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FUZZY TOPSIS APPROACH TO IDENTIFY THE FLOOD VULNERABILITY REGION IN SOUTH CHENNAI

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Abstract: The technique for order preference by similarity to ideal solution (TOPSIS) is a well-known and very simple ranking method for solving Multi Attribute Decision Making (MADM). In many real life situations, the decision data of human judgments are often vague so that the conventional ways of using crisp numbers are inadequate. Also, using fuzzy numbers such as triangular, trapezoidal, etc. are not suitable when the uncertainties arise at six different points. Therefore, Hexagonal fuzzy number and its arithmetic operations, linguistic values are used to extend the TOPSIS method to analyze the flood vulnerability region in south Chennai.

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Key Words: linguistic variable, fuzzy sets and fuzzy logic, fuzzy-TOPSIS, hexagonal fuzzy number

1. Introduction

The fuzzy set theory was introduced by Zadeh LA [17] to deal with vagueness

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and uncertainty of the human thoughts and language in making proper decisions. To determine such vagueness, fuzzy set theory has been combined with many Multiple attribute decision making (MADM) methods. MADM models such as DEMATEL, VIKOR, ANP, TOPSIS etc, are powerful tools for the evaluation of service quality in different fields. The technique for order preference by similarity to ideal solution (TOPSIS), is a simple ranking method for solving Multi Attribute Decision Making (MADM), was proposed by Hwang & Yoon (1981) [6]. According to this technique, the positive ideal solution is a solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria [6,7,11]. In decision making problems experts might provide uncertain linguistic terms to express their view when they have no clear information. The uncertain linguistic term is frequently used as input in decision analysis. In the last decade, some fuzzy TOPSIS methods have been constructed for creating application in different areas such as order selection when orders exceed production [11], Taiwan's Air Force Academy for choosing optimal initial training aircraft [15], supplier selection problem in supply chain system [1], computer retailers in a purchasing decision problem [6], the best place to locate a manufacturing facility [10], Benefits of the practices of Islam [2] and Impact of Periyar's Philosophy in the Society [4].

This present study is interested to investigate the flood vulnerability region in south Chennai, Tamil Nadu. Some of the researchers have used the Multi criteria decision making model to analyze the flood vulnerability region in the different parts of the continent. Recently researchers have used the different fuzzy multi criteria decision making model to analyze water resources of red river valley region [13], assess flood disaster risk in Kelantan, Malaysia [9], to decrease the evacuation time of people from the affected areas before flood occurrence [13], for development of flood risk management plans [14], for vulnerability measurement that incorporates both socio-economic and flood [8]. From this review, it is observed that the effective research can be done to identify the vulnerability region in Chennai due to torrential rainfall.

2. Preliminaries

Definition 2.1. A fuzzy set $\widetilde{A}$ in X is characterized by a membership function $\mu_{\widetilde{A}}(x)$ which associates each point in X, to a real number in the interval $[0,1]$. The value of $\mu_{\widetilde{A}}(x)$ represents "grade of membership"of $x \in \mu_{\widetilde{A}(x)}$. More general representation for a fuzzy set is $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x))\}$

Definition 2.2. A fuzzy set $\widetilde{A}$ defined on the set of real numbers $\mathbb{R}$ is said to be a fuzzy number if its membership function $\widetilde{A} : \mathbb{R} \to [0,1]$ has the following characteristics.

(i) $\widetilde{A}$ is convex
$\mu_{\widetilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\widetilde{A}}(x_1),\ \mu_{\widetilde{A}}(x_2)),\ \forall x \in [0,1],\ \lambda \in [0,1]$

(ii) $\widetilde{A}$ is normal, (i.e.) $\exists$ an $x \in \mathbb{R}$ such that if $\max \mu_{\widetilde{A}}(x) = 1$.

(iii) $\widetilde{A}$ is piecewise continuous.

Definition 2.3. A Hexagonal fuzzy number [12] $\widetilde{H}$ can be defined as $(a_1, a_2, a_3, a_4, a_5, a_6)$, and the membership function $\mu_{\widetilde{H}}(x)$ is defined as

$$\mu_{\widetilde{A}}(x) = \begin{cases} \frac{1}{2}\frac{x-a_1}{a_2-a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}\frac{x-a_2}{a_3-a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 1, & \text{for } a_3 \leq x \leq a_4 \\ 1 - \frac{1}{2}\frac{x-a_4}{a_5-a_4}, & \text{for } a_4 \leq x \leq a_5 \\ \frac{1}{2}\frac{a_6-x}{a_6-a_5}, & \text{for } a_5 \leq x \leq a_6 \\ 0, & \text{elsewhere.} \end{cases}$$

Definition 2.4. A Linguistic variable / term is variable whose value is not crisp number but word or sentence linguistic in a natural language.

Definition 2.5. Let $\widetilde{H_1} = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\widetilde{H_2} = (b_1, b_2, b_3, b_4, b_5, b_6)$ be two Hexagonal fuzzy numbers. The addition, subtraction, multiplication operations of $\widetilde{H_1}$ and $\widetilde{H_2}$, denoted by $\widetilde{H_1} \oplus \widetilde{H_2}$, $\widetilde{H_1} \ominus \widetilde{H_2}$ and $\widetilde{H_1} \otimes \widetilde{H_2}$ respectively, yield another Hexagonal fuzzy number.

(i) $\widetilde{H_1} \oplus \widetilde{H_2} = a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6$

(ii) $\widetilde{H_1} \ominus \widetilde{H_2} = a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 + b_3, a_5 + b_2, a_6 + b_1$

(iii) $k \otimes \widetilde{H_1} = ka_1, ka_2, ka_3, ka_4, ka_5, ka_6,\ k > 0$ a crisp number

(iv) $\widetilde{H_1} \otimes \widetilde{H_2} = a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5, a_6 \times b_6$

3. Extended TOPSIS Method

This section provides the extension of TOPSIS under fuzzy environment.

Step-1: Construct the fuzzy decision matrix and determine the fuzzy weight for each criterion. Let us consider that there are K experts in the decision group. Then the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as

$$\tilde{x}_{ij} = \frac{1}{K}\left[x_{ij}^{1} + x_{ij}^{2} + \ldots + x_{ij}^{K}\right]$$
$$\tilde{w}_{ij} = \frac{1}{K}\left[w_{ij}^{1} + w_{ij}^{2} + \ldots + w_{ij}^{K}\right]$$

where $\tilde{x}_{ij}^{K}$ and $\tilde{w}_{ij}^{K}$ are the rating and the weight of the K^{th} decision maker. A fuzzy multi criteria group decision-making problem which can be concisely expressed in matrix format as,

$$\widetilde{D} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

$$\widetilde{W} = [\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n]$$

where $\tilde{x}_{ij}$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$ and $\tilde{w}_j$ are linguistic variables, which are described by Hexagonal fuzzy number in the following Table

Table 1: Linguistic variables for the Alternative's and Weighting's Criteria

Linguistic Variable	CODE	Fuzzy Value
No Influence	NI	(0, 0, 0, 0.06, 0.12, 0.18)
Very Low	VL	(0.06, 0.12, 0.18, 0.24, 0.3, 0.36)
Low	L	(0.24, 0.3, 0.36, 0.42, 0.48, 0.54)
Medium	M	(0.42, 0.48, 0.54, 0.6, 0.66, 0.72)
High	H	(0.6, 0.66, 0.72, 0.78, 0.84, 0.9)
Very High	VH	(0.78, 0.84, 0.9, 1, 1, 1)

Step 2 Normalized decision matrix $\tilde{R} = [\tilde{r}_{ij}]_{m\times n}$ can be computed from $\tilde{D}$ in two ways,

$$\tilde{r}_{ij} = (a_1,\, a_2,\, a_3,\, a_4,\, a_5,\, a_6) \div \max_i a_6,\; j \in B$$
$$\tilde{r}_{ij} = \min_i a_1 \div (a_1,\, a_2,\, a_3,\, a_4,\, a_5,\, a_6),\; j \in C$$

where B and C are in the respective of cost criteria and benefit criteria.

Step 3 Construct the weighted normalized fuzzy decision matrix as

$$\widetilde{V} = [\tilde{v}_{ij}]_{n\times m},\ i = 1, 2, \ldots, m,\ j = 1, 2, \ldots, n, \text{ where } \tilde{v}_{ij} = \tilde{r}_{ij} \otimes \tilde{w}_j.$$

Step 4 Determine Fuzzy Positive Ideal Solution (FPIS) and Fuzzy Negative Ideal Solution (FNIS). FPIS A^+ and FNIS A^- are defined as,

$$A^+ = \left(\tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_n^+\right) \text{ and } A^- = \left(\tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^-\right),$$

$$\text{where } \tilde{v}_j^+ = \left(max(\tilde{v}^6), max(\tilde{v}6), \ldots, max(\tilde{v}^6)\right) \text{ and}$$
$$\tilde{v}_j^- = \left(min(\tilde{v}^1), min(\tilde{v}^1), \ldots, min(\tilde{v}^1)\right)\ \forall\ j = 1, 2, \ldots, n.$$

Step 5 Calculate the distance of each alternative from FPIS and FNIS, respectively.
The distance of each alternative from A^+ and A^- is given by

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+),\ j = 1, 2, \ldots, m$$
$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-),\ j = 1, 2, \ldots, m.$$

The distance between two fuzzy number is given by

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{6}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + \ldots + (a_6 - b_6)^2\right]}$$

Step 6 Calculate the closeness coefficient of each alternative.

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-}\ \forall\ i = 1, 2, \ldots, m.$$

Step 7 According to the closeness coefficient, the ranking order of all alternatives can be determined.

4. Adaptation of the Problem to the Proposed Model

Chennai is the bustling south Indian metropolis. It is a topographically flat city with altitude ranging from 2 meters to 15 meter above sea level. The city is increasingly vulnerable to flooding because of torrential of rainfall in the mid of November of every year. According to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, extreme weather events (the mid-November rains) are expected to increase in the coming years. However, it is not as if the annual number of depressions, cyclonic storms and severe cyclonic storms in the Bay of Bengal region have increased over the years. To witness this, Most of the Chennai region was being effected extremely due to torrential rainfall and cyclone in the year 2015 and 2016. Therefore, this present study investigates the flood vulnerability region in Chennai. Here, the following 12 regions of south Chennai and the important factors are chosen for our study:

R_1-Tambaram / R_2-Vandalur / R_3-Velachery / R_4-Palavakkam / R_5-Thoraipakkam / R_6-Pallikaranai / R_7-Medavakkam / R_8-Kelambakkam/ R_9-Pallavaram / R_{10}-Perungudi / R_{11}-Adayar / R_{12}-Sholinganallur

C_1-Population growth / C_2-Residensial and industrial Area / C_3-population density C_4-Rain fall C_5- Number of flood and disaster prevention institution (number/ years) C_6-River / lake important ratio.

By using the proposed algorithm, First, the Decision Matrix was constructed with aid of three different experts. The decision-makers use linguistic rating variables and linguistic weighting variables (shown in Table-1) to assess the importance of the criteria to evaluate the rating of alternatives with respect to each criterion. Then, transform the linguistic variable in to Hexagonal fuzzy number and take the average of them. Next, the decision maatrix is normalized and weighted normalized matrix is obtained by multiplying the weighted matrix with Normalized matrix. Finally, the distance of each alternative from FPIS and FNIS and closeness coefficient of each alternative are computed as below in table-2,

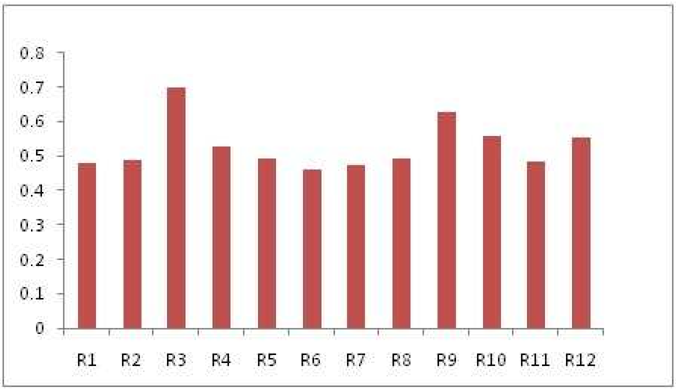
5. Conclusion

According to the closeness coefficient, the ranking order of the flood vulnerability regions are $R_3 > R_9 > R_{10} > R_{12} > R_4 > R_8 > R_5 > R_2 > R_{11} > R_1 > R_7 > R_6$. From this analysis, it is observed that Velachery is the most vulnerable region to flooding due to torrential of rainfall.

Table 2: Fuzzy distance measurement and closeness coefficient

	d^+	d^-	CC_i	Rank
R_1	3.989139	3.654021	0.478077	10
R_2	3.933049	3.753977	0.488352	8
R_3	2.312773	5.393135	0.69987	1
R_4	3.653586	4.097561	0.528639	5
R_5	3.888774	3.761101	0.491655	7
R_6	4.066815	3.494014	0.46212	12
R_7	4.071332	3.649939	0.472712	11
R_8	3.912927	3.816726	0.493777	6
R_9	2.802586	4.775707	0.630182	2
R_{10}	3.426317	4.340209	0.558835	3
R_{11}	3.90235	3.668547	0.484559	9
R_{12}	3.483342	4.350468	0.555345	4

Figure 1: Closeness coefficients



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