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A NOTE ON THE "TRANSMUTED TRANSMUTED–G FAMILY" OF CUMULATIVE DISTRIBUTION FUNCTIONS

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ABSTRACT: In [1] the authors proposed a new "transmuted transmuted – G (TT–G) family" of distributions.

The authors' assertion that probability distribution (in some particular cases) produces very good results in approximating specific data from different fields such as population dynamics, biostatistics, survival analysis and others has encouraged us to conduct further studies on " saturation " in Hausdorff sense of the corresponding commutative function to the horizontal asymptote.

We also analyze some experimental data.

The experiments show that in some cases the use of the model proposed in [1] and analyzed in this article with "respect to the Hausdorff distance" is satisfactory.

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

AMS Subject Classification: 41A46
Key Words: "transmuted transmuted – G family" (TT–G) of distributions, Heaviside function, Hausdorff approximation

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1. INTRODUCTION AND PRELIMINARIES

It is well known that many data sett from finance, reliability analysis, biochemical sciences and other fields do not follow known continuous distributions.

In this regard, in [1], the authors examined a new family of continuous distributions called the "transmuted transmuted – G (TT–G) family" which extends the quadratic rank transmutation map [2].

The cumulative distribution function (cdf) of the traditional transmuted – G (TG) family is given as

$$H(t;\lambda,\phi) = (1+\lambda)G(t;\phi) - \lambda G^2(t;\phi),$$

where $G(t;\phi)$ is the baseline cumulative distribution function.

Some extensions and modifications of the (TG) family can be found in [3]–[10].

Definition 1. The cdf of the new (TT–G) family can be expressed as [1]:

$$\begin{aligned} F(t;a,\lambda,\phi) &= (1+a)H(t;\lambda,\phi) - aH^2(t;\lambda,\phi), \\ H(t;\lambda,\phi) &= (1+\lambda)G(t;\phi) - \lambda G^2(t;\phi), \end{aligned} \tag{1}$$

where $G(t;\phi)$ is the baseline cumulative distribution function.

Definition 2. The *shifted Heaviside step function* is defined by

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases}$$

Definition 3. The Hausdorff distance [11] (the H–distance) $\rho(f,g)$ between two interval functions f,g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$.

More precisely,

$$\rho(f,g) = \max\{\sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A=(t_A,x_A)$, $B=(t_B,x_B)$ in $\mathbb{R}^2$ is $||A-B|| = \max(|t_A-t_B|,|x_A-x_B|)$.

We study the "saturation" in Hausdorff sense of (cdf) of the new (TT–G) family of type (1) to the horizontal asymptote.

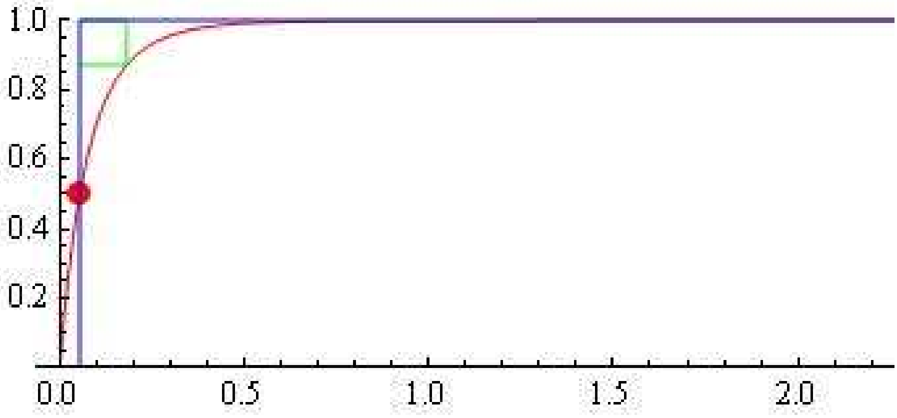


Figure 1: The model (1) for $a = 0.5$, $\lambda = 0.1$, $\theta = 8.1$ and $t_0 = 0.0548958$; H–distance $d = 0.127494$.

2. MAIN RESULTS AND NUMERICAL EXAMPLES

1. For example, let $G(t) = 1 - e^{-\theta t}$.

Let t_0 is the value for which $F(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the (cdf) $F(t)$ (1) satisfies the relation

$$F(t_0 + d) = 1 - d. \tag{2}$$

For given $a > 0$, $\lambda > 0$, $\theta > 0$ and t_0, the nonlinear equation $F(t_0 + d) - 1 + d = 0$ has unique positive root – d.

The model (1) for $a = 0.5$, $\lambda = 0.1$, $\theta = 8.1$ and $t_0 = 0.0548958$ is visualized on Fig. 1.

From the nonlinear equation (2) we have: $d = 0.127494$.

The model (1) for $a = 0.4$, $\lambda = 0.02$, $\theta = 25$ and $t_0 = 0.0203606$ is visualized on Fig. 2.

From the nonlinear equation (2) we have: $d = 0.0684804$.

Some computational examples are presented in Table 1.

2. Some comparisons between (cdf) of traditional (TG) family - $H(t)$ and the (cdf) of the new (TT–G) family - $F(t)$.

Let $a = 0.8$, $\lambda = 0.2$ and $\theta = 10$ are fixed.

Evidently, for $t_0 = 0.0598729$ we have $H(t_0) = \frac{1}{2}$.

From the nonlinear equation $H(t_0 + d_1) - 1 + d_1 = 0$ for the one–sided Hausdorff distance d_1 between the function $h_{t_0}(t)$ and the (cdf) $H(t)$ we have $d_1 = 0.127524$.

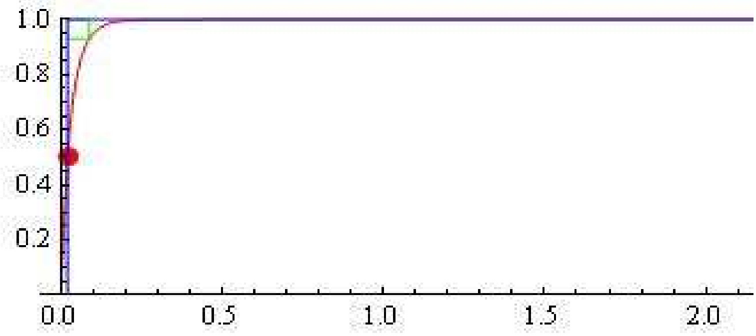


Figure 2: The model (1) for $a = 0.4$, $\lambda = 0.02$, $\theta = 25$ and $t_0 = 0.0203606$; H–distance $d = 0.0684804$.

a	λ	θ	t_0	$H - distance$
0.5	0.1	8.1	0.0548958	0.127494
0.5	0.01	20	0.0238704	0.0761122
0.6	0.2	5	0.0765814	0.153787
0.4	0.02	25	0.0203606	0.0684804
0.1	0.001	45	0.0143111	0.0500238
0.01	0.0001	70	0.00983014	0.0370924
0.0001	0.0001	100	0.00693047	0.0286082

Table 1: The Hausdorff distance d computed by nonlinear equation (2)

With the parameters thus fixed, the corresponding values for t_0 and d for the (cdf) $F(t)$ are respectively: $t_0 = 0.0333999$; $d = 0.0938953$ (see, Fig. 3).

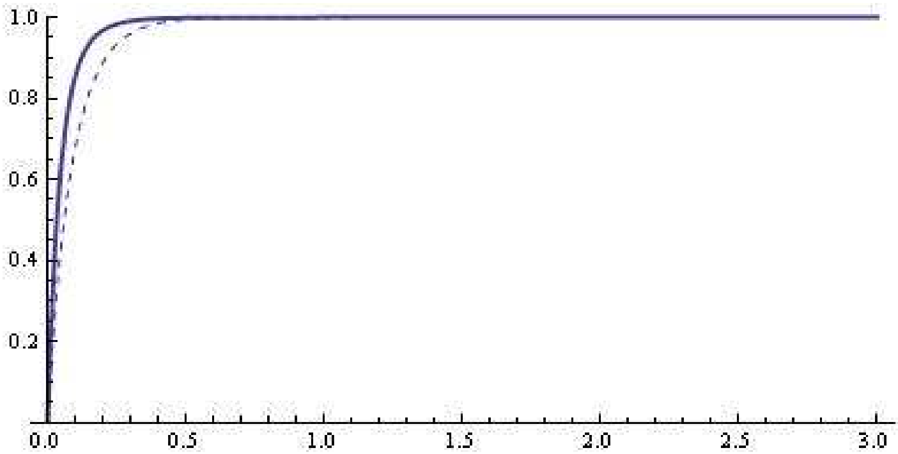
From the above examples, it can be seen that the "supersaturation" by the (cdf) $F(t)$ is faster.

Obviously, this "advantage" can actually be used to approximate some specific data.

In the next Section, we will support what is said by analyzing real datasets.

3. APPLICATIONS

1. We consider the following data "cdf of the number of Bitcoin received per address" (see, [12]:



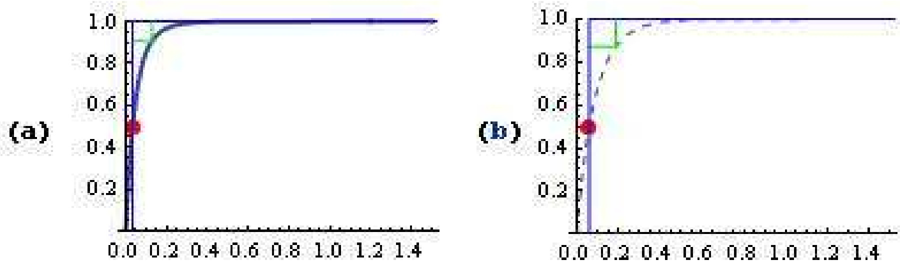


Figure 3: Comparisons between $F(t)$ – thick and $H(t)$ – dashed. a) H–distance $d = 0.0938953$; b) H–distance $d_1 = 0.127524$.

$$data\_CDF\_of\_Bitcoin\_received\_(inransoms)\_per\_address\_in\_C_{CL}$$

$$:= \{\{0.1, 0.0857\}, \{2, 0.1238\}, \{3, 0.6571\}, \{4, 0.6854\}, \{5, 0.8381\},$$

$$\{6, 0.8476\}, \{7, 0.8810\}, \{8, 0.9095\}, \{9, 0.9143\}, \{10, 0.9333\},$$

$$\{12, 0.9429\}, \{14, 0.9571\}, \{18, 0.9667\}, \{20, 0.9762\}, \{23, 0.9810\},$$

$$\{27, 0.9857\}, \{40, 0.9905\}, \{46, 0.9952\}, \{59, 0.9981\}\}.$$

The (cdf) $F(t)$ for $a = 0.005$, $\lambda = 0.0001$, $\theta = 0.2735$ is visualized on Fig. 4.

2. Storm worm was one of the most biggest cyber threats of 2008 [13].

We consider the following data:

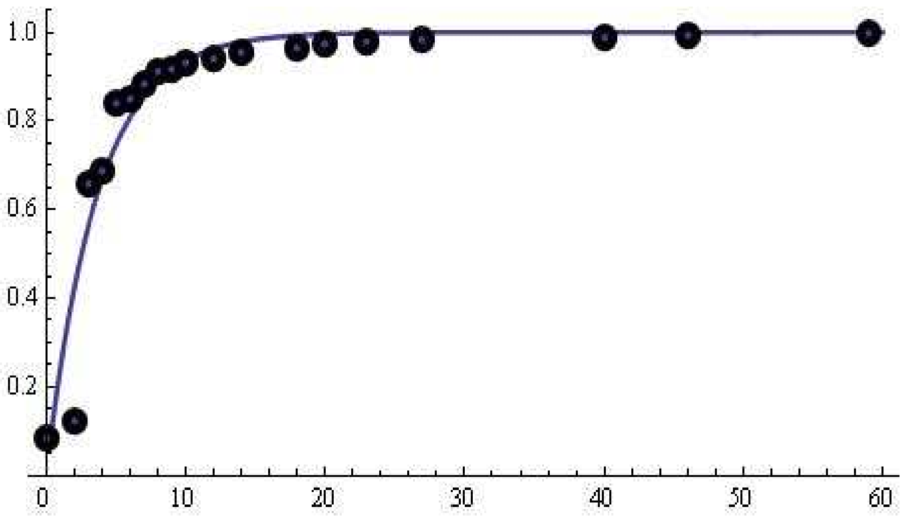


Figure 4: The fitted model for approximation of the data: "cdf of the number of Bitcoin received per address" [12].

$data\_Storm\_IDs$

$:= \{\{1.8, 0.843\}, \{4, 0.926\}, \{5, 0.954\}, \{6, 0.967\}, \{7, 0.976\},$

$\{8, 0.981\}, \{9, 0.985\}, \{10, 0.991\}, \{22, 0.995\}, \{38, 0.997\}, \{51, 0.998\},$

$\{64, 0.9985\}, \{74, 0.999\}, \{83, 1\}, \{100, 1\}\}$

The (cdf) $F(t)$ for $a = 0.99$, $\lambda = 0.88$ and $\theta = 0.259621$ is visualized on Fig. 5.

4. CONCLUSIONS.

In [1] the authors considered the special case of the family (1) with baseline (cdf) of Lindley–type

$$G(t) = 1 - \frac{1 + \theta + \theta t}{1 + \theta} e^{-\theta t}$$

Similar results related to the study of the important characteristic - "supersaturation" in the Hausdorff sense can also be carried out for the so-defined new family (TTL-G) (see, for example, Figures 6 and 7).

A comparison between (cdf) of traditional (TG) family - $H(t)$ and the (cdf) of the new (TTL–G) family - $F(t)$ for fixed: $a = 0.9$, $\lambda = 0.1$, $\theta = 15$ is illustrated in Figure

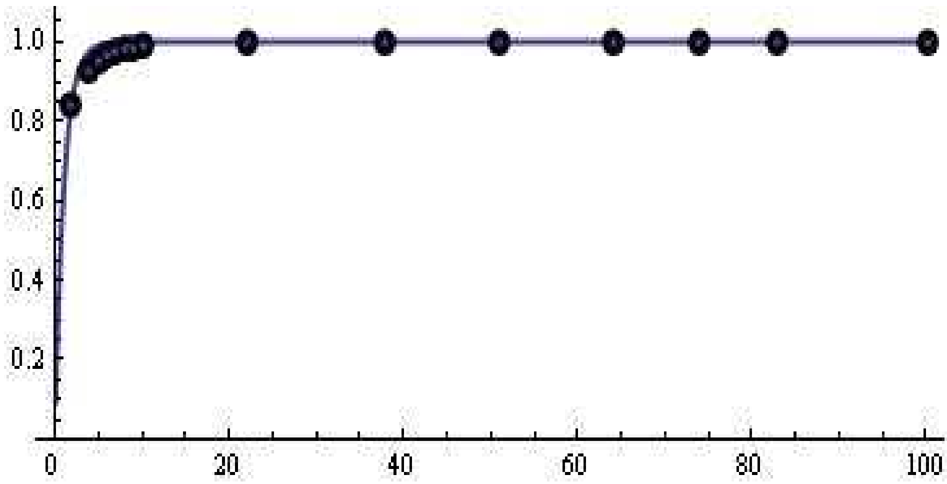


Figure 5: The fitted model for approximation of the data: "data-Storm".

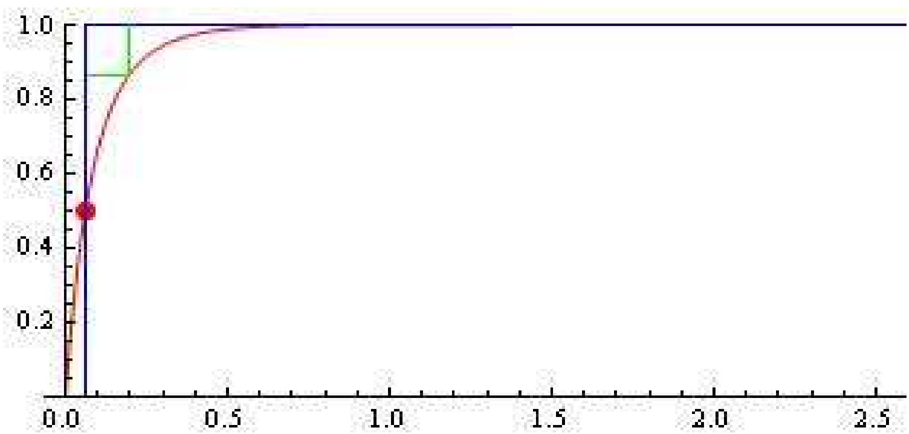


Figure 6: (cdf) of the (TTL-G) family for $a = 0.5$, $\lambda = 0.1$, $\theta = 8.1$ and $t_0 = 0.0614727$; H–distance $d = 0.134979$.

8.

From the experiments it can be concluded that for some specific datasets, the new transmuted transmuted cumulative family produces satisfactory results.

For some approximation, computational and modelling aspects, see [14]–[40].

Obviously, such studies are a must for the experimenter in the search for dialectical unity "data–model".

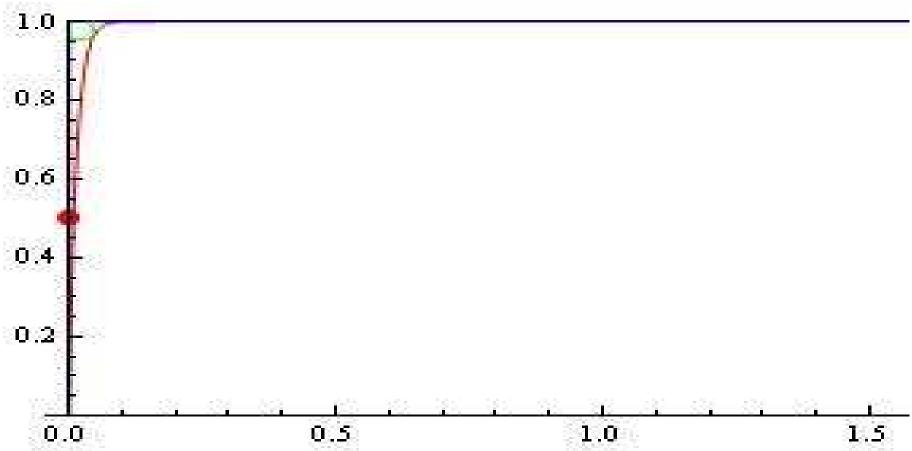


Figure 7: (cdf) of the (TTL-G) family for $a = 0.01$, $\lambda = 0.0001$, $\theta = 70$ and $t_0 = 0.000996988$; H–distance $d = 0.0440816$.

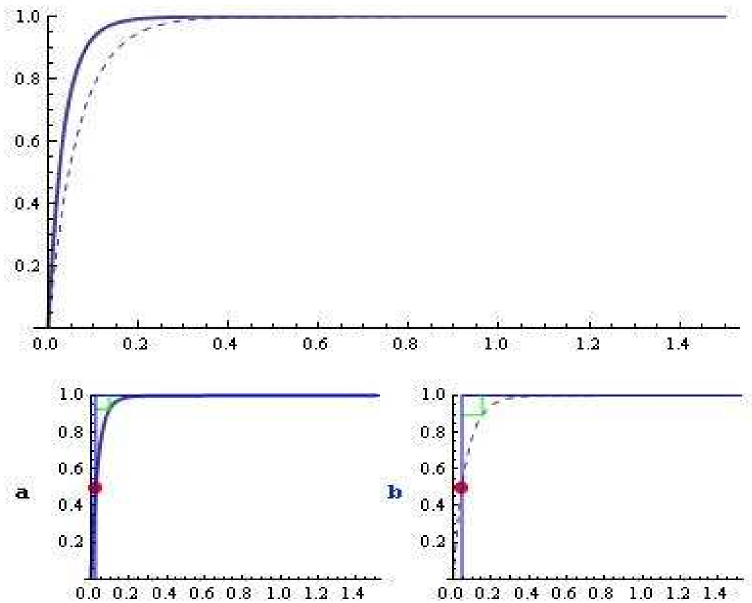


Figure 8: Comparisons between $F(t)$ – thick and $H(t)$ – dashed. a) H–distance $d = 0.073129$; b) H–distance $d_1 = 0.106355$.

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