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EDGE TRIMAGIC TOTAL LABELINGS OF SOME SHADOW AND SPLITTING GRAPHS

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Abstract: An edge trimagic total labeling of a graph $G = (V, E)$ with p vertices and q edges is a bijection $f : V(G) \cup E(G) \to \{1, 2, \ldots, p + q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is either k_1 or k_2 or k_3. In this paper, we find the edge trimagic total labeling of some shadow and splitting graphs.

AMS Subject Classification: 05C50, 05C25
Key Words: graph, bijective function, shadow graphs, splitting graphs, trimagic total labeling

1. Introduction

Based on certain criteria, the vertices or sometimes edges of the graphs are assigned with integers that is known to be as graph labeling. Rosa, Graham and Sloane are the pioneers of graph labeling. Rosa [7] in 1967 and Graham and Sloane [2] in 1980 introduced the concept of labeling.

Magic labeling was introduced by Kotzing and Rosa [7] defined by a graph G with a bijection $f : V \cup E \to \{1, 2, \ldots, p + q\}$ such that, for each edge $uv \in E(G), f(u) + f(uv) + f(v)$ is a magic constant. Edge bimagic labeling

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of graphs was introduced by J. Baskar Babujee [1] in 2004. An edge bimagic labeling of a (p, q) graph G is a bijection $f : V \cup E \to \{1, 2, \ldots, p+q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is either k_1 or k_2.

In 2013, Jayasekaran et al. [3] introduced the edge trimagic total labeling of graphs. An edge trimagic total labeling of a (p, q) graph G is a bijection $f : V \cup E \to \{1, 2, \ldots, p + q\}$ such that for each edge $uv \in E(G)$, the value of $f(u) + f(uv) + f(v)$ is equal to any of the distinct constants k_1 or k_2 or k_3. An edge trimagic total labeling is called a super edge trimagic total labeling of a graph G, if the vertices are labeled with the smallest possible integers i.e. $1, 2, \ldots, p$.

The shadow graph $D_2(G)$ [6] of a connected graph G is constructed by taking two copies of G, G' and G'' and joining each vertex u' in G' to the neighbors of the corresponding vertex u'' in G''. For each point v of G, take a new point v'. Join v' to all points of G adjacent to v. The graph $S(G)$ thus obtained is called the splitting graph [8] of G.

In this paper we prove that the shadow and splitting graph of path P_n, star $K_{1,n}$ and cycle C_n are edge trimagic total and super edge trimagic total labeling. For further reference we can look at J.A. Gallians [4] dynamic survey of graph labeling.

2. Main Results

Theorem 2.1. *The shadow graph $D_2(K_1, n)$ has an edge trimagic total labeling for all n.*

Proof. Let $v, v_1, v_2, \ldots, v_n$ and $u, u_1, u_2, \ldots, u_n$ be the vertices of two copies of $K_{1,n}$, where v and u are respective central vertices. Join v with u_i and u with $v_i, 1 \leq i \leq n$ and the resultant graph $D_2(K_{1,n})$ has vertex set $V = \{v, u\} \cup \{u_i, v_i / 1 \leq i \leq n\}$ and the edge set $E = \{vv_i, uu_i / 1 \leq i \leq n\} \cup \{vu_i, uv_i / 1 \leq i \leq n\}$. Thus $D_2(K_{1,n})$ has $2n + 2$ vertices and $4n$ edges.

Define a bijection $f : V \cup E \to \{1, 2, \ldots, 6n+2\}$ such that $f(v) = 1; f(u) = 2n+2; f(v_i) = i+1, 1 \leq i \leq n; f(u_i) = n+i+1, 1 \leq i \leq n; f(vv_i) = 6n+3-i, 1 \leq i \leq n; f(uv_i) = 3n + 3 - i, 1 \leq i \leq n; f(uu_i) = 4n + 3 - i, 1 \leq i \leq n$ and $f(vu_i) = 5n + 3 - i, 1 \leq i \leq n$.

Now we show that this labeling is an edge trimagic total labeling. Consider the edges $vv_i, 1 \leq i \leq n$.
$f(v) + f(vv_i) + f(v_i) = 1 + 6n + 3 - i + i + 1 = 6n + 5 = \lambda_1$.
Consider the edges $vu_i, 1 \leq i \leq n$.
$f(v) + f(vu_i) + f(u_i) = 1 + 5n + 3 - i + n + i + 1 = 6n + 5 = \lambda_1$.

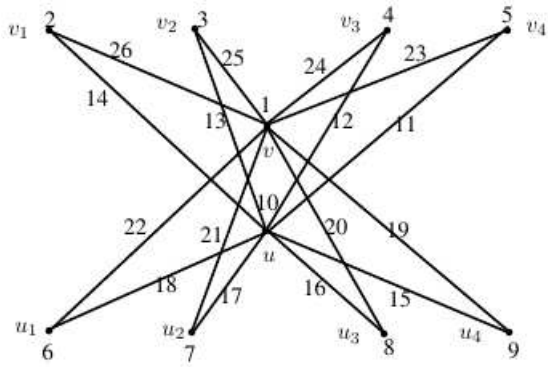

Figure 1: $D_2(K_{1,4})$ with $\lambda_1 = 29, \lambda_2 = 34$ and $\lambda_3 = 26$

Consider the edges $uu_i, 1 \leq i \leq n$.
$f(u) + f(uu_i) + f(u_i) = 2n + 2 + 4n + 3 - i + n + i + 1 = 7n + 6 = \lambda_2$.
Consider the edges $uv_i, 1 \leq i \leq n$.
$f(u) + f(uv_i) + f(vi) = 2n + 2 + 3n + 3 - i + i + 1 = 5n + 6 = \lambda_3$.
Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 6n + 5, \lambda_2 = 7n + 6$ and $\lambda_3 = 5n + 6$. Therefore, the shadow graph $D_2(K_{1,n})$ admits an edge trimagic total labeling for all $n \geq 3$. Hence the result. □

Corollary 2.2. The shadow graph $D_2(K_{1,n})$ admits super edge trimagic total labeling for all $n \geq 3$ as shown in Figure 1.

Theorem 2.3. *The shadow graph $D_2(P_n)$ admits an edge trimagic total labeling for all n.*

Proof. Let $v_1v_2 \ldots v_n$ and $u_1u_2 \ldots u_n$ be two paths of length n. Join u_i with v_{i+1} and v_i with $u_{i+1}, 1 \leq i \leq n-1$ the resultant graph is $D_2(P_n)$ with the vertex set $V = \{u_i, v_i/1 \leq i \leq n\}$ the edge set $E = \{u_iu_{i+1}, v_iv_{i+1}, u_iv_{i+1}, v_iu_{i+1}/1 \leq i \leq n-1\}$. Thus $D_2(P_n)$ has $2n$ vertices and $4n-4$ edges.

Define a bijection $f : V \cup E \to \{1, 2, \ldots, 6n-4\}$ such that $f(u_i) = 2i-1, 1 \leq i \leq n; f(v_i) = 2i, 1 \leq i \leq n; f(u_iu_{i+1}) = 6n - 4i - 1, 1 \leq i \leq n-1; f(v_iv_{i+1}) = 6n - 4i, 1 \leq i \leq n-1; f(u_iv_{i+1}) = 6n - 4i - 3, 1 \leq i \leq n-1$ and $f(v_iu_{i+1}) = 6n - 4i - 2, 1 \leq i \leq n-1$.

Now we show that this labeling is an edge trimagic total labeling.
For the edges $u_iu_{i+1}, 1 \leq i \leq n-1$,
$f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = 2i - 1 + 6n - 4i - 1 + 2i + 1 = 6n - 1 = \lambda_1$.

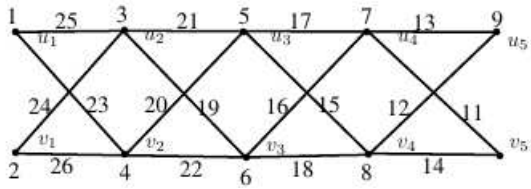

Figure 2: $D_2(P_5)$ with $\lambda_1 = 29, \lambda_2 = 28$ and $\lambda_3 = 32$

For the edges $u_iv_{i+1}, 1 \leq i \leq n-1$,
$f(u_i) + f(u_iv_{i+1}) + f(v_{i+1}) = 2i - 1 + 6n - 4i - 3 + 2i + 2 = 6n - 2 = \lambda_2$.
For the edges $v_iu_{i+1}, 1 \leq i \leq n-1$,
$f(v_i) + f(v_iu_{i+1}) + f(u_{i+1}) = 2i + 6n - 4i - 2 + 2i + 1 = 6n - 1 = \lambda_1$.
For the edges $v_iv_{i+1}, 1 \leq i \leq n-1$,
$f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = 2i + 6n - 4i + 2i + 2 = 6n + 2 = \lambda_3$.
Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 6n - 1, \lambda_2 = 6n - 2$ and $\lambda_3 = 6n + 2$. Therefore, the shadow graph $D_2(P_n)$ admits an edge trimagic total labeling for all n as shown in Figure 2. Hence the result. □

Corollary 2.4. The shadow graph $D_2(P_n)$ admits super edge trimagic total labeling for all n.

Theorem 2.5. *The shadow graph $D_2(C_n)$ admits an edge trimagic total labeling for every positive odd integer $n \geq 3$.*

Proof. Let $v_1v_2\ldots, v_nv_1$ and $u_1u_2\ldots u_nu_1$ be two copies of $C_n, n \geq 3$ is odd. Join u_i with v_{i+1}, v_i with u_{i+1} and join u_1v_n and v_1u_n. Then the resultant graph is $D_2(C_n)$ with vertex set $V = \{u_i, v_i/1 \leq i \leq n\}$ and the edge set $E = \{u_iu_{i+1}, v_iv_{i+1}/1 \leq i \leq n-1\} \cup \{u_1u_n, v_1v_n\} \cup \{u_iv_{i-1}, /2 \leq i \leq n\} \cup \{u_1v_n, u_nv_1\} \cup \{u_iv_{i+1}/1 \leq i \leq n-1\}$. Thus $D_2(C_n)$ has $2n$ vertices and $4n$ edges.

Define a bijection $f : V \cup E \to \{1, 2, \ldots, 6n\}$ as follows:

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n$,

$$f(v_i) = \begin{cases} n + \frac{i+1}{2}, & i \text{ is odd} \\ n + \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n-1, f(u_iu_{i+1}) = 4n - i; f(v_iv_{i+1}) = 6n - i;$

$$f(u_iv_{i+1}) = \begin{cases} 5n - i, & i \text{ is odd} \\ 3n - i, & i \text{ is even} \end{cases}$$

$$f(v_iu_{i+1}) = \begin{cases} 3n - i, & i \text{ is odd} \\ 5n - i, & i \text{ is even} \end{cases}$$

And $f(u_1u_n) = 4n; f(v_1v_n) = 6n; f(u_nv_1) = 3n$ and $f(u_1v_n) = 5n$.
Now we show that this labeling is an edge trimagic total labeling.
Consider the edges $u_iu_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 4n - i + \frac{n+i+2}{2} = \frac{9n+3}{2} = \lambda_1$.
For even $i, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{n+i+1}{2} + 4n - i + \frac{i+2}{2} = \frac{9n+3}{2} = \lambda_1$.
For the edge $u_1u_n, f(u_1) + f(u_1u_n) + f(u_n) = 1 + 4n + \frac{n+1}{2} = \frac{9n+3}{2} = \lambda_1$
For the edge $u_nv_1, f(u_n) + f(u_nv_1) + f(v_1) = \frac{n+1}{2} + 3n + n + 1 = \frac{9n+3}{2} = \lambda_1$.
Consider the edges $v_iu_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(v_i) + f(v_iu_{i+1}) + f(u_{i+1}) = n + \frac{i+1}{2} + 3n - i + \frac{n+i+2}{2} = \frac{9n+3}{2} = \lambda_1$.
For even $i, f(v_i) + f(v_iu_{i+1}) + f(u_{i+1}) = n + \frac{n+i+1}{2} + 5n - i + \frac{i+2}{2} = \frac{13n+3}{2} = \lambda_2$.
Consider the edges $u_iv_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(u_i) + f(u_iv_{i+1}) + f(v_{i+1}) = \frac{i+1}{2} + 5n - i + n + \frac{n+i+2}{2} = \frac{13n+3}{2} = \lambda_2$.
For even $i, f(u_i) + f(u_iv_{i+1}) + f(v_{i+1}) = \frac{n+i+1}{2} + 3n - i + n + \frac{i+2}{2} = \frac{9n+3}{2} = \lambda_1$.
For the edge $v_{n-1}u_n, f(v_{n-1}) + f(v_{n-1}u_n) + f(u_n) = 2n + 4n + 1 + \frac{n+1}{2} = \frac{13n+3}{2} = \lambda_2$.
For the edge $u_1v_n, f(u_1) + f(u_1v_n + f(v_n) = 1 + 5n + \frac{3n+1}{2} = \frac{13n+3}{2} = \lambda_2$.
Consider the edges $v_iv_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = n+\frac{i+1}{2}+6n-i+n+\frac{n+i+2}{2} = \frac{17n+3}{2} = \lambda_3$.
For even $i, f(v_i)+f(v_iv_{i+1})+f(v_{i+1}) = n+\frac{n+i+1}{2}+6n-i+n+\frac{i+2}{2} = \frac{17n+3}{2} = \lambda_3$.
For the edge $v_1v_n, f(v_1)+f(v_1v_n)+f(v_n) = n+1+6n+n+\frac{n+1}{2} = \frac{17n+3}{2} = \lambda_3$.

Hence for each edge $uv \in E, f(u)+f(uv)+f(v)$ yields any one of the constants $\lambda_1 = \frac{9n+3}{2}$, $\lambda_2 = \frac{13n+3}{2}$ and $\lambda_3 = \frac{17n+3}{2}$. Therefore, the shadow graph $D_2(C_n)$ has an edge trimagic total labeling for every positive odd integer $n \geq 3$ as shown in Figure 3. Hence the result. □

Corollary 2.6. *The shadow graph $D_2(C_n)$ has a super edge trimagic total labeling for odd $n \geq 3$.*

Theorem 2.7. *The splitting graph of $K_{1,n}, S(K_{1,n})$ has an edge trimagic total labeling for all n.*

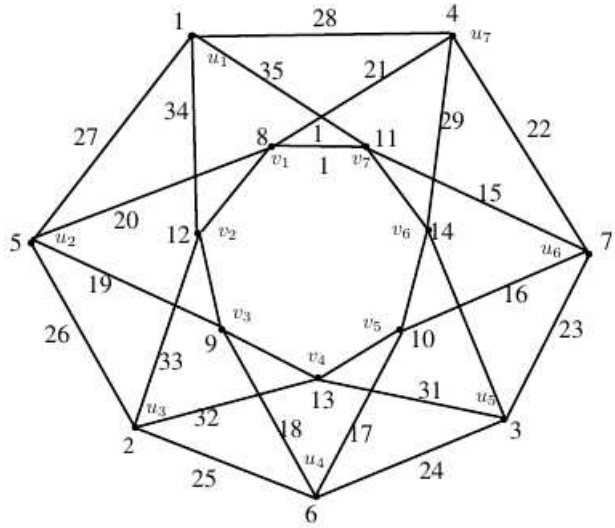

Figure 3: $D_2(C_7)$ with $\lambda_1 = 33, \lambda_2 = 47$ and $\lambda_3 = 61$

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of $K_{1,n}$ with centre u. For each vertex u_i add vertex $v_i, 1 \leq i \leq n$ such that v_i is adjacent to u and for the vertex u, add a new vertex v which is adjacent to each $u_i, 1 \leq i \leq n$. The resultant graph is $S(K_{1,n})$ with the vertex set $V = \{v, u\} \cup \{u_i, v_i / 1 \leq i \leq n\}$ and the edge set $E = \{vu_i, uu_i, uv_i / 1 \leq i \leq n\}$. Thus $S(K_{1,n})$ has $2n + 2$ vertices and $3n$ edges. Define a bijection $f : V \cup E \to \{1, 2, \ldots, 5n + 2\}$ such that $f(u) = 1,\ f(v) = n + 2, f(u_i) = i + 1, 1 \leq i \leq n, f(v_i) = n + i + 2, 1 \leq i \leq n; f(uu_i) = 4n - i + 3, 1 \leq i \leq n; f(vu_i) = 5n - i + 3, 1 \leq i \leq n$ and $f(uv_i) = 3n - i + 3, 1 \leq i \leq n$.

Now we show that this labeling is an edge trimagic total labeling.
For the edges $vu_i, 1 \leq i \leq n, f(v) + f(vu_i) + f(u_i) = n + 2 + 5n - i + 3 + i + 1 = 6(n + 1) = \lambda_1$.
For the edges $uui, 1 \leq i \leq n, f(u) + f(uu_i) + f(u_i) = 1 + 4n - i + 3 + i + 1 = 4n + 5 = \lambda_2$.
For the edges $uvi, 1 \leq i \leq n, f(u) + f(uv_i) + f(vi) = 1 + 3n - i + 3 + n + i + 2 = 4n + 6 = \lambda_3$.
Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = 6(n + 1), \lambda_2 = 4n + 5$ and $\lambda_3 = 4n + 6$. Therefore, the shadow graph $S(K_{1,n})$ has an edge trimagic total labeling for all n as shown in figure 4. Hence the result. $\square$

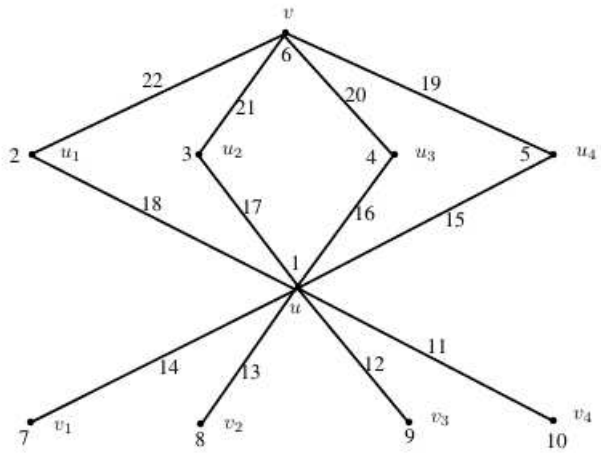

Figure 4: $S(K_{1,4})$ with $\lambda_1 = 30, \lambda_2 = 21$ and $\lambda_3 = 22$

Corollary 2.8. The splitting graph of $K_{1,n}, S(K_{1,n})$ has a super edge trimagic total labeling for all n.

Theorem 2.9. *The splitting graph of a path $P_n, S(P_n)$ has an edge trimagic total labeling for all n.*

Proof. Let $u_1u_2 \dots u_n$ be a path P_n. For each vertex u_i, add a vertex $v_i, 1 \leq i \leq n$ so that v_i is adjacent to the vertices which are adjacent to u_i. The resulting graph is $S'(P_n)$ with the vertex set $V = \{u_i, v_i / 1 \leq i \leq n\}$ and the edge set $E = \{u_iu_{i+1}, u_iv_{i+1}, v_iu_{i+1} / 1 \leq i \leq n-1\}$. Thus $S(P_n)$ has $2n$ vertices and $3n-3$ edges. Now we have the following two cases:

Case 1: n is odd

Define a bijection $f : V \cup E \to \{1, 2, \dots, 5n-3\}$ as follows:

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} n + \frac{i+1}{2}, & i \text{ is odd} \\ n + \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n, f(u_iu_{i+1}) = 3n - i; f(u_iv_{i+1}) = 4n - i - 1$ and $f(v_iu_{i+1}) = 5n - i - 2$.

Now we show that this labeling is an edge trimagic total labeling.

Consider the edges $u_iu_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 3n - i + \frac{n+i+2}{2} = \frac{7n+3}{2} = \lambda_1$.
For even $i, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{n+i+1}{2} + 3n - i + \frac{i+2}{2} = \frac{7n+3}{2} = \lambda_1$.
Consider the edges $v_iu_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(v_i)+f(v_iu_{i+1})+f(u_{i+1}) = n+\frac{i+1}{2}+5n-i-2+\frac{n+i+2}{2} = \frac{13n-1}{2} = \lambda_2$.
For even $i, f(v_i)+f(v_iu_{i+1})+f(u_{i+1}) = n+\frac{n+i+1}{2}+5n-i-2+\frac{i+2}{2} = \frac{13n-1}{2} = \lambda_2$.
Consider the edges $u_iv_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(u_i)+f(u_iv_{i+1})+f(v_{i+1}) = \frac{i+1}{2}+4n-i-1+n+\frac{n+i+2}{2} = \frac{11n+1}{2} = \lambda_3$.
For even $i, f(u_i)+f(u_iv_{i+1})+f(v_{i+1}) = \frac{n+i+1}{2}+4n-i-1+n+\frac{i+2}{2} = \frac{11n+1}{2} = \lambda_3$.
Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = \frac{7n+3}{2}, \lambda_2 = \frac{13n-1}{2}$ and $\lambda_3 = \frac{11n+1}{2}$. Therefore, the graph $S(P_n)$ has an edge trimagic total labeling for odd n.

Case 2: n is even

Define a bijection $f : V \cup E \to \{1, 2, \ldots, 5n-3\}$ as follows:

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i}{2}, & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} n + \frac{i+1}{2}, & i \text{ is odd} \\ n + \frac{n+i}{2}, & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n-1, f(u_iu_{i+1}) = 3n - i; f(u_iv_{i+1}) = 4n - i - 1$ and $f(v_iu_{i+1}) = 5n - i - 2$.
Now we show that this labeling is an edge trimagic total labeling.
Consider the edges $u_iu_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{i+1}{2} + 3n - i + \frac{n+i+1}{2} = \frac{7n+2}{2} = \lambda_1$.
For even $i, f(u_i) + f(u_iu_{i+1}) + f(u_{i+1}) = \frac{i+n}{2} + 3n - i + \frac{i+2}{2} = \frac{7n+2}{2} = \lambda_1$.
Consider the edges $v_iu_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(v_i)+f(v_iu_{i+1})+f(u_{i+1}) = n+\frac{i+1}{2}+5n-i-2+\frac{n+i+1}{2} = \frac{13n-2}{2} = \lambda_2$.
For even $i, f(v_i)+f(v_iu_{i+1})+f(u_{i+1}) = n+\frac{n+i}{2}+5n-i-2+\frac{i}{2}+1 = \frac{13n-2}{2} = \lambda_2$.
Consider the edges $u_iv_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(u_i)+f(u_iv_{i+1})+f(v_{i+1}) = \frac{i+1}{2}+4n-i-1+n+\frac{n+i+1}{2} = \frac{11n}{2} = \lambda_3$.
For even $i, f(u_i)+f(u_iv_{i+1})+f(v_{i+1}) = \frac{n+i}{2}+4n-i-1+n+\frac{i}{2}+1 = \frac{11n}{2} = \lambda_3$.
Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = \frac{7n+2}{2}, \lambda_2 = \frac{13n-2}{2}$ and $\lambda_3 = \frac{11n}{2}$. Therefore, the graph S(Pn) admits an edge trimagic total labeling for even n. Hence S(Pn) has an edge trimagic total labeling for all n as shown in Figure 5. Hence the result. □

Corollary 2.10. The splitting graph of a path $P_n, S(P_n)$ has a super edge trimagic total labeling for all n.

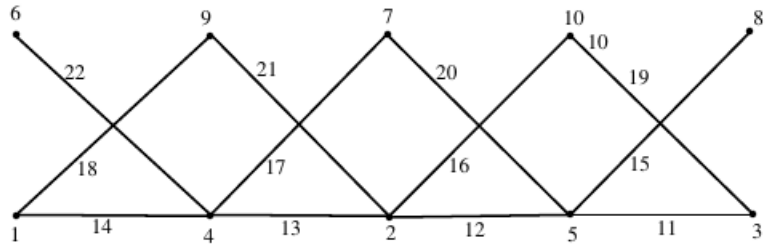

Figure 5: $S(P_5)$ with $\lambda_1 = 19, \lambda_2 = 32$ and $\lambda_3 = 28$

Theorem 2.11. *The splitting graph of cycle $C_n, S(C_n)$ has an edge trimagic total labeling for odd $n \geq 3$.*

Proof. Let $v_1v_2 \ldots v_nv_1$ be the cycle C_n. For $1 \leq i \leq n$, add u_i such that u_i is adjacent to the vertices which are adjacent to v_i. The resultant graph is $S'(C_n)$ with vertex set $V = \{v_i, u_i/1 \leq i \leq n\}$ and edge set $E = \{v_iv_{i+1}, u_iv_{i+1}/1 \leq i \leq n-1\} \cup \{v_1v_n, v_1u_n, u_1v_n\}$. Thus $S(C_n)$ has $2n$ vertices and $3n$ edges.
Define a bijection $f : V \cup E \to \{1, 2, \ldots, 5n\}$ as follows:

For $1 \leq i \leq n$,

$$f(u_i) = \begin{cases} \frac{i+1}{2}, & i \text{ is odd} \\ \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} n + \frac{i+1}{2}, & i \text{ is odd} \\ n + \frac{n+i+1}{2}, & i \text{ is even} \end{cases}$$

For $1 \leq i \leq n-1$,

$$f(u_iv_{i+1}) = \begin{cases} 4n - i, & i \text{ is odd} \\ 3n - i, & i \text{ is even} \end{cases}$$

$$f(v_iu_{i+1}) = \begin{cases} 3n - i, & i \text{ is odd} \\ 4n - i, & i \text{ is even} \end{cases}$$

$f(v_iv_{i+1}) = 5n - i$. And $f(v_1v_n) = 5n; f(u_nv_1) = 3n$ and $f(v_nu_1) = 4n$.
Now we show that this labeling is an edge trimagic total labeling.
Consider the edges $v_iu_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(v_i) + f(v_iu_{i+1}) + f(u_{i+1}) = n + \frac{i+1}{2} + 3n - i + \frac{n+i+2}{2} = \frac{9n+3}{2} = \lambda_1$.
For even $i, f(v_i) + f(v_iu_{i+1}) + f(u_{i+1}) = n + \frac{n+i+1}{2} + 4n - i + \frac{i+2}{2} = \frac{11n+3}{2} = \lambda_2$.

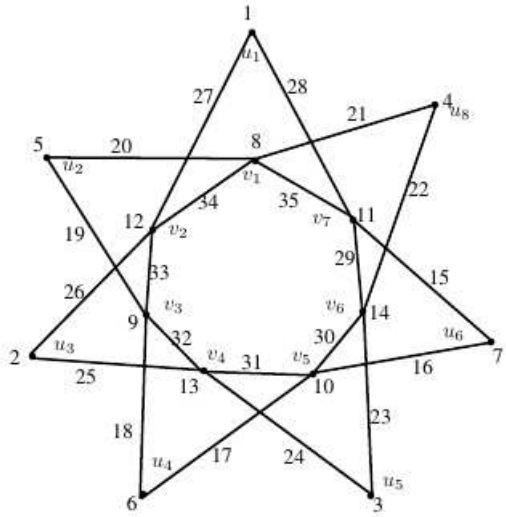

Figure 6: $S(C_7)$ with $\lambda_1 = 33, \lambda_2 = 40$ and $\lambda_3 = 54$

Consider the edges $u_iv_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(u_i) + f(u_iv_{i+1}) + f(v_{i+1}) = \frac{i+1}{2} + 4n - i + n + \frac{n+i+2}{2} = \frac{11n+3}{2} = \lambda_2$
For even $i, f(u_i) + f(u_iv_{i+1}) + f(v_{i+1}) = \frac{n+i+1}{2} + 3n - i + n + \frac{i}{2} + 1 = \frac{9n+3}{2} = \lambda_1$.
For the edge $v_1u_n, f(v_1) + f(v_1u_n) + f(u_n) = n + 1 + 3n + \frac{n+1}{2} = \frac{9n+3}{2} = \lambda_1$.
For the edge $u_1v_n, f(u_1) + f(u_1v_n) + f(v_n) = 1 + 4n + n + \frac{n+1}{2} = \frac{11n+3}{2} = \lambda_2$.

Consider the edges $v_iv_{i+1}, 1 \leq i \leq n-1$.
For odd $i, f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = n + \frac{i+1}{2} + 5n - i + n + \frac{n+i+2}{2} = \frac{15n+3}{2} = \lambda_3$.
For even $i, f(v_i) + f(v_iv_{i+1}) + f(v_{i+1}) = n + \frac{n+i+1}{2} + 5n - i + n + \frac{i+2}{2} = \frac{15n+3}{2} = \lambda_3$.
For the edge $v_1v_n, f(v_1) + f(v_1v_n) + f(v_n) = n + 1 + 5n + n + \frac{n+1}{2} = \frac{15n+3}{2} = \lambda_3$.

Hence for each edge $uv \in E, f(u) + f(uv) + f(v)$ yields any one of the magic constants $\lambda_1 = \frac{9n+3}{2}, \lambda_2 = \frac{11n+3}{2}$ and $\lambda_3 = \frac{15n+3}{2}$. Therefore, the graph $S(C_n)$ is an edge trimagic total labeling for odd $n \geq 3$ as shown in Figure 6. Hence the result. □

Corollary 2.12. *The splitting graph of cycle $C_n, S(C_n)$ has a super edge trimagic total labeling for odd $n \geq 3$.*

3. Conclusion

In this paper we have shown that the shadow and splitting graphs of path P_n, star $K_{1,n}$ and cycle C_n are edge trimagic total and super edge trimagic total labeling.

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