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OPERATION ON SEMI GENERALIZED OPEN SETS WITH ITS SEPARATION AXIOMS

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Abstract: The aim of this paper is to introduce the concept of an operation γ on the class of all semi-generalized open sets in topological spaces. Using this operation, we define a new concept called semi-generalized-γ-open (sg-γ-open) sets and study some of their related properties. We found that the relation between this new concept and sg-open set are independent. In addition, we study some separation axioms called sg-γ-T_i for $i = 0, 1, 2$. Some basic properties and relations of these separation axioms are obtained.

AMS Subject Classification: 54A05, 54A10, 54D10
Key Words: Operation γ on τ_{sg}, sg-γ-open sets, sg-γ-T_i spaces ($i \in \{0, 1, 2\}$)

1. Introduction and Preliminaries

Levine [7] introduced the concept of semiopen sets in topology. In 1987, Bat-

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tacharyya and Lahiri [3] used semiopen sets to define the notion of semi-generalized closed sets. Kasahara [8] introduced the notion of an α operation approaches on a class τ of sets and studied the concept of α-continuous functions with α-closed graphs and α-compact spaces. After this, Jankovic [6] introduced the concept of α-closure of a set in X via α-operation and investigated further characterizations of function with α-closed graph. Later, Ogata [10] defined and studied the concept of γ-open sets, and applied it to investigate operation-functions and operation-separation. Recently, several researchers developed many concepts of operation γ in a space (X, τ). Krishnan, Ganster and Balachandran [9] introduced and studied the concept of the operation γ on the class of all semiopen sets of (X, τ), and defined the notion of semi γ-open sets and investigated some of their properties. An, Cuong and Maki [1] defined and investigated an operation γ on the class of all preopen sets of (X, τ) and introduced the notion of pre-γ-open sets, and developed some of their properties. Tahiliani [11] defined an operation γ on the class of all β-open sets of (X, τ), and described the notion of β-γ-open sets. Carpintero, Rajesh and Rosas [4] studied the operation γ on the class of all b-open sets of (X, τ), and defined the notion of b-γ-open sets. Asaad [2] defined the notion of an operation γ on the class of all generalized open sets in (X, τ) and study some of its applications.

The aim of this paper is to define the concept of an operation γ on τ_{sg} and to introduce the notion of sg-γ-open sets of a topological space (X, τ) by using the operation γ on τ_{sg}. Also, some notions of sg-γ-open sets with their relationships are investigated. In Section 3, sg-γ-T_i spaces where $i \in \{0, 1, 2\}$ by utilizing the operation γ on τ_{sg} are introduced and studied.

In this study, the space (X, τ) represent nonempty topological space on which no separation axioms are assumed, unless otherwise mentioned. Let S be a subset of a topological space (X, τ). The closure of S and the interior of S in (X, τ) are denoted by $Cl(S)$ and $Int(S)$, respectively. A subset S of a space X is said to be semiopen [7] if $S \subseteq Cl(Int(S))$. The complement of a semiopen set is said to be semiclosed [5]. We denote by $SO(X)$ the set of all semiopen sets in (X, τ). The semi-closure of S is defined as the intersection of all semiclosed sets containing S and it is denoted by $sCl(S)$ [5]. A subset S of a space (X, τ) is said to be semi-generalized closed (in short sg-closed) [3] if $sCl(S) \subseteq U$ whenever $S \subseteq U$ and U is a semiopen set in X. The complement of an sg-closed set of X is sg-open. The family of all sg-open subsets of a space (X, τ) is denoted by τ_{sg}. In general, every semiclosed set of a space X is sg-closed. A space (X, τ) is semi-$T_{\frac{1}{2}}$ [3] if every sg-closed subset of X is semiclosed. A topological space (X, τ) is semi-$T_{\frac{1}{2}}$ if and only if $\tau_{sg} = SO(X)$ [3].

An operation γ on $SO(X)$ on X is a mapping $\gamma\colon SO(X) \to P(X)$ such that $U \subseteq \gamma(U)$ for each $U \in SO(X)$, where $\gamma(U)$ denotes the value of γ at U and $P(X)$ is the power set of X. A nonempty subset S of a space (X, τ) with an operation γ on $SO(X)$ is said to be semi γ-open [9] if for each $x \in S$, there exists a semiopen set U containing x such that $\gamma(U) \subseteq S$. The complement of a semi γ-open subset of a space X as semi γ-closed. The family of all semi γ-open sets of a space (X, τ) is denoted by $SO(X)_\gamma$. A point $x \in X$ is in the semi γ-closure [9] of a set $S \subseteq X$ if $\gamma(U) \cap S \neq \phi$ for each semiopen set U containing x. We denote by $sCl_\gamma(S)$ the set of all semi γ-closure points of A which is called semi γ-closure of S. A subset S of a topological space (X, τ) is called semi γ-g.closed [9] if $sCl_\gamma(S) \subseteq U$ whenever $S \subseteq U$ and U is a semi γ-open set in (X, τ). A topological space (X, τ) with an operation γ on $SO(X)$ is called (i) semi γ-T_0 [9] if for any two points x, y in X such that $x \neq y$, there exists a semiopen set U such that $x \in U$ and $y \notin \gamma(U)$ or $y \in U$ and $x \notin \gamma(U)$, (ii) semi γ-T_1 [9] if for any two points x, y in X such that $x \neq y$, there exist two semiopen sets U and V containing x and y respectively such that $y \notin \gamma(U)$ and $x \notin \gamma(V)$, (iii) semi γ-T_2 [9] if for any two points x, y in X such that $x \neq y$, there exist two semiopen sets U and V containing x and y respectively such that $\gamma(U) \cap \gamma(V) = \phi$ and (iv) semi γ-$T_{\frac{1}{2}}$ [9] if every semi γ-g.closed set in X is semi γ-closed.

2. sg-γ-Open Sets

Definition 2.1. An operation γ on τ_{sg} is a mapping $\gamma\colon \tau_{sg} \to P(X)$ such that $U \subseteq \gamma(U)$ for every $U \in \tau_{sg}$. From this, for any operation $\gamma\colon \tau_{sg} \to P(X)$, we have $\gamma(X) = X$. A nonempty set S of X is called sg-γ-open if for each $x \in S$, there exists an sg-open set U such that $x \in U$ and $\gamma(U) \subseteq S$. The complement of an sg-γ-open set of X is sg-γ-closed. Assume that the empty set ϕ is also sg-γ-open set for any operation $\gamma\colon \tau_{sg} \to P(X)$.

The family of all sg-γ-open subsets of a space (X, τ) is denoted by $\tau_{sg\gamma}$.

Theorem 2.2. *The union of any class of sg-γ-open sets in a topological space X is sg-γ-open.*

Proof. Let $x \in \bigcup_{\lambda\in\Lambda}\{A_\lambda\}$, where $\{A_\lambda\}_{\lambda\in\Lambda}$ be a class of sg-γ-open sets in X. Then $x \in A_\lambda$ for some $\lambda \in \Lambda$. Since A_λ is sg-γ-open set in X, then there exists a sg-open set V such that $x \in V \subseteq \gamma(V) \subseteq A_\lambda \subseteq \bigcup_{\lambda\in\Lambda}\{A_\lambda\}$. Therefore, $\bigcup_{\lambda\in\Lambda}\{A_\lambda\}$ is sg-γ-open set in X. □

Example 2.3. The intersection of any two sg-γ-open sets in (X,τ) is generally not an sg-γ-open set. To see this, let $X=\{a,b,c\}$ and $\tau=P(X)=\tau_{sg}$. Let $\gamma\colon \tau_{sg}\to P(X)$ be an operation on τ_{sg} defined as follows:
For every $S\in\tau_{sg}$

$$\gamma(S)=\begin{cases} S & \text{if } S\neq\{c\}\\ \{b,c\} & \text{if } S=\{c\}\end{cases}$$

Thus, $\tau_{sg\gamma}=P(X)\backslash\{c\}$. Then $\{a,c\}\in\tau_{sg\gamma}$ and $\{b,c\}\in\tau_{sg\gamma}$, but $\{a,c\}\cap\{b,c\}=\{c\}\notin\tau_{sg\gamma}$.

Remark 2.4. Since the union of two sg-open sets is generally not an sg-open set. So the concept of sg-open set and sg-γ-open set are independent (That is, $\tau_{sg}\neq\tau_{sg\gamma}$). To show this, let the space (X,τ) as in Example 2.3, then the set $\{c\}$ is sg-open, but it is not sg-γ-open.

Definition 2.5. A topological space (X,τ) with an operation γ on τ_{sg} is called sg-γ-regular if for each $x\in X$ and for each sg-open set U containing x, there exists an sg-open set W such that $x\in W$ and $\gamma(W)\subseteq U$.

Theorem 2.6. *Let (X,τ) be a topological space and $\gamma\colon\tau_{sg}\to P(X)$ be an operation on τ_{sg}. Then the following conditions are equivalent:*

1. $\tau_{sg}\subseteq\tau_{sg\gamma}$.
2. *(X,τ) is an sg-γ-regular space.*
3. *For every $x\in X$ and for every sg-open set U of (X,τ) containing x, there exists an sg-γ-open set W of (X,τ) containing x such that $W\subseteq U$.*

Proof. (1) $\Rightarrow$ (2) Let $x\in X$ and U be an sg-open set in X such that $x\in U$. It follows from assumption that U is an sg-γ-open set. This implies that there exists an sg-open set W such that $x\in W$ and $\gamma(W)\subseteq U$. Therefore, the space (X,τ) is sg-γ-regular. (2) $\Rightarrow$ (3) Let $x\in X$ and U be an sg-open set in (X,τ) containing x. Then by (2), there is an sg-open set W such that $x\in W\subseteq\gamma(W)\subseteq U$. Again, by using (2) for the set W, it is shown that W is sg-γ-open. Hence W is an sg-γ-open set containing x such that $W\subseteq U$. (3) $\Rightarrow$ (1) By applying the part (3) and Theorem 2.2, it follows that every sg-open set of X is sg-γ-open in X. That is, $\tau_{sg}\subseteq\tau_{sg\gamma}$. □

Remark 2.7. Since every semiopen set is sg-open. Then it is easy to show that every semi γ-open set is sg-γ-open (this means that $SO(X)_\gamma\subseteq\tau_{sg\gamma}$), but the converse is not true in general. For instance, consider the space $X=\{a,b,c\}$ and $\tau=\{\phi,X,\{a\},\{b,c\}\}=SO(X)$. Then $\tau_{sg}=P(X)$. Define an operation $\gamma\colon\tau_{sg}\to P(X)$ by $\gamma(S)=S$ for all $S\in\tau_{sg}$. Here, $\tau_{sg\gamma}=P(X)$ and $SO(X)_\gamma=$

$SO(X)$. Then, the set $\{b\} \in \tau_{sg\gamma}$, but the set $\{b\} \notin SO(X)_\gamma$. This shows that $\tau_{sg\gamma} \nsubseteq SO(X)_\gamma$.

Lemma 2.8. *If the space (X, τ) is semi-$T_{\frac{1}{2}}$, then the concept of sg-γ-open set and semi γ-open set coincide (That is $\tau_{sg\gamma} = SO(X)_\gamma$).*

Proof. Follows from their definitions and the fact that $\tau_{sg} = SO(X)$ since (X, τ) is semi-$T_{\frac{1}{2}}$. □

Definition 2.9. Let (X, τ) be any topological space. An operation γ on τ_{sg} is called

1. sg-open if for each $x \in X$ and for every sg-open set U containing x, there exists an sg-γ-open set W containing x such that $W \subseteq \gamma(U)$.
2. sg-regular if for each $x \in X$ and for every pair of sg-open sets U_1 and U_2 such that both containing x, there exists an sg-open set W containing x such that $\gamma(W) \subseteq \gamma(U_1) \cap \gamma(U_2)$.

Proposition 2.10. *Let a mapping γ be sg-regular operation on τ_{sg}. If the subsets S and T are sg-γ-open in a topological space (X, τ), then $S \cap T$ is also sg-γ-open set in (X, τ).*

Proof. Suppose $x \in S \cap T$ for any sg-γ-open subsets S and T in (X, τ) both containing x. Then there exist sg-open sets U_1 and U_2 such that $x \in U_1 \subseteq S$ and $x \in U_2 \subseteq T$. Since γ is a sg-regular operation on τ_{sg}, then there exists a sg-open set W containing x such that $\gamma(W) \subseteq \gamma(U_1) \cap \gamma(U_2) \subseteq S \cap T$. Therefore, $S \cap T$ is sg-γ-open set in (X, τ). □

By applying Proposition 2.10, it is easy to show that $\tau_{sg\gamma}$ forms a topology on X for any sg-regular operation γ on τ_{sg}.

Definition 2.11. The point $x \in X$ is in the sg-closure$_\gamma$ of a set S if $\gamma(U) \cap S \neq \phi$ for each sg-open set U containing x. The set of all sg-closure$_\gamma$ points of S is called sg-closure$_\gamma$ of S and is denoted by $sgCl_\gamma(S)$.

Definition 2.12. Let S be any subset of a topological space (X, τ) and γ be an operation on τ_{sg}. The sg-γ-closure of S is defined as the intersection of all sg-γ-closed sets of X containing S and it is denoted by $sg_\gamma Cl(S)$. That is,

$$sg_\gamma Cl(S) = \bigcap\{F : S \subseteq F,\ X\backslash F \in \tau_{sg\gamma}\}.$$

Theorem 2.13. *Let S be any subset of a topological space (X, τ) and γ be an operation on τ_{sg}. Then $x \in sg_\gamma Cl(S)$ if and only if $S \cap U \neq \phi$ for every sg-γ-open set U of X containing x.*

Proof. Let $x \in sg_\gamma Cl(S)$ and let $A \cap U = \phi$ for some sg-γ-open set U of X containing x. Then $S \subseteq X \backslash U$ and $X \backslash U$ is sg-γ-closed set in X. So $sg_\gamma Cl(S) \subseteq X \backslash U$. Thus, $x \in X \backslash U$. This is a contradiction. Hence $S \cap U \neq \phi$ for every sg-γ-open set U of X containing x.

Conversely, suppose that $x \notin sg_\gamma Cl(S)$. So there exists an sg-γ-closed set F such that $S \subseteq F$ and $x \notin F$. Then $X \backslash F$ is an sg-γ-open set such that $x \in X \backslash F$ and $S \cap (X \backslash F) = \phi$. Contradiction of hypothesis. Therefore, $x \in sg_\gamma Cl(S)$. □

Lemma 2.14. *The following statements are true for any subsets A and T of a topological space (X, τ) with an operation γ on τ_{sg}.*

1. *$sg_\gamma Cl(S)$ is sg-γ-closed set in X and $sgCl_\gamma(S)$ is sg-closed set in X.*
2. *$S \subseteq sgCl_\gamma(S) \subseteq sg_\gamma Cl(S)$.*
3. *$sg_\gamma Cl(\phi) = sgCl_\gamma(\phi) = \phi$ and $sg_\gamma Cl(X) = sgCl_\gamma(X) = X$.*
4. *(a) S is sg-γ-closed if and only if $sg_\gamma Cl(S) = S$ and,*
   (b) S is sg-γ-closed if and only if $sgCl_\gamma(S) = S$.
5. *If $S \subseteq T$, then $sg_\gamma Cl(S) \subseteq sg_\gamma Cl(T)$ and $sgCl_\gamma(S) \subseteq sgCl_\gamma(T)$.*
6. *(a) $sg_\gamma Cl(S \cap T) \subseteq sg_\gamma Cl(S) \cap sg_\gamma Cl(T)$ and,*
   (b) $sgCl_\gamma(S \cap T) \subseteq sgCl_\gamma(S) \cap sgCl_\gamma(T)$.
7. *(a) $sg_\gamma Cl(S) \cup sg_\gamma Cl(T) \subseteq sg_\gamma Cl(S \cup T)$ and,*
   (b) $sgCl_\gamma(S) \cup sgCl_\gamma(T) \subseteq sgCl_\gamma(S \cup T)$.
8. *$sg_\gamma Cl(sg_\gamma Cl(S)) = sg_\gamma Cl(S)$.*

Proof. Straightforward. □

Theorem 2.15. *For any subsets S, T of a topological space (X, τ). If γ is an sg-regular operation on τ_{sg}, then*

1. *$sg_\gamma Cl(S) \cup sg_\gamma Cl(T) = sg_\gamma Cl(S \cup T)$.*
2. *$sgCl_\gamma(S) \cup sgCl_\gamma(T) = sgCl_\gamma(S \cup T)$.*

Proof. (1) It is enough to proof that $sg_\gamma Cl(S \cup T) \subseteq sg_\gamma Cl(S) \cup sg_\gamma Cl(T)$ since the other part follows directly from Lemma 2.14 (7). Let $x \notin sg_\gamma Cl(S) \cup sg_\gamma Cl(T)$. Then there exist two sg-γ-open sets U and V containing x such that $S \cap U = \phi$ and $T \cap V = \phi$. Since γ is an sg-regular operation on τ_{sg}, then by Proposition 2.10, $U \cap V$ is sg-γ-open in X such that

$$(U \cap V) \cap (S \cup T) = \phi.$$

Therefore, we have $x \notin sg_\gamma Cl(S \cup T)$ and hence

$$sg_\gamma Cl(S \cup T) \subseteq sg_\gamma Cl(S) \cup sg_\gamma Cl(T).$$

(2) Let $x \notin sgCl_\gamma(S) \cup sgCl_\gamma(T)$. Then there exist sg-open sets U_1 and U_2 such that $x \in U_1$, $x \in U_2$, $S \cap \gamma(U_1) = \phi$ and $S \cap \gamma(U_2) = \phi$. Since γ is an sg-regular operation on τ_{sg}, then there exists an sg-open set W containing x such that $\gamma(W) \subseteq \gamma(U_1) \cap \gamma(U_2)$. Thus, we have

$$(S \cup T) \cap \gamma(W) \subseteq (S \cup T) \cap (\gamma(U_1) \cap \gamma(U_2)).$$

This implies that $(S \cup T) \cap \gamma(W) = \phi$ since $(S \cup T) \cap (\gamma(U_1) \cap \gamma(U_2)) = \phi$. This means that $x \notin sgCl_\gamma(S \cup T)$ and hence $sgCl_\gamma(S \cup T) \subseteq sgCl_\gamma(S) \cup sgCl_\gamma(T)$. Using Lemma 2.14 (7), we have the equality. □

Theorem 2.16. *Let S be any subset of a topological space (X, τ). If γ is an sg-open operation on τ_{sg}, then $sgCl_\gamma(S) = sg_\gamma Cl(S)$, $sgCl_\gamma(sgCl_\gamma(S)) = sgCl_\gamma(S)$ and $sgCl_\gamma(S)$ is sg-γ-closed set in X.*

Proof. First we need to show that $sg_\gamma Cl(S) \subseteq sgCl_\gamma(S)$ since by Lemma 2.14 (2), we have $sgCl_\gamma(S) \subseteq sg_\gamma Cl(S)$. Now let $x \notin sgCl_\gamma(S)$, then there exists an sg-open set U containing x such that $S \cap \gamma(U) = \phi$. Since γ is an sg-open on τ_{sg}, then there exists an sg-γ-open set W containing x such that $W \subseteq \gamma(U)$. So $S \cap W = \phi$ and hence by Theorem 2.13, $x \notin sg_\gamma Cl(S)$. Therefore, $sg_\gamma Cl(S) \subseteq sgCl_\gamma(S)$. Hence $sgCl_\gamma(S) = sg_\gamma Cl(S)$. Moreover, using the above result and by Lemma 2.14 (8), we get $sgCl_\gamma(sgCl_\gamma(S)) = sgCl_\gamma(S)$ and by Lemma 2.14 (4b), we obtain $sgCl_\gamma(S)$ is sg-γ-closed set in X. □

Theorem 2.17. *Let S be any subset of a topological space (X, τ) and γ be an operation on τ_{sg}. Then the following statements are equivalent:*

1. *S is sg-γ-open set.*
2. *$sgCl_\gamma(X \backslash S) = X \backslash S$.*
3. *$sg_\gamma Cl(X \backslash S) = X \backslash S$.*
4. *$X \backslash S$ is sg-γ-closed set.*

Proof. Clear. □

Lemma 2.18. *Let (X, τ) be a topological space and γ be an sg-regular operation on τ_{sg}. Then $sg_\gamma Cl(S) \cap U \subseteq sg_\gamma Cl(S \cap U)$ holds for every sg-γ-open set U and every subset S of X.*

Proof. Suppose that $x \in sg_\gamma Cl(S) \cap U$ for every sg-γ-open set U, then $x \in sg_\gamma Cl(S)$ and $x \in U$. Let V be any sg-γ-open set of X containing x. Since γ is sg-regular on τ_{sg}. So by Proposition 2.10, $U \cap V$ is sg-γ-open set containing x. Since $x \in sg_\gamma Cl(S)$, then by Theorem 2.13, we have $S \cap (U \cap V) \neq \phi$. This means that $(S \cap U) \cap V \neq \phi$. Therefore, again by Theorem 2.13, we obtain that $x \in sg_\gamma Cl(S \cap U)$. Thus, $sg_\gamma Cl(S) \cap U \subseteq sg_\gamma Cl(S \cap U)$. □

3. sg-γ- Separation Axioms

In this section, we introduce some types of sg-γ- separation axioms called sg-γ-T_i for $i \in \{0, 1, 2\}$. Some basic properties of these spaces are investigated.

Definition 3.1. A topological space (X, τ) with an operation γ on τ_{sg} is called:

1. sg-γ-T_0 if for any two points x, y in X such that $x \neq y$, there exists an sg-open set U such that $x \in U$ and $y \notin \gamma(U)$ or $y \in U$ and $x \notin \gamma(U)$.
2. sg-γ-T_1 if for any two points x, y in X such that $x \neq y$, there exist two sg-open sets U and V containing x and y respectively such that $y \notin \gamma(U)$ and $x \notin \gamma(V)$.
3. sg-γ-T_2 if for any two points x, y in X such that $x \neq y$, there exist two sg-open sets U and V containing x and y respectively such that $\gamma(U) \cap \gamma(V) = \phi$.

Theorem 3.2. *Let γ be an sg-open operation on τ_{sg}. Then (X, τ) is an sg-γ-T_0 space if and only if $sgCl_\gamma(\{x\}) \neq sgCl_\gamma(\{y\})$, for every pair x, y of X with $x \neq y$.*

Proof. Let x, y be any two distinct points of an sg-γ-T_0 space (X, τ). Then by definition, we assume that there exists an sg-γ-open set U such that $x \in U$ and $y \notin \gamma(U)$. Since γ is an sg-open operation on τ_{sg}, then there exists an sg-γ-open set W such that $x \in W$ and $W \subseteq \gamma(U)$. Hence $y \in X \backslash \gamma(U) \subseteq X \backslash W$. Since $X \backslash W$ is an sg-γ-closed set in (X, τ). Then we obtain that $sgCl_\gamma(\{y\}) \subseteq X \backslash W$ and therefore $sgCl_\gamma(\{x\}) \neq sgCl_\gamma(\{y\})$. Conversely, suppose for any $x, y \in X$ with $x \neq y$, we have $sgCl_\gamma(\{x\}) \neq sgCl_\gamma(\{y\})$. Now, we assume that there exists $z \in X$ such that $z \in sgCl_\gamma(\{x\})$, but $z \notin sgCl_\gamma(\{y\})$. If $x \in sgCl_\gamma(\{y\})$, then $\{x\} \subseteq sgCl_\gamma(\{y\})$, which implies that $sgCl_\gamma(\{x\}) \subseteq sgCl_\gamma(\{y\})$ (by Lemma 2.14 (5)). This implies that $z \in sgCl_\gamma(\{y\})$. This contradiction shows that $x \notin sgCl_\gamma(\{y\})$. This means that by Definition 2.11, there exists an sg-open set U such that $x \in U$ and $\gamma(U) \cap \{y\} = \phi$. Thus, we have that $x \in U$ and $y \notin \gamma(U)$. It gives that the space (X, τ) is sg-γ-T_0. □

Theorem 3.3. *The space (X, τ) is sg-γ-T_1 if and only if for every point $x \in X$, $\{x\}$ is an sg-γ-closed set.*

Proof. Let x be a point of an sg-γ-T_1 space (X, τ). Then for any point $y \in X$ such that $x \neq y$, there exists an sg-open set V_y such that $y \in V_y$ but $x \notin \gamma(V_y)$. Thus, $y \in \gamma(V_y) \subseteq X \backslash \{x\}$. This implies that $X \backslash \{x\} = \cup \{\gamma(V_y) : y \in X \backslash \{x\}\}$. It is shown that $X \backslash \{x\}$ is sg-γ-open set in (X, τ). Hence $\{x\}$ is sg-γ-closed set in (X, τ). Conversely, let $x, y \in X$ such that $x \neq y$. By hypothesis, we get $X \backslash \{y\}$ and $X \backslash \{x\}$ are sg-γ-open sets such that $x \in X \backslash \{y\}$ and $y \in X \backslash \{x\}$. Therefore, there exist sg-open sets U and V such that $x \in U$, $y \in V$, $\gamma(U) \subseteq X \backslash \{y\}$ and $\gamma(V) \subseteq X \backslash \{x\}$. So, $y \notin \gamma(U)$ and $x \notin \gamma(V)$. This implies that (X, τ) is sg-γ-T_1. □

Theorem 3.4. *For any topological space (X, τ) and any operation γ on τ_{sg}, the following properties hold.*

1. *Every sg-γ-T_2 space is sg-γ-T_1.*

2. *Every sg-γ-T_1 space is sg-γ-T_0.*

Proof. The proofs are obvious by their definitions. □

Remark 3.5. By Theorem 3.4, Remark 2.7, and Remark 4.12 in [9], we obtain the following diagram of implications. Moreover, the Examples 3.6, 3.7 and 3.8 below show that the reverse implications are not true in general.

$$\begin{array}{ccccccc}
sg\text{-}\gamma\text{-}T_2 & \longrightarrow & sg\text{-}\gamma\text{-}T_1 & & \longrightarrow & & sg\text{-}\gamma\text{-}T_0 \\
\uparrow & & \uparrow & & & & \uparrow \\
\text{semi } \gamma\text{-}T_2 & \longrightarrow & \text{semi } \gamma\text{-}T_1 & \longrightarrow & \text{semi } \gamma\text{-}T_{\frac{1}{2}} & \longrightarrow & \text{semi } \gamma\text{-}T_0
\end{array}$$

Where $S \rightarrow T$ represents S implies T.

Example 3.6. Let $X = \{a, b, c\}$ and $\tau = \{\phi, X\} = SO(X)$. Then $\tau_{sg} = P(X)$. Define an operation $\gamma\colon \tau_{sg} \to P(X)$ by $\gamma(S) = S$ for all $S \in \tau_{sg}$. Here, $\tau_{sg\gamma} = P(X)$ and $SO(X) = SO(X)_\gamma$. Then the space (X, τ) is sg-γ-T_i ($i \in \{0, 1, 2\}$), but not semi γ-T_i ($i \in \{0, 1, 2\}$).

Example 3.7. Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a\}, \{b, c\}\}$. Then $\tau_{sg} = P(X)$. Let $\gamma\colon \tau_{sg} \to P(X)$ be an operation on τ_{sg} defined as follows:
For every set $S \in \tau_{sg}$

$$\gamma(S) = \begin{cases} S & \text{if } b \in S \\ Cl(S) & \text{if } b \notin S \end{cases}$$

Then the space (X, τ) is sg-γ-T_0, but (X, τ) is not sg-γ-T_1.

Example 3.8. Suppose $X = \{a, b, c\}$ and $\tau = P(X)$. Define an operation γ on τ_{sg} as follows: For every $S \in \tau_{sg}$

$$\gamma(S) = \begin{cases} S & \text{if } S = \{a, b\} \text{ or } \{a, c\} \text{ or } \{b, c\} \\ X & \text{otherwise} \end{cases}$$

Then (X, τ) is sg-γ-T_1 space, but (X, τ) is not sg-γ-T_2.

Lemma 3.9. *Let (X, τ) be a semi-$T_{\frac{1}{2}}$ space. Then (X, τ) is sg-γ-T_i if and only if it is semi γ-T_i, where $i \in \{0, 1, 2\}$.*

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