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ON THE BURR XII–WEIBULL SOFTWARE RELIABILITY MODEL

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Abstract: In this paper we study the Hausdorff approximation of the Heaviside step function by Burr XII–Weibull cumulative distribution function. The results have independent significance in the study of issues related to debugging theory. Numerical examples, illustrating our results are presented using programming environment Mathematica. We give also real examples with data provided in [1] using Burr XII–Weibul software reliability model. Dataset included [2] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing.

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1. Introduction

Within the hierarchical models in the procedure for quantifying the quality of

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software products, an important role is played by the so-called computational method based on the theoretical and empirical dependencies (usually at an early stage in their development), statistical data accumulated during tests, exploitation and the accompaniment of the program product. An important measure of reliability assessment (completeness, accuracy and consistency) is the so-called metric (asymptotic metrics). Numerous studies have been devoted to this overarching theme. We will only note that, depending on the test data selected and the expected results, testing is divided into: deterministic testing and stochastic testing. Detailed description of all elements in the area of debugging theory may be found in the following books [3]–[5]. For some degradation models with applications to reliability and survival analysis, see [6]. In the book [7], we pay particular attention to both deterministic approaches and probability models for debugging theories. A Hausdorff metric was chosen to evaluate the test data which are fitted to the sigmoid models proposed in this book. Some of the existing cumulative distributions (Gompertz–Makeham, Yamada-exponential, Yamada–Rayleigh, Yamada–Weibull, transmuted inverse exponential, transmuted Log-Logistic, Kumaraswamy–Dagum, Kumaraswamy Quasi Lindley and Log–logistic) are considered in the light of modern debugging and test theories. Some software reliability models, can be found in [8]–[51].

In this note we study the one–sided Hausdorff approximation of the Heaviside step function by the Burr XII–Weibull cumulative distribution function [52].

We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis.

The models have been tested with real-world data.

2. Preliminaries

Definition 1. The (basic) step function is:

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0, \end{cases}$$

usually known as *shifted Heaviside step function.*

Definition 2. The traditional form of the Burr XII–Weibull cumulative distribution function (c.d.f.) is given as follows (see, [52]):

$$M(t) = 1 - \frac{1}{(1+t^c)^k e^{at^b}} \tag{1}$$

where $c, k, a, b > 0$, and $t \geq 0$.

For other extensions of the Burr and Weibull (c.d.f.), see [53] – [56].

Definition 3. [57] The Hausdorff distance (the H–distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

3. Main Results

3.1. A note on the extended Burr XII–Weibull (c.d.f.)

We consider the following model:

$$M^*(t) = 1 - \frac{1}{(1+t^c)^k e^{at^b}}, \tag{2}$$

with $M^*(t_0) = \frac{1}{2}$ where t_0 is the positive root of the nonlinear equation:

$$(1+t_0^c)^k e^{at_0^b} - 2 = 0. \tag{3}$$

The one–sided Hausdorff distance d between the Heaviside step function and the c.d.f. ((2)–(3)) satisfies the relation

$$M^*(t_0 + d) = 1 - d. \tag{4}$$

The following theorem gives upper and lower bounds for d

Theorem 1. Let

$$p = -\frac{1}{2},$$

$$q = 1 + \frac{1}{2}ck\left(1 + t_0^c\right)^{-1} t_0^{c-1} + \frac{1}{2}abt_0^{b-1}.$$

For the one–sided Hausdorff distance d between $h_{t_0}(t)$ and function ((2)–(3)) the following inequalities hold for:

$$2.1q > e^{1.05}$$

$$d_l = \frac{1}{2.1q} < d < \frac{\ln(2.1q)}{2.1q} = d_r. \quad (5)$$

Proof. Let us examine the function:

$$F(d) = M^*(t_0 + d) - 1 + d. \quad (6)$$

From $F'(d) > 0$ we conclude that function F is increasing.

Consider the function

$$G(d) = p + qd. \quad (7)$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $2.1q > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

The model ((2)–(3)) for $a = 0.6$, $b = 7$, $c = 8$, $k = 0.2$; $t_0 = 0.990666$ is visualized on Fig. 2. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.157017$, $d_l = 0.141146$.

The model ((2)–(3)) for $a = 7.5$, $b = 18$, $c = 24$, $k = 5$; $t_0 = 0.864373$ is visualized on Fig. 3. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.0640576$, $d_l = 0.0547099$.

3.2. Application in the field of debugging and test theory.

We give real examples with data provided in [1].

The operating time of the software is 167,900 days. 115 failures are detected for these days which contain 71 unique failures.

Table 1 shows the failures data which are united for each of the 13 months.

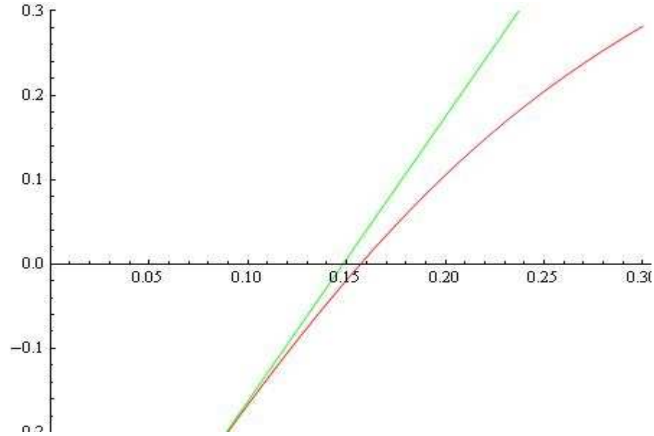

Figure 1: The functions $F(d)$ and $G(d)$.

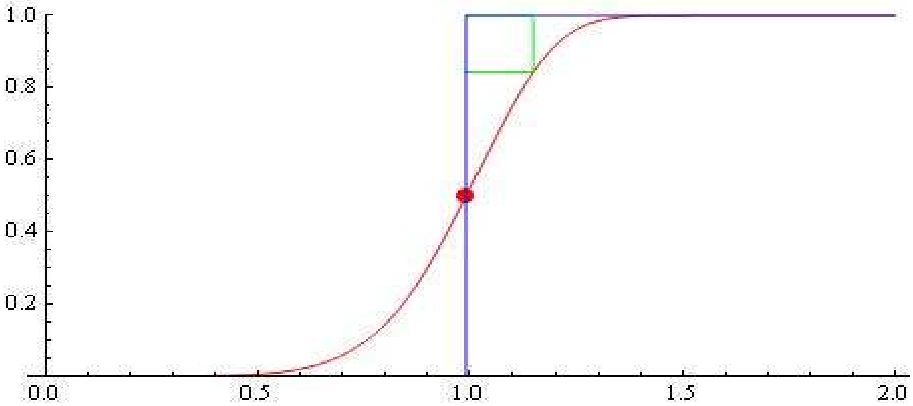

Figure 2: The model ((2)–(3)) for $a = 0.6$, $b = 7$, $c = 8$, $k = 0.2$; $t_0 = 0.990666$; H–distance $d = 0.157017$, $d_l = 0.141146$.

Dataset included [2] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing.

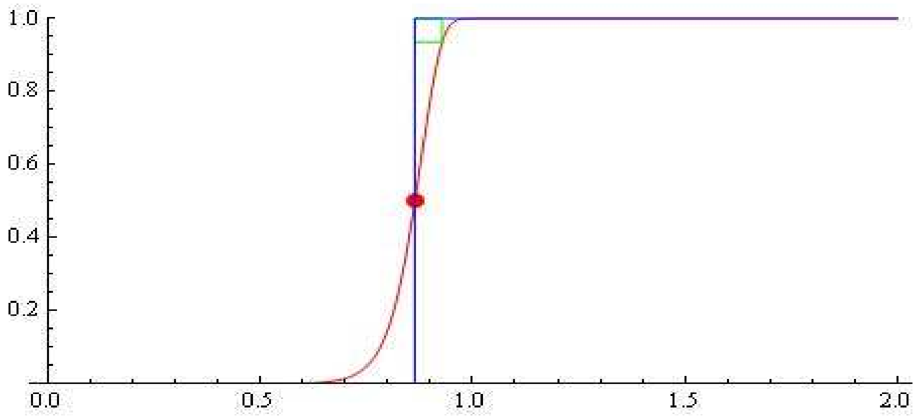

Figure 3: The model ((2)–(3)) for $a = 7.5$, $b = 18$, $c = 24$, $k = 5$; $t_0 = 0.864373$ H–distance $d = 0.0640576$, $d_l = 0.0547099$.

Month Index	System Days (Days)	System Days (Cumulative)	Failures	Cumulative Failures
1	961	961	7	7
2	4170	5131	3	10
3	8789	13,920	14	24
4	11,858	25,778	8	32
5	13,110	38,888	11	43
6	14,198	53,086	8	51
7	14,265	67,351	7	58
8	15,175	82,526	19	77
9	15,376	97,902	17	94
10	15,704	113,606	6	100
11	18,182	131,788	11	111
12	17,760	149,548	4	115
13	18,352	167,900	0	115

Table 1. Field failure data [1].

The fitted model

$$M^*(t) = \omega \left(1 - \frac{1}{(1+t^c)^k e^{at^b}}\right)$$

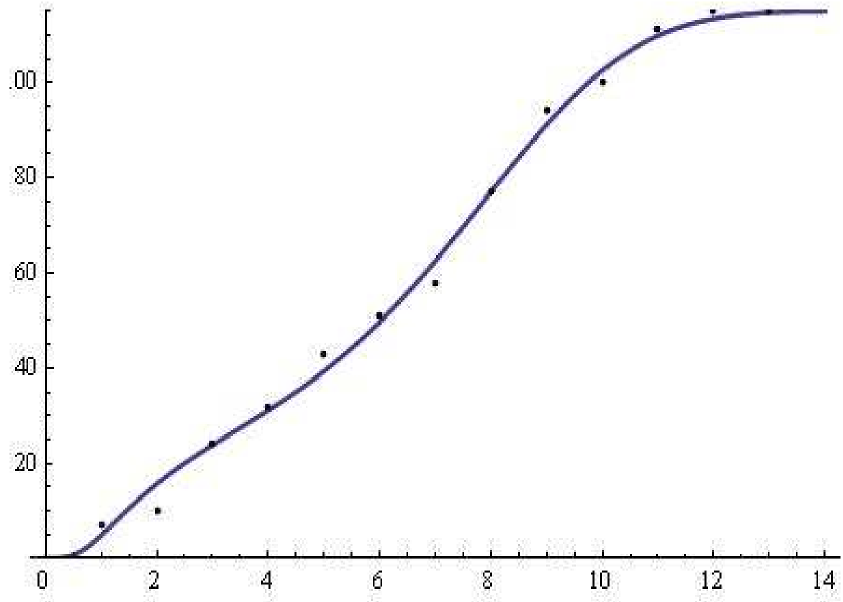

Figure 4: The fitted model $M^*(t)$.

based on the data of Table 1 for the estimated parameters:

$$\omega = 115;\ \ a = 0.000114933;\ \ b = 4.18575;\ \ c = 3.22386;\ \ k = 0.0618996$$

is plotted on Fig. 4.

The reliability or the survival function for the fitted model is plotted on Fig. 5

4. Conclusions

The Burr XII – Weibull model has algebraic tails which are effective for modeling failures that occur with lesser frequency than with corresponding models based on exponential tails.

The Burr XII – Weibull distribution gives the reliability practitioner another model for representing failure data [53].

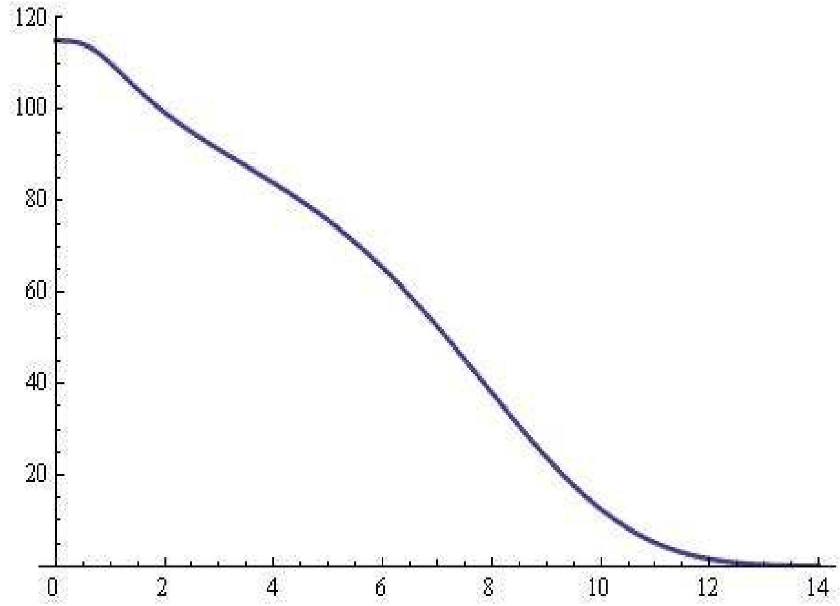

Figure 5: The survival function for the model $M^*(t)$.

From the above examples, it can be seen that the proven bottom estimate (see Theorem 1) for the value of the Hausdorff approximation is reliable when assessing the important characteristic - ”saturation”.

This characteristic as it is already known has its equal participation together with the other two characteristics - ”confidence intervals” and ”confidence bounds” in the area of the Software Reliability Theory.

Mudholkar and Srivastava [56] pioneered and studied the exponentiated Weibull distribution to analyze bathtub failure data.

The traditional form of the mixture of Burr XII and Weibull cumulative distribution functions is (see for instance [55]):

$$M_2(t) = p_1\left(1-\left(1+\left(\frac{t}{s}\right)^c\right)^{-k}\right)+p_2\left(1-e^{-\left(\frac{t}{\beta}\right)^\alpha}\right)$$

where $k, s, c, \alpha, \beta > 0$ and $p_1 + p_2 = 1$.

This model is also used for analysis in the field of debugging theory.

In [52] the following new Burr XII–Weibull–Logarithmic cumulative distribution function is proposed:

$$M_3(t) = 1 - \frac{\ln\left(1 - \theta(1+t^c)^{-k} e^{-\alpha t^\beta}\right)}{\ln(1-\theta)}$$

where $c, k, \alpha, \beta > 0$ and $0 \leq \theta < 1$.

The reader can get accurate bounds for the saturation feature using the technique described in this article.

Other important in practice activation functions [58] and possibility of their recurrent generations are explored in [59].

We hope that the results will be useful for specialists in this scientific area.

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