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CREEP TRANSITION IN TRANSVERSELY ISOTROPIC COMPOSITE CIRCULAR CYLINDER SUBJECTED TO INTERNAL PRESSURE

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Abstract: Creep stresses occur in cylinder comprised with transversely isotropic composite material subjected to internal pressure. The Seth's transition theory, based on principal Almansi's strain measure is basically applied to simplify mechanical equations by mentioning the order of the measure related to deformations.The Results have been discussed analytically and numerically. From the results, it has been concluded that highly non-homogeneous transversely isotropic thick circular cylinder subjected to internal pressure for nonlinear stain measure is better alternative for designing point of view as compared to less non-homogeneous transversely isotropic thick circular cylinder.

AMS Subject Classification: 74A10, 74C20, 74C99
Key Words: creep, pressure, cylinder, functionally graded transversely isotropic material

1. Introduction

The thick-walled hollow circular cylinders are widely used in nuclear industries, helicopter rotors and submarines etc. In general, the cylinders which are highly pressurized need a strict optimum design analysis for accurate and se-

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cure operational performance. Therefore, efforts are continually made to extend the reliability of such kinds of cylinders. Creep deformation in a thick-walled cylinders subjected to internal pressure has been discussed by Altenbach and Skrzypek [1]. A computational model for the investigation of elastic-plastic and residual stresses in functionally graded rotating solid shafts has been developed by Argeso and Eraslan [2]. Singh and Gupta [3] investigated creep stresses in a thick-walled circular cylinder under internal pressure. A closed form analytical solution for the steady state creep stresses in functionally graded rotating thick cylindrical pressure vessel has been obtained by Zahra et al. [4] using Nortons law. The authors concluded that functionally graded material has significant effect on the stresses in pressure vessels. All the above authors used the concept of infinitesimal strain theory, strain laws etc. Seth [5] defined transition theory using the concept of generalized strain measures which eliminates the use of above mentioned assumptions. This theory has been applied by many authors [6-10] to the problems of cylinder, shell, disk etc. i.e. thermal creep stresses in a transversely isotropic thick-walled rotating cylinder under internal pressure has been evaluated by Sharma et al. [6] . Sharma et al. [7] investigated creep stresses in thick-walled non-homogeneous circular cylinder under external pressure and concluded that cylinder with nonlinear measure is best alternative for the designing purpose whereas safety analysis of thermal creep non-homogeneous thick-walled pressurized circular cylinder has been discussed by Sharma et al. [8]. Thermal creep stresses have been investigated by Sharma and Panchal [9] in functionally graded thick-walled rotating shell. Sharma [10] determine thermal plastic stresses in functionally graded thick-walled cylinder and discussed that yielding in the material starts at any radius in between internal and external surface and full plasticity occurs at the internal or at the external surface.

2. Objective

In order to enhance the life of the cylinder, the failure characteristics of a cylinder under pressure should be considered carefully. In this paper, our main aim is to eliminate the need of assuming creep-strain laws, semi-empirical laws, jump conditions etc. to evaluate the creep stresses for transversely isotropic circular cylinder made up of non-homogeneous material subjected to internal pressure.

3. Mathematical Formulation

Consider a thick-walled circular cylinder made up of non-homogeneous transversely isotropic material with inner and outer radii 'r_i' and 'r_o' respectively subjected to pressure p on the internal surface. The components of the displacement are given by

$$u = r(1-\kappa), \;\; v = 0, \;\; w = dz, \tag{1}$$

where κ is a function of $r = \sqrt{x^2+y^2}$ only and d is a constant. The strain components are,

$$\begin{aligned} e_{rr} &= \frac{1}{n}[1-(r\kappa' + \kappa)^n], \quad e_{\theta\theta} = \frac{1}{n}[1-\kappa^n], \\ e_{zz} &= \frac{1}{n}[1-(1-d)^n], \quad e_{r\theta} = e_{\theta z} = e_{zr} = 0, \end{aligned} \tag{2}$$

where n is the measure and $\kappa' = \frac{d\kappa}{dr}$.

The stress-strain relations for transversely isotropic material are,

$$\begin{aligned} T_{rr} &= (C_{11}/n)\left[1-\left(\kappa + r\kappa'\right)^n\right] + \left[\left(C_{11}-2C_{66}\right)/n\right]\left[1-\kappa^n\right] + C_{13}e_{zz}, \\ T_{\theta\theta} &= \left[\left(C_{11}-2C_{66}\right)/n\right]\left[1-\left(\kappa + r\kappa'\right)^n\right] + (C_{11}/n)\left[1-\kappa^n\right] + C_{13}e_{zz}, \\ T_{zz} &= (C_{13}/n)\left[1-\left(\kappa + r\kappa'\right)^n\right] + (C_{13}/n)\left[1-\kappa^n\right] + C_{33}e_{zz}, \\ T_{r\theta} &= T_{\theta z} = T_{rz} = 0. \end{aligned} \tag{3}$$

Taking k is the non-homogeneity in the material as

$$C_{11} = C_{011}\left(\frac{r}{b}\right)^{-k}, \; C_{13} = C_{013}\left(\frac{r}{b}\right)^{-k}, \; C_{66} = C_{066}\left(\frac{r}{b}\right)^{-k}, \; C_{33} = C_{033}\left(\frac{r}{b}\right)^{-k}, \tag{4}$$

where k is non-homogeneity parameter. Equations of equilibrium are all satisfied except,

$$\frac{d}{dr}(T_{rr}) + \left(\frac{T_{rr}-T_{\theta\theta}}{r}\right) = 0. \tag{5}$$

At transition, the differential system defining the elastic state should attain some sort of criticality. The nonlinear differential equation at the transition state is obtained by substituting eqns (4) in eqn (5),

$$[-nTC_{011}\kappa^{n+1}(1+T)^{n-1}]\frac{dT}{d\kappa} = nTC_{011}\kappa^n(1+T)^n + \left(C_{011}-2C_{066}\right)nT\kappa^n$$

$$+ (-kC_{011} + 2C_{066}) \left[1 - \kappa^n (1+T)^n\right] + (C_{011} + 2C_{066})\, k(1 - \kappa^n) \\ + C_{013} e_{zz} + 2C_{066}(1 - \kappa^n), \quad (6)$$

where $r\kappa' = \kappa T$, $T = T(r)$.

The critical points or transitional points of eqn (6) are $T \to -1$ and $T \to \pm\infty$. The boundary conditions considered for the problem are

$$T_{rr} = -p \quad \text{at} \quad r = r_i \quad \text{and} \quad T_{rr} = 0 \quad \text{and} \quad r = r_o. \quad (7)$$

The resultant axial force is given by

$$2\pi \int_{r_i}^{r_o} r T_{zz} dr = 0. \quad (8)$$

4. Mathematical Approach

It is proved by many authors [6-10] that transition through principal stress difference leads to creep state for critical point $T \to -1$. Thus, the transition function R is defined as

$$R = T_{rr} - T_{\theta\theta} = (2C_{066}/n) \left(\frac{r}{b}\right)^{-k} \left[\kappa^n (1 - (T+1)^n)\right]. \quad (9)$$

After substituting eqn (6) in logarithmic differentiation of eqn (9) with respect to r and taking the asymptotic value of the obtained eqn as $T \to -1$ and integrating, we get

$$R = AF, \quad (10)$$

where A is a constant of integration and $F = e^{\int f\, dr}$ and

$$f = \frac{-2(k+n)}{r} + \frac{2nC_{066}}{rC_{011}} + \frac{r^{n-1}C_{013}}{D^n C_{011}} kne_{zz} + \frac{2r^{n-1}C_{066}}{D^n C_{011}}.$$

The asymptotic value of κ as $T \to -1$ is D/r, where$D$ being a constant. Using this value of κ in above equation, the value of f reduce to the following equation as,

$$f = \frac{-2(k+n)}{r} + \frac{2nC_{066}}{rC_{011}} + \frac{r^{n-1}C_{013}}{D^n C_{011}} kne_{zz} + \frac{2r^{n-1}C_{066}}{D^n C_{011}}.$$

From eqn (6) and eqn (10) , we get

$$T_{rr} = B - \int \frac{AF}{r} dr, \quad T_{\theta\theta} = T_{rr} - AF, \quad (11)$$

where A, B are constants of integration. To determine the constants A and B, use the boundary conditions from eqn (8) in eqn (11), we get

$$A = \frac{-p}{X_1}, \quad B = \frac{-p\left(\int \frac{AF}{r} dr\right)_{r=r_o}}{X_1}, \text{ where } X_1 = \int_{r_i}^{r_o} \frac{AF}{r} dr.$$

Substituting the value of A and B in eqn (11), we get

$$T_{rr} = \frac{-p \int_r^{r_o} \frac{F}{r} dr}{X_1}, \quad T_{\theta\theta} = \frac{-p\left[\int_r^{r_o} \frac{F}{r} dr - F\right]}{X_1}, \tag{12}$$

$$T_{zz} = \frac{C_{013}(T_{rr} - T_{\theta\theta})}{2(C_{011} - C_{066})} - \frac{(C_{013})^2 - C_{033}(C_{011} - C_{066})e_{zz}}{2(C_{011} - C_{066})}$$

Introducing the following non-dimensional components as:

$$R = (r/r_o), \quad R_0 = (r_i/r_o),$$

$$\sigma_{rr} = (T_{rr}/C_{066}), \quad \sigma_{\theta\theta} = (T_{\theta\theta}/C_{066}), \quad \sigma_{zz} = (T_{zz}/C_{066}).$$

The radial, circumferential and axial creep stresses given by eqn (12) in non-dimensional form for functionally graded cylinder can be rewritten as

$$\sigma_{rr} = \frac{(-p/C_{066}) \int_R^1 \frac{F^*}{R} dR}{X_2}, \sigma_{\theta\theta} = \frac{(-p/C_{066})\left[\int_R^1 \frac{F^*}{R} dR - F^*\right]}{X_2},$$
$$\sigma_{zz} = \frac{C_{013}(\sigma_{rr} - \sigma_{\theta\theta})}{2(C_{011} - C_{066})} - \frac{(C_{013})^2 - C_{033}(C_{011} - C_{066})e^*_{zz}}{2(C_{011} - C_{066})} \tag{13}$$

where

$$X_2 = \int_{R_0}^1 \frac{F^*}{R} dR,$$

$$F^* = e^{\int f^* dr},$$

and

$$f^* = \frac{-2(k+n)}{r_o R} + \frac{2(n+k-1)C_{066}}{r_o R C_{011}} + \frac{r_o^{n-1} R^{n-1} C_{013}}{D^n C_{011}} k n e^*_{zz} - \frac{2(k-2) r_o^{n-1} R^{n-1} C_{066}}{D^n C_{011}}.$$

These equations are same as obtained by Sharma et al. [8] without considering the effect of temperature and non-homogeneity.

Materials	C_{11}	C_{12}	C_{13}	C_{33}	C_{44}
Steel (Isotropic Material)	2.908	1.27	1.27	2.908	0.819
Magnesium (Transversely Isotropic Material)	5.97	2.62	2.17	6.17	1.64
Beryl (Transversely Isotropic Material)	2.746	0.98	0.67	4.69	0.883

Table 1: Elastic constants C_{ij} used (in units of 10^{10} N/m^2)

5. Numerical Discussion and Conclusion

For obtaining the stresses on the basis of above discussions, the values of measure $N = 1, 5$; $D = 1$and the values of internal pressure are $P =$ 5 and 10. Elastic constants C_{ij} for isotropic material (Steel) and transversely isotropic materials (Magnesium and Beryl) are given in Table 1.

Graphs are drawn in figures (1-6) between the stresses and radii ratios ($R = r\}r_o$ for transversely isotropic material (Beryl and Magnesium) and isotropic material (Steel) subjected to internal pressure.

It is observed in figure 1 that circumferential creep stresses are tensile in nature and found to be maximum at the internal surface for homogeneous circular cylinders made up of isotropic as well as transversely isotropic materials with linear measure under internal pressure. These circumferential creep stresses are maximum in the case of cylinder made of isotropic material (Steel) as compared to the cylinders made of transversely isotropic materials (Beryl and Magnesium). These circumferential creep stresses increase significantly as the measure changes from linear to nonlinear. It has also been noticed from figure 1 that circumferential creep stresses are less for cylinder made up of transversely isotropic material (Magnesium) as compared to cylinder made of isotropic material (Steel) and transversely isotropic material (Beryl). From figure 2, it is observed that circumferential creep stresses are tensile in nature and maximum at the internal surface when non-homogeneity factor is introduced. Also these circumferential creep stresses increase at the internal surface for cylinder with linear measure while these stresses decrease for cylinder with nonlinear measure. As the non-homogeneity increases, these circumferential creep stresses increase for cylinder with linear measure while these circumferential creep stresses decrease for cylinder with nonlinear measure as observed (figure 3). Also, these circumferential creep stresses are again less for functionally graded transversely isotropic cylinder (Magnesium) as compared to cylinder made up of isotropic material (Steel) and transversely isotropic material (Beryl) with nonlinear mea-

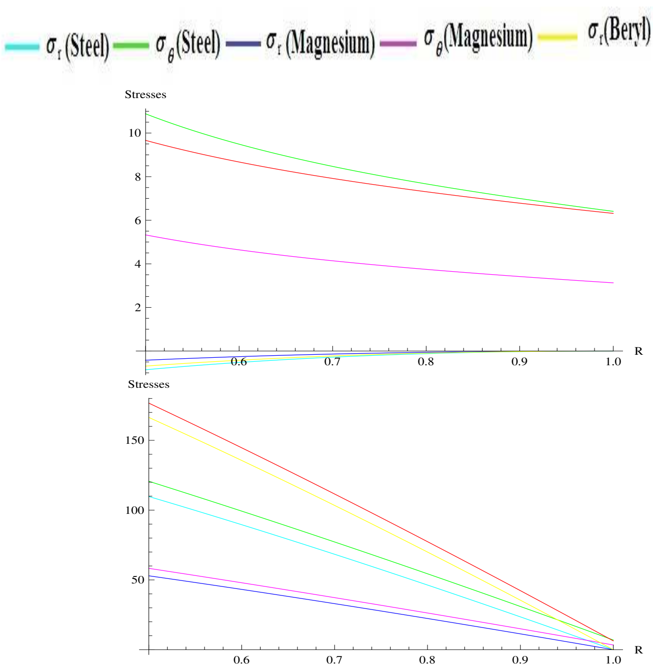

Figure 1: Creep stresses for Steel, Magnesium and Beryl with $P = 5$, $N = 1, 5$ and $k = 0$.

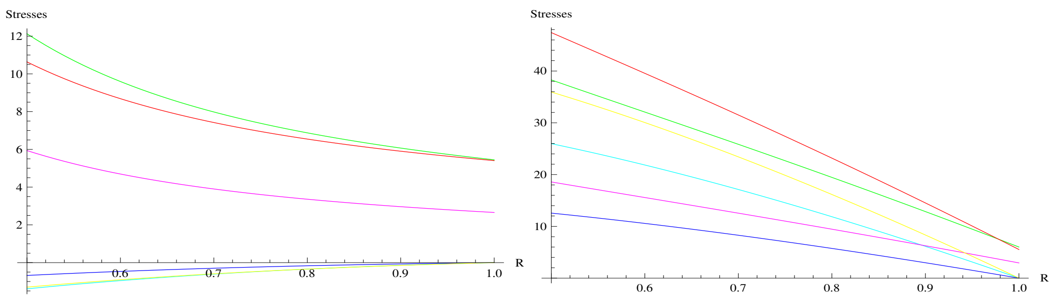

Figure 2: Creep stresses for Steel, Magnesium and Beryl with $P = 5$, $N = 1, 5$ and $k = 0.25$.

sure. It can be seen in figure 4 that, as the internal pressure increases, circumferential creep stresses increase at the internal surface cylinders made up of isotropic and transversely isotropic materials. It is also observed from figure 5 that with the introduction of non-homogeneity, these circumferential creep stresses increase with linear measure while decreases with nonlinear measure.

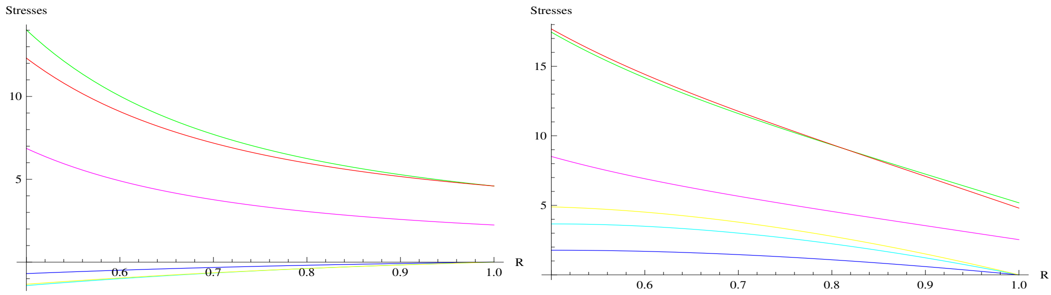

Figure 3: Creep stresses for Steel, Magnesium and Beryl with $P = 5$, $N = 1$, 5 and $k = 0.5$.

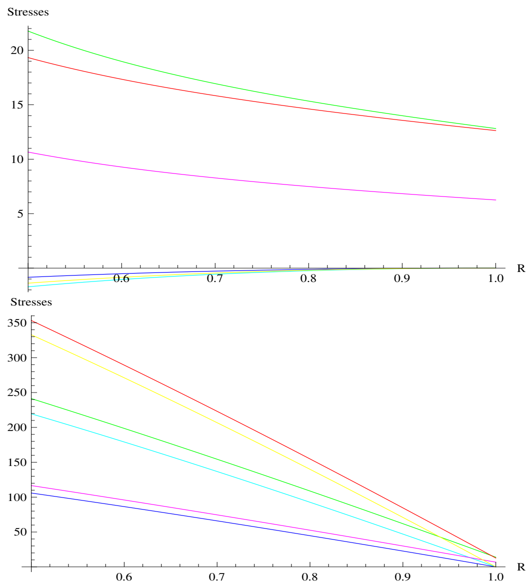

Figure 4: Creep stresses for Steel, Magnesium and Beryl with $P = 10$, $N = 1$, 5 and $k = 0$.

These stresses increased significantly with increase in internal pressure in case of circular cylinder made up of functionally graded materials. As the factor of non-homogeneity increases, circumferential stresses increase for cylinder with linear measure while decrease for cylinder with nonlinear measure which can be seen in figure 6. Also, these stresses increases significantly with the increase in internal pressure.

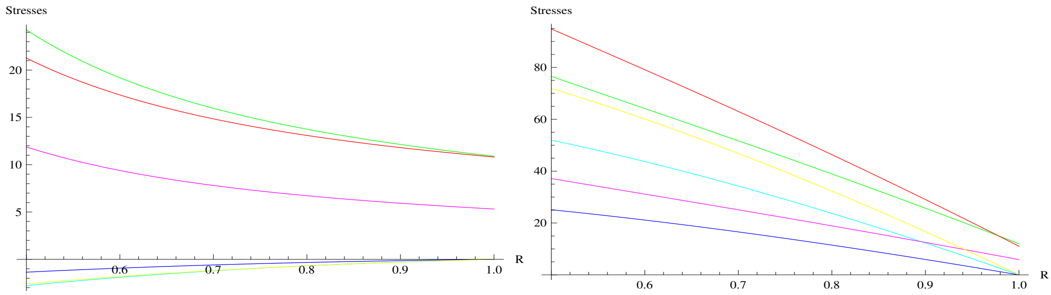

Figure 5: Creep stresses for Steel, Magnesium and Beryl with $P = 10$, $N = 1$, 5 and $k = 0.25$.

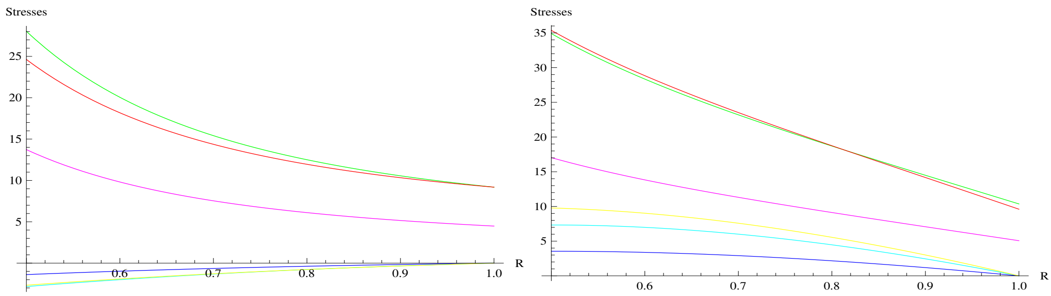

Figure 6: Creep stresses for Steel, Magnesium and Beryl with $P = 10$, $N = 1$, 5 and $k = 0.5$.

6. Conclusion

From the analysis it is concluded that circular cylinder made up of highly functionally graded transversely isotropic material (Magnesium) with nonlinear measure under internal pressure is better choice for design engineers as compared to cylinder made up of functionally graded transversely isotropic material (Beryl) and isotropic material (Steel). This is because of the reason that the circumferential stresses are less for Magnesium as compared to Steel and Beryl.

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