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THE GROWTH OF VAPOR SHELL AROUND HEATED PARTICLE IN A LIQUID

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Abstract: The asymptotic stage of vapor bubble growth around heated particle in liquid is con-sidered. This problem is connected with vapor explosions which occur particularly in the cooling systems of nuclear-power stations, when, as result of some accident, the heated particles of the nuclear fuel settle in the water. This leads to the explosive boiling of the liquid and to a rapid increase of the pressure. In the present work the simple formula for the asymptotic stage of bubble growth rate around heated particle is obtained.

AMS Subject Classification: 76T10
Key Words: heated particle, bubble growth, liquid, vapor explosion

1. Basic equations

Formulation of the problem of vapor explosion at the micro scale is done in [1]. Numerical simulation of the dynamics of two-phase bubble containing a heated particle was done in [2]. Let us consider the growth of vapor shell around heated particle due to evaporation of the surrounding liquid. We assume that the particle is spherical and processes of heat and mass transfer around the particle we will study in the frame of spherically symmetry. The origin of

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spherical system is located at the center of the particle. Estimates show that temperature inside the particle is practically uniform. Then we can write the following equation describing the process of cooling of heated particle

$$c_d\rho_d\frac{4}{3}\pi R_d^3\frac{dT_d}{dt} = 4\pi R_d^2\lambda_V\left.\frac{\partial T_V}{\partial r}\right|_{R_d} \tag{1.1}$$

Here subscripts d and V are corresponding to the parameters of particle and vapor; R_d is radius of the particle, c is the heat capacity, ρ is the density, T is the temperature, λ is the heat conductivity. For asymptotic stage of vapor layer growth around the heated particle the quazi-steady temperature distribution will be realized:

$$T_V = (T_d - T_S)\frac{R_d}{r - R_d}\left(\frac{R}{r} - 1\right) + T_{VR} \tag{1.2}$$

Here r is radial coordinate, R is the radius of the vapor shell, T_{VR} is the vapor temperature at the external surface of the vapor layer.

The evaporation process is taking place at the surface. For simplification we assume that the liquid is at the saturation temperature $T_{VR} = T_S$. The subscript S corresponds to the parameters at the saturation.

Let us substitute (1.2) to the equation (1.1). Taking into account that at the asymptotic stage of vapor bubble growth $R >> R_d$ we will get

$$\frac{dT_d}{dt} = -\frac{3\lambda_V}{c_d\rho_d R_d^2}(T_d - T_S) \tag{1.3}$$

Assuming that temperature of liquid and thermophysical parameters of the particle and vapor are constants we can solve equation equation (1.3). Solution will have a form:

$$T_d - T_S = (T_{d_0} - T_S)e^{-At}, \quad A = \frac{3\lambda_V}{c_d\rho_d R_d^2} \tag{1.4}$$

Here T_{d_0} is the initial temperature of the particle. Boundary condition at the external surface of the vapor layer have form [3]

$$r = R : q_R = jl, \; q_R = -\lambda_V\left.\frac{\partial T_V}{\partial r}\right|_R \tag{1.5}$$

Here l is latent heat of evaporation, j is rate of phase change per unit surface.

Calculating the heat flux to the surface of vapor bubble from the heated particle using formulas (1.2) and (1.4) we obtain

$$q_R = \lambda_V(T_d - T_S)\frac{R_d}{R^2} = \lambda_V\frac{R_d}{R^2}(T_{d_0} - T_S)e^{-At}, \tag{1.6}$$

Mass balance equation for the vapor layer has a form:

$$\frac{dm}{dt} = 4\pi R^2 j \tag{1.7}$$

$$m = 4\pi \int\limits_{R_d}^{R} \rho_V r^2 dr \tag{1.8}$$

We assume that the vapor can be described by the equation of state for ideal gas [3]

$$p_V = \rho_V(r,t) B T_V(r,t)/\mu_V \tag{1.9}$$

Here ρ is the pressure, μ is the molecular mass, B - the gas constant. We assume that the pressure inside the vapor shell is uniform $p_V = p_V(t)$. This is correct when the velocity of the bubble surface is much less then the sound speed in the vapor [3]. Let us substitute (1.5), (1.6), (1.8) and (1.9) to the equation (1.7). Then we will get

$$\frac{d}{dt}\int\limits_{R_d}^{R} \frac{r^2 dr}{T_V(r)} = \frac{B R_d \lambda_V}{p_V \mu_V l}(T_{d_0} - T_S)e^{-At} \tag{1.10}$$

Let us substitute to (1.10) vapor temperature distribution (1.2) and integrate. Then we will obtain

$$\frac{(R-R_d)^3}{3} + \frac{3}{2}R_d\,(R-R_d)^2 + 3R_d^2\,(R-R_d) +$$
$$+ R_d^2 \ln\left[1 + \frac{T_S\,(R-R_d)}{T_d R_d}\right] = Ja\frac{R_d^3}{3}(1-e^{-At}) \tag{1.11}$$
$$Ja = \frac{c_d \rho_d (T_{d_0} - T_S)}{l \rho_V(R)}$$

For asymptotic stage of vapor bubble growth when $R >> R_d$ the first term in the left side of equation (1.11) is dominant. In this case

$$\frac{R}{R_d} = \left[Ja(1-e^{-At})\right]^{1/3} \tag{1.12}$$

We can see from this formula that radius of vapor shell is approaching asymptotically to the value $R_d Ja^{1/3}$.

For $A << 1$ the time is short and we can neglect the effect of cooling of the particle. The formula (1.12) is simplified.

$$\frac{R}{R_d} = (JaAt)^{1/3} \tag{1.13}$$

2. Conclusion

The simple analytical expressions describing asymptotic stage of vapor shell growth around heated solid particle in the liquid are is obtained.

References

[1] Nail S.Khabeev, *Simulation of vapor explosions,* Applied Energy, **64**,(1999), 317-321. DOI: 10.1016/S0306-2619(99)00052-5.

[2] O.R.Ganiev, N.S.Khabeev, *Dynamics of a vapor shell around a heated particle in a liquid,* Journal of Applied Mechanics and Technical Physics, **48**, N4, (2007), 525-533. DOI: 10.1007/s10808-007-0066-z.

[3] R.I. Nigmatulin, N.S.Khabeev, F.B.Nagiev, *Dynamics, heat and mass transfer of vapour-Gas bubbles in liquid,* Int.J.Heat Mass Trans., **24**, N6, (1981), 1033-1044. DOI: 10.1016/0017-9310(81)90134-4.