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EFFECT OF TRANSMISSION OF HIGH-POWER MICROWAVE SIGNALS ON THE THERMOELASTIC STATE OF A WAVEGUIDE

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Abstract: We performed experimental investigations aimed at studying in more detail the interdisciplinary problem of the effect of power loss for microwave signals transmitted through waveguides on the thermoelastic state of the latter. The waveguide temperature field was estimated using the developed mathematical models with differential equations of heat balance taking into account different operation conditions. The obtained temperature distribution may be applied to calculate deflected mode of waveguide as a whole and its conductive coating to ensure strength and hardness conditions.

AMS Subject Classification: 80-05, 70-05
Key Words: electromagnetic energy, temperature field, convective heat exchange, loss factor, skin layer

1. Introduction

Waveguides are intended to transmit microwave electromagnetic signals between antennas and transceiver blocks in aerospace technology, radiolocation, military and civil communications service, etc. In the course of operation, the waveguides are subject to different external loads: deformation, force and temperature actions of solar radiation as well as those from loss of power when transmitting microwave signals [1].

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To ensure general strength and hardness of waveguides, special calculation procedures have been developed and used [2, 3, 4], with allowance made for deformation and force external actions. At the same time, now the power of signals transmitted through waveguides is being increased along with growing their active service life to 15 years and more. This requires refinement of the existing methods of waveguide calculation to take account of the effect of those external factors that earlier have been considered minor. For example, our ground tests of waveguides showed that the waveguides were heated to $40-60°C$ when transmitting through them microwave signals of increased power (10 kW and more). Since transmission of high-power signals through waveguides occurs regularly during their operation, such heating is an additional thermocycling that should be taken into account along with periodic solar-radiation heating and other thermal actions.

Distinction between the mechanical (modulus of elasticity, Poisson coefficient, etc.) and thermal (coefficient of thermal expansion, heat capacity, etc.) properties of waveguide construction material (aluminum, copper, alloys), on the one hand, and those properties of the conductive coating on its inside wall (gold, silver, copper, etc.), on the other hand, leads to appearance of tangential stresses at their contact. These tangential stresses will vary periodically with time in line with periodicity of heating caused by signals transmission. Such thermocycling may lead to appearance of microcracks in the construction. Together with periodical action of tangential stresses, this will promote flaking of the layer of conductive material and failure of waveguide leakproofness [5].

The problem becomes complicated by the fact that in the outer space the possibilities to dissipate excess heat are very limited, especially in an unsealed spacecraft. In this case, all its elements and systems are exposed to solar radiation, while there is practically no convective exchange. All these facts hamper efficient heat removal and require to take into account the effect of transmission of microwave signals through waveguides on the thermoelastic state of the latter.

2. Results of Analytical Review of Literature

The performed literature review [6, 7, 8, 9, 10, 11] on the considered problem showed that most of the authors mentioned only very small loss of transmitted signals in waveguides as compared with other types of guide systems. Consequently a conclusion is made that there is no need to take into account heat release.

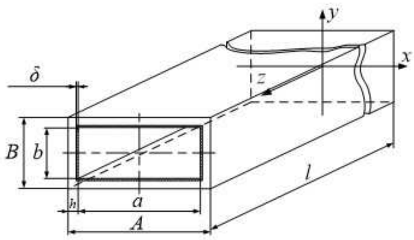

Figure 1: Design of a waveguide straight section

In our opinion, such situation is acceptable only for low power microwave signals. However, when transmitting high-power signals, one should not neglect waveguide heating. In this connection, it seems necessary to improve the methods of calculating strength of thin-walled waveguide construction to take into account periodic heating when transmitting signals of heightened power.

2.1. Signal Power Loss

According to the existing procedures [6, 7, 8, 9, 10, 11], attenuation of a transmitted signal occurs in the course of electromagnetic wave propagation through waveguide because of partial scattering of signal energy over the thin-walled construction. The reasons for signal attenuation are as follows:

1) loss in the conductive material layer on the inside walls of waveguide;

2) loss in dielectric in the waveguide inner cavity;

3) radiation loss due to loose joint between separate elements of a section of wave propagation system, etc.

As a result, all components of the electromagnetic field along the waveguide axial line (z-axis in Fig. 1) will decrease exponentially [6, 7, 8, 9, 10, 11], so that energy flux decreasing along the waveguide may be characterized by a change δP of signal power:

$$\Delta P = P_{in} - P_{out} = P_{in} - P_{in}e^{-2\alpha l} = P_{in}\left(1 - e^{-2\alpha l}\right)[W] \qquad (1)$$

where P_{in} is signal power at the waveguide input, W; P_{out} is signal power at the waveguide output, W; α is loss factor, dB/m; l is waveguide length, m. According to Eq. (1), loss factor α for signal power in a waveguide serves to estimate power dissipation of electromagnetic energy at its propagation along waveguide, with allowance made for all possible reasons of this phenomenon.

2.2. Loss Factor

The value of loss factor α can be determined either empirically or by using theoretical dependences [6, 7, 8, 9, 10, 11]. They take into account the conditions of signal propagation: waveguide type and size, properties of material of waveguide walls and skin layer, presence/absence of dielectric, characteristics of transmitted electromagnetic wave, etc.

For example, in the most widespread case of transmitting H_{10} electromagnetic wave through a waveguide of rectangular cross-section, the theoretical value of loss factor α can be determined as [6]

$$\alpha = 10^{-3}\sqrt{0.1 \cdot \pi \cdot f \cdot \mu_\alpha \cdot \rho} \cdot \frac{1 + (2b/a)(\lambda/2a}{bZ \cdot \sqrt{1 - (\lambda/2a)^2}} [\text{dB/m}]. \quad (2)$$

Here f is frequency of the transmitted signal,Hz; μ_a is absolute magnetic permeability of the medium, H/m; ρ is resistivity of the skin layer material, Ω/m; $a(b)$ is width (height) of the waveguide inner cavity, m ($> b$); λ is wavelength of transmitted signal, m; Z is wave impedance of the medium in which signal propagates, Ω.

However, the real values of loss factor will exceed the calculated ones by 1030% because of neglect of such factors as loss due to loose joints, transient resistance, lacquering, etc. [6, 7].

3. Waveguide Heating Simulation

The heating process can be divided into two steps: heating of a thin skin layer at a short-time signal and heating of the whole waveguide construction at long-term transmission of microwave signals. The previous investigations showed that time of heating conductive coating is small, so the first step may be neglected. As to the second step, however, it cannot be considered adiabatic because heat exchange with environment will have effect on waveguide temperature field in the course of heating waveguide walls.

3.1. Heat Conduction Equation

A difference between temperatures of inner and outer waveguide walls will lead to appearance of heat flux through the waveguide walls [12]. When transmitting electromagnetic signals, heat will spread in a waveguide at a normal $\vec{n}$ to the outer waveguide walls (Fig. 2a). We shall neglect the edge effects at the corners of plate joints.

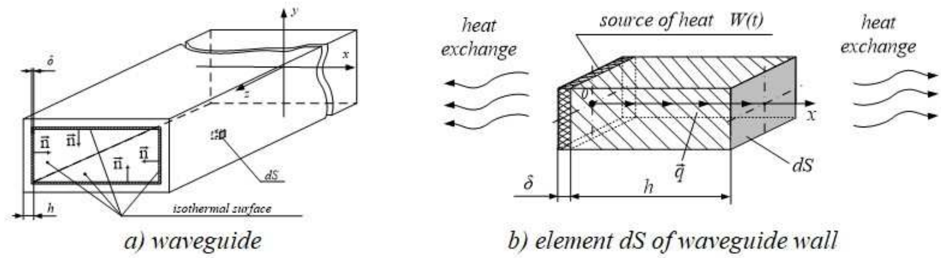

Figure 2: Design model of heat transfer at a waveguide wall

The approach accepted by us permits to take a 1D spreading model for calculation of waveguide temperature field. Heat release ΔQ in the skin layer at electromagnetic signal flow through it is simulated by a heat source of intensity $W(t)$. Then the waveguide temperature field can be described by a 1D parabolic differential heat conduction equation with heat source as [13]

$$\frac{\partial T(x,t)}{\partial t} = a_2 \frac{\partial^2 T(x,t)}{\partial x^2} + \frac{W(t)}{c_1 \gamma_1}, \tag{3}$$

where a_2 is temperature diffusivity coefficient of the waveguide wall material:

$$a_2 = \frac{\lambda_2}{c_2 \gamma_2} \; [m^2/s].$$

Here λ_2 is heat conduction coefficient of the waveguide wall material, $W/m\cdot^\circ C$; c_2 is specific heat capacity of the waveguide wall material, $J/kg \cdot^\circ C$; γ_2 is specific density of the waveguide wall material, kg/m^3; c_1 is specific heat capacity of the skin layer material, $J/kg \cdot^\circ C$; γ_1 is specific density of the skin layer material, kg/m^3; is heat source intensity in the skin layer, W/m^3:

$$W(t) = \frac{\Delta P(t)}{V} = \frac{\Delta P(t)}{S \cdot \delta} = \frac{\left(1 - e^{-2\alpha l}\right) P_{in}(t)}{S \cdot \delta},$$

where S is side face area of all waveguide walls, m^2.

To obtain an unambiguous solution of differential Eq. (3), it is necessary to set edge conditions that involve initial and boundary conditions at the outer and inner waveguide walls.

3.2. Boundary Conditions

We take a uniform temperature field as initial condition. For a 1D model, it is equality of temperature at all points of a waveguide in the initial moment $t = 0$:

$$T\,(x,\, t = 0) = T_0 = \text{const.} \tag{4}$$

The boundary conditions are specified at the inner and outer surfaces of waveguide wall (Fig. 3) at which convective heat exchange occurs with environment of temperature T_{env} (boundary conditions of the third kind). The heat exchange condition for the inner surface of waveguide wall ($x = 0$) is

$$\lambda_1 \frac{\partial T_1\,(x = 0, t)}{\partial x} + \alpha_1\,[T_1\,(x = 0, t) - T_{env1}] = 0, \tag{5}$$

where $\lambda_1 = \frac{Bm}{m \cdot {}^\circ C}$ is coefficient of thermal conductivity of the skin layer, $T_1(x = 0, t)$ is temperature at the surface of the inner waveguide wall; α_1 is heat exchange coefficient between the surface of the inner waveguide wall and the medium inside of the waveguide, $W/m^2 \cdot {}^\circ C$; T_{env1} is temperature inside of the waveguide, ${}^\circ C$.

The second boundary condition specifies heat exchange between the outer waveguide wall ($x = h$) and environment:

$$\lambda_2 \frac{\partial T_2\,(x = h, t)}{\partial x} + \alpha_2\,[T_2\,(x = h, t) - T_{env2}] = 0, \tag{6}$$

where $T_2(x = h, t)$ is temperature at the surface of the outer waveguide wall; α_2 is heat exchange coefficient between the surface of the outer waveguide wall and environment outside of the waveguide, $W/m^2 \cdot {}^\circ C$; T_{env2} is temperature of environment outside of the waveguide, ${}^\circ C$. The main mechanism of heat exchange with environment in the boundary conditions Eqs. (5, 6) is convection; this corresponds to waveguide operation in a gas (air) environment, i.e., on the Earth.

The waveguide operation conditions at a preset orbit in a communication spacecraft with an open platform presume functioning in vacuum. In this case, there is practically no convection. The predominant mechanism of transfer and scattering thermal energy by waveguide construction will be radiative heat exchange. So, instead of the convection condition Eq. (6) at the outer surfaces of waveguide walls, the condition of radiative heat exchange should be set. According to the Stefan-Boltzmann law, it is of the form of a set heat flux:

$$\lambda_2 \frac{\partial T_2\,(x = h, t)}{\partial x} - \sigma_2\left[T_{env2}^4 - T_2^4\,(x = h, t)\right] = 0, \tag{7}$$

where σ_2 is reduced radiation factor for the outer waveguide walls, $W/m^2 \cdot^\circ C$.

An analysis of heat propagation in a waveguide shows that its inner walls represent a closed surface made by orthogonal plates. Complicated nonlinear inter-heating between them occurs by radiative heat exchange. With accuracy sufficient for practical calculations, it is possible to set for inner waveguide walls the following condition of absence of heat exchange with environment:

$$\frac{\partial T_1\,(x=0,t)}{\partial x} = 0. \tag{8}$$

The obtained differential Eq. (3) with edge conditions Eqs. (47) makes it possible to determine temperature fields of waveguide straight section for any its size, material and microwave signal characteristics (occurring in practice) with known loss factor α.

The methods to solve differential Eq. (3) with preset initial Eq. (4) and boundary Eqs. (57) conditions are well understood by now [12]. This equation can be solved by both analytical (variable separation method, method of sources, etc.) and numerical (finite difference method, finite element method, etc.) methods. Thermal calculation of waveguides with curvilinear longitudinal axis is hampered, first of all, by complexity of choosing the values of loss factors α because there were no such data in the existing literature [6, 7, 8, 9, 10, 11]. However, in recent years the works appear in which propagation of electromagnetic fields in curvilinear waveguides is calculated by numerical methods [14].

Temperature fields for straight sections of waveguides of different dimension types of cross section and wall thickness were calculated in accordance with the developed model and obtained resolvent equations. Special programs were developed to get solution using the analytical variable separation method and explicit scheme of the numerical finite difference method [15, 16].

4. Deflected Mode of Waveguide

Temperature variations of waveguide as a whole relative to other construction elements of antenna-feeder system lead to appearance of temperature stresses whose value and distribution strongly depend on construction geometry and fixation method. To take all these features into account, a procedure based on the finite elements method has been developed [17] as well as a program to calculate deflected modes of waveguide straight sections [18] that uses the data of temperature field precalculated with the programs from [15, 16].

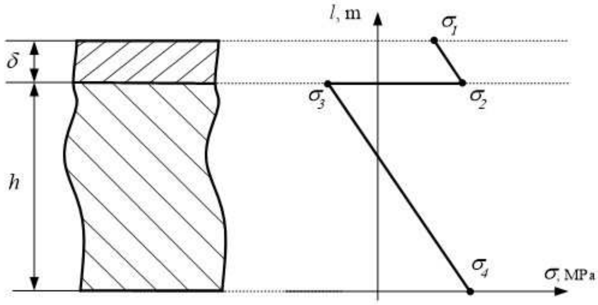

Figure 3: Stress distribution over a waveguide wall thickness

It is proposed to estimate stress state in the waveguide conductive coating using refined Stoney's formula [19] that takes into account distinctions in coefficients of thermal expansions and temperatures of layers. As a result, the diagram of normal stresses over the waveguide wall thickness has a typical distribution often called as "z-system" (Fig. 3).

After Stoney's formula refinement [20], the values of stresses at the typical points of the diagram (Fig. 3) are determined by the formulae:

$$\sigma_1 = \frac{E \cdot \Delta\alpha \cdot \Delta t \cdot h}{h + \delta} - \frac{E \cdot (h + \delta)}{2\rho} \; [Pa], \tag{9}$$

$$\sigma_2 = \frac{E \cdot \Delta\alpha \cdot \Delta t \cdot h}{h + \delta} - \frac{E \cdot (h - \delta)}{2\rho} \; [Pa], \tag{10}$$

$$\sigma_3 = -\frac{E \cdot \Delta\alpha \cdot \Delta t \cdot \delta}{h + \delta} - \frac{E \cdot (h - \delta)}{2\rho} \; [Pa], \tag{11}$$

$$\sigma_4 = -\frac{E \cdot \Delta\alpha \cdot \Delta t \cdot \delta}{h + \delta} + \frac{E \cdot (h + \delta)}{2\rho} \; [Pa]. \tag{12}$$

Here is the average Young's modulus of material and coating (assuming that their values are close to each other); $\Delta\alpha$ is a difference between the coefficients of thermal expansion of the conducting coating and waveguide wall; Δt is temperature difference, $^{\circ}C$; ρ is radius of curvature of the wall due to a difference between the coefficients of thermal expansion of conducting coating and waveguide wall. It can be determined as [21]:

$$\rho = \frac{1}{\Delta\alpha \cdot \Delta t} \left[\frac{h + \delta}{2} + \frac{1}{6 \cdot (h + \delta)} \left(\frac{E_1 \delta^3}{1 - \mu_1} + \frac{E_2 h^3}{1 - \mu_2} \right) \times \right.$$

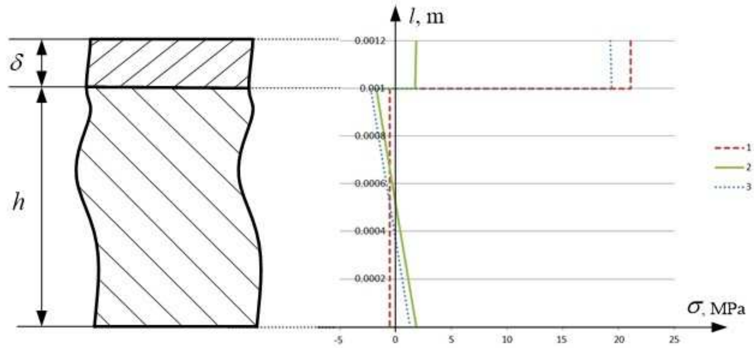

Figure 4: Stressed state of a conductive coating

$$\times \left(\frac{1-\mu_1}{E_1}\delta + \frac{1-\mu_2}{E_2}h \right) \Bigg] \; [m], \quad (13)$$

where E_1, μ_1 and E_2, μ_2 are the Young's modulus and Poisson coefficient of the conducting coating and waveguide wall, respectively.

After substitution of the data for a waveguide with dimension type of 15×35 mm, the diagrams of normal stresses over the waveguide wall thickness become of the form given in Fig. 4. Here the lines 1, 2 and 3 correspond to temperature differences Δt of $20°C$, $75°C$ and $100°C$, respectively. The material of waveguide is aluminum, that of coating is silver.

A considerable stress jump over the area of joining conducting coating and substrate is typical for all cases of calculation. This stress jump leads to appearance of tangential stresses in this area. Periodic action of tangential stresses may result, after a while, in a fatigue failure of a joint between coating and substrate. To avoid this, the corresponding choice of the method and technological parameters of coating deposition is required. For example, to reduce the maximal value of tensile stresses over the area of joining coating and substrate (lines 2 and 3 in Fig. 4), one can choose a regime of coating deposition at which residual compressive stresses of required value will exist in that area after making.

5. Conclusion

In our work a method is proposed to estimate the effect of power loss of microwave signals transmitted through a waveguide on its thermoelastic state.

We developed mathematical models of waveguide heating and obtained a resolvent differential equation of the problem with initial and boundary conditions that correspond to the waveguide operating conditions on the Earth and in the outer space. Based on the developed methods and obtained dependences, we developed a computer program to calculate thermal fields and to take into account their effect on deflected mode of waveguide as a whole.

The dependencies are obtained that determine stress distribution over the waveguide wall thickness, with allowance made for conducting coating. The obtained temperature and stress distributions may be applied to calculate the deflected mode of a waveguide as a whole, as well as of its conducting coating, for ensuring strength and hardness conditions.

Based on the results obtained, a possibility to develop methods of calculating fatigue strength of material of waveguide skin layer.

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