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SOME RESULTS ON PRIME NUMBERS

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Abstract: In this article using the functions $f_1(k) = 10k + 1$, $k \neq \overset{\circ}{3}+2$; $f_2(k) = 10k + 3$, $k \neq \overset{\circ}{3} - \{0\}$; $f_3(k) = 10k + 7$, $k \neq \overset{\circ}{3}+2$, $k \neq \overset{\circ}{7} - \{0\}$; and $f_4(k) = 10k + 9$, $k \neq \overset{\circ}{3}$, where $k \in \mathbb{N}_0$, we obtain two important results on prime numbers. The first result indicates that if p is a prime number that ends in 7, then $p + 10l$ will be a prime number under certain conditions. The second result states that if k is a number ending in 7, then $k + 10$ will be also a prime number under certain conditions.

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1. Introduction

There are no general methods in the literature, to our knowledge, on how to generate a prime number starting from a given prime number.

We know of the existence of large prime numbers and that to determine the

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primality of these numbers there are special algorithms, and that there are also prizes for those who show the largest prime number; encouraged by these facts, in this work, we obtain two important results on prime numbers.

The first result states that if p is a large number that ends in 7, then for $l \in \mathbb{N}$, $10l + p$ is a prime number provided that

$$\frac{p-27}{90}, \frac{p-27}{30}, \frac{p+10l-27}{90}, \frac{p+10l-27}{30}, \frac{p+10l-7}{70}, \frac{p-7}{70},$$

are not integers and the (x, y) natural numbers belonging to the intervals

$$\left\langle \frac{p-27}{30}, \frac{p+10l-27}{30} \right\rangle \times \left\langle \frac{p-27}{90}, \frac{p+10l-27}{90} \right\rangle,$$

are not integer solutions of the diophantine equation:

$$p + 10l = (10x + 9)(10y + 3)$$

and, in addition, the (x, y) natural numbers belonging to the intervals

$$\left\langle \frac{p-7}{70}, \frac{p+10l-7}{70} \right\rangle \times \left[\frac{p-7}{10}, \frac{p+10l-7}{10} \right],$$

are not solutions of the diophantine equation:

$$p + 10l = (10x + 1)(10y + 7).$$

The second result states that if k is a large number ending in 7, $k \neq \overset{\circ}{3}$, $\frac{k-27}{30}, \frac{k-27}{90} \notin \mathbb{N}$ and there exists an unique $(a, b) \in \mathbb{N} \times \mathbb{N}$ such that

$$k = (10a + 9)(10b + 3),$$

then $k + 10$ is a prime number provided that the equation

$$k + 10 = (10x + 1)(10y + 7), x \neq \overset{\circ}{3} + 2, y \neq \overset{\circ}{3} + 2, y \neq \overset{\circ}{7},$$

does not have an integer solution.

In this work $\mathbb{N}$ represents the set of natural numbers, $\overset{\circ}{n}$ represents the set of multiples of n, and $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$.

If (X, d) is a metric space, the distance between subsets of X is defined as:

$$d(A, B) = \inf_{\substack{x \in A \\ y \in B}} d(x, y).$$

2. Some results on prime numbers

In this section we state the Lemma (2.1) and the Lemma (2.2), and then we obtain the results mentioned in the introduction.

Lemma 2.1. *Let $f_1, f_2, f_3, f_4 : \mathbb{N}_0 \to \mathbb{N}$ be functions defined by:*

$$\begin{aligned} f_1(k) &= 10k+1 \quad , \quad k \neq \overset{\circ}{3}+2 \\ f_2(k) &= 10k+3 \quad , \quad k \neq \overset{\circ}{3}-\{0\} \\ f_3(k) &= 10k+7 \quad , \quad k \neq \overset{\circ}{3}+2,\ k \neq \overset{\circ}{7}-\{0\} \\ f_4(k) &= 10k+9 \quad , \quad k \neq \overset{\circ}{3}\,. \end{aligned}$$

So, let p be a natural number ending in seven, and the following diophantine equations

$$\begin{aligned} p = (10x+1)(10y+7) \quad &; \quad x \neq \overset{\circ}{3}+2, x \geq 1, y \neq \overset{\circ}{7}-\{0\}, y \neq \overset{\circ}{3}+2 \\ p = (10z+3)(10w+9) \quad &; \quad z \neq \overset{\circ}{3}-\{0\}, w \neq \overset{\circ}{3}, \end{aligned}$$

do not possess integer solutions, then p is a prime number.

Proof. Is inmediate, see [1]. □

Lemma 2.2. *If p and l are fixed numbers,*

$$\mathcal{C}_1 = \left\{(x,y) \in \mathbb{R}^2 : (10x+9)(10y+3) = p, x \geq 0, y \geq 0\right\},$$

$$\mathcal{C}_2 = \left\{(x,y) \in \mathbb{R}^2 : (10x+9)(10y+3) = p+10l, x \geq 0, y \geq 0\right\},$$

$$\mathcal{C}_3 = \left\{(x,y) \in \mathbb{R}^2 : (10x+1)(10y+7) = p, x \geq 0, y \geq 0\right\},$$

$$\mathcal{C}_4 = \left\{(x,y) \in \mathbb{R}^2 : (10x+1)(10y+7) = p+10l, x \geq 0, y \geq 0\right\},$$

subsets of $\mathbb{R}^2$ and d the euclidean metric, then

$$d(\mathcal{C}_1,\mathcal{C}_2) = \inf_{\substack{x\in\mathcal{C}_1\\ y\in\mathcal{C}_2}} d(x,y) = \frac{\sqrt{2}}{10}\left(\sqrt{p+10l}-\sqrt{p}\right).$$

$$d(\mathcal{C}_3,\mathcal{C}_4) = \inf_{\substack{x\in\mathcal{C}_3\\ y\in\mathcal{C}_4}} d(x,y) = \frac{\sqrt{2}}{10}\left(\sqrt{p+10l}-\sqrt{p}\right).$$

Proof. Just use the Lagrange multipliers, see [2]. □

Theorem 1. *Let p be a large prime number that ends in 7, then $p+10l$ is a prime number, where $l \in \mathbb{N}$, l is a fixed natural number, smaller than p, provided that*

$$\frac{p-27}{90}, \frac{p-27}{30}, \frac{p+10l-27}{90}, \frac{p+10l-27}{30}, \frac{p+10l-7}{70}, \frac{p-7}{70},$$

are not integers, and the (x,y) natural numbers belonging to intervals

$$\left\langle \frac{p-27}{30}, \frac{p+10l-27}{30} \right\rangle \times \left\langle \frac{p-27}{90}, \frac{p+10l-27}{90} \right\rangle,$$

are not solutions of the equation $p+10l=(10x+9)(10y+3)$, in addition, the (x,y) natural numbers belonging to the intervals

$$\left\langle \frac{p-7}{70}, \frac{p+10l-7}{70} \right\rangle \times \left[\frac{p-7}{10}, \frac{p+10l-7}{10} \right],$$

are not integer solutions of the equation $p+10l=(10x+1)(10y+7)$.

Proof. Suppose that $p+10l, l \geq 1, l \in \mathbb{N}$, is not a prime number. Then there exists $(A,B) \in \mathbb{N} \times \mathbb{N}$ such that the next two eqs. (1) or (2) will happen

$$p+10l=(10A+9)(10B+3) \tag{1}$$

$$p+10l=(10A+1)(10B+7). \tag{2}$$

If it happens to occur (1) we have that the (x,y) natural numbers belonging to the intervals

$$\left\langle \frac{p-27}{30}, \frac{p+10l-27}{30} \right\rangle \times \left\langle \frac{p-27}{90}, \frac{p+10l-27}{90} \right\rangle,$$

are not integer solutions of the equation $p+10l=(10x+9)(10y+3)$, thus

$$A \in \left\langle 0, \frac{p-27}{30} \right\rangle, B \in \left\langle 0, \frac{p-27}{90} \right\rangle.$$

The straight line $y=B$ intersects the equation

$$p=(10x+9)(10y+3),$$

so, we have

$$p=(10x+9)(10B+3). \tag{3}$$

From relations (1), (3) and lemma (2.2) we have

$$\frac{\sqrt{2}}{10}\left[\sqrt{p+10l}-\sqrt{p}\right] \leq A-x=\frac{l}{10B+3}. \tag{4}$$

Similar analysis shows us that

$$\frac{\sqrt{2}}{10}\left[\sqrt{p+10l}-\sqrt{p}\right] \leq B-y=\frac{l}{10A+9}. \tag{5}$$

From the equation (1) we have two possibilities

$$\sqrt{p+10l} \leq 10A+9 \tag{6}$$

$$\sqrt{p+10l} \leq 10B+3. \tag{7}$$

If the relation (6) is true, then from this relation and (5) we have

$$\frac{\sqrt{2}}{10}\left[\sqrt{p+10l}-\sqrt{p}\right] \leq B-y \leq \frac{l}{\sqrt{p+10l}}. \tag{8}$$

If the relation (7) is true, then from this relation and (4) we have

$$\frac{\sqrt{2}}{10}\left[\sqrt{p+10l}-\sqrt{p}\right] \leq A-x \leq \frac{l}{\sqrt{p+10l}} \tag{9}$$

Therefore we have that either (8) or (9) is true.
Since p is a large prime number, we have that $(A,B) \in \mathbb{N}\times\mathbb{N}$ would be a solution of the equation (3), which is false, since p is a prime number.
Similarly, if it happens to be the case (2) we will have

$$A \in \left\langle 0, \frac{p-7}{70}\right\rangle, B \in \left\langle 0, \frac{p-7}{10}\right\rangle,$$

$$\frac{\sqrt{2}}{10}\left(\sqrt{p+10l}-\sqrt{p}\right) \leq A-x \leq \frac{l}{\sqrt{p+10l}} \text{ or}$$

$$\frac{\sqrt{2}}{10}\left(\sqrt{p+10l}-\sqrt{p}\right) \leq B-y \leq \frac{l}{\sqrt{p+10l}}.$$

For large p we have that (A,B) would be a solution of the equation

$$p=(10x+1)(10y+7)$$

which is false, since p is a prime number. □

Theorem 2. *Let k be a large natural number ending in 7, $k \neq \overset{\circ}{3}$. If $\frac{k-27}{30}, \frac{k-27}{90}, \frac{k-17}{90}, \frac{k-17}{30} \notin \mathbb{N}$ and there exists an unique $(a,b) \in \mathbb{N} \times \mathbb{N}$ such that*

$$k = (10a+9)(10b+3),$$

then $k+10$ is a prime number, provided that the equation

$$k+10 = (10x+1)(10y+7), x \neq \overset{\circ}{3}+2, y \neq \overset{\circ}{3}+2, y \neq \overset{\circ}{7},$$

does not possess an integer solution.

Proof. Suppose $k+10$ is not a prime number, so there exists $(A,B) \in \mathbb{N} \times \mathbb{N}$ such that

$$k+10 = (10A+9)(10B+3). \tag{10}$$

So one has

$$\frac{k-27}{30}, \frac{k-27}{90}, \frac{k-17}{90}, \frac{k-17}{30} \notin \mathbb{N} \;\Rightarrow\; 0 < A < \frac{k-27}{30}, 0 < B < \frac{k-27}{90}.$$

The straight line $y = B$ intersects the equation

$$k = (10x+9)(10y+3),$$

which implies that

$$k = (10x+9)(10B+3). \tag{11}$$

From (10) and (11) we have

$$A - x = \frac{1}{10B+3}. \tag{12}$$

Similarly we have

$$B - y = \frac{1}{10A+9}. \tag{13}$$

Given that $\sqrt{k} \leq 10A+9$ or $\sqrt{k} \leq 10B+3$ then from (12) and lemma (2.2) or (13) and lemma (2.2) we have that

$$\frac{\sqrt{2}}{10}\left[\sqrt{k+10} - \sqrt{k}\right] \leq A - x \leq \frac{1}{\sqrt{k}} \quad \text{or} \quad \frac{\sqrt{2}}{10}\left[\sqrt{k+10} - \sqrt{k}\right] \leq B - y \leq \frac{1}{\sqrt{k}};$$

so, as k is large we might conclude that (A,B) is another solution of the equation

$$k = (10x+9)(10y+3),$$

which is a contradiction, since there exists an unique solution by hypothesis. □

Theorem 3. *Let k be a large natural number, ending in 7, $k \neq \overset{\circ}{3}$. If $\frac{k-7}{70}, \frac{k+3}{70} \notin \mathbb{N}$ and there exists an unique $(a,b) \in \mathbb{N} \times \mathbb{N}$ such that*

$$k = (10a+1)(10b+7), a \geq 1,$$

then $k+10$ is a prime number, provided that the equation

$$k + 10 = (10x+9)(10y+3),\ x \neq \overset{\circ}{3},\ y \neq \overset{\circ}{3} - \{0\},$$

does not possess an integer solution, and $y_0 = \frac{k-7}{10}$ is not a solution of the equation

$$k + 10 = (10x+1)(10y+7).$$

Proof. Analogous to the above. □

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