

4. An increasing magnetic parameter ceases the flow velocity whereas raises the temperature.

5. Temperature of the biofluid within the channel increases with increasing Prandtl number and increasing Brinkman number.

6. Wall shear stress decreases with increasing visco-elasticity, shear-thinning and magnetic intensity, whereas, it increases with increasing porous permeability.

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