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COMMENTS ON A NEW HYPERBOLIC SINE–WEIBULL MODEL WITH APPLICATIONS TO THE THEORY OF COMPUTER VIRUSES PROPAGATION. VI

Todorka Terzieva[1], Anton Iliev[2],
Asen Rahnev[3], Nikolay Kyurkchiev[4]
[1,2,3,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

ABSTRACT: In [1] the authors look at a new Hyperbolic Sine–Weibull (HSW) family of distributions with cdf. of type

$$M(t;a,\beta,\lambda)=\frac{2e^a}{(e^a-1)^2}\left(cosh\left(a\left(1-e^{-\lambda t^\beta}\right)\right)-1\right)$$

where $t>0$, $a>0$, $\beta>0$, $\lambda>0$.

The authors illustrated that this distribution provides a better fit to real data sets that Weibull, Hyperbolic Sine Exponential and other distributions.

Also of interest to the specialists is the task of approximating the Heaviside function

$$h_{t_0}(t)=\begin{cases} 0, & \text{if} \quad t<t_0,\\ [0,1], & \text{if} \quad t=t_0,\\ 1, & \text{if} \quad t>t_0 \end{cases}$$

where t_0 is the median, i.e. $M(t_0)=\frac{1}{2}$ with the new cdf in the Hausdorff sense.

We will show that the proposed model can be successfully used with success (of course, after extensive research) in the field of analysis of Computer Viruses Propagation.

We also analyze some experimental data: "data of Conficker propagation in 2008", data "Blaster worm" and data "Witty worm".

Numerical examples, illustrating our results are presented using programming environment *CAS Mathematica*.

AMS Subject Classification: 41A46
Key Words: Hyperbolic Sine–Weibull (HSW) family of cdf, Heaviside step–function $h_{t_0}(t)$, Hausdorff distance

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1. INTRODUCTION AND PRELIMINARIES

In this note we study the Hausdorff approximation of the Heaviside function $h_{t_0}(t)$ by the cdf of the Hyperbolic Sine family of distributions with baseline cdf. of Weibull–type (HSW), defined by Kharazmi, Saadatinik and M. Tamandi [1].

Definition 1. Kharazmi, Saadatinik and M. Tamandi [1] developed the following cdf of the new (HSW) family of distribution for $t \geq 0$:

$$M(t) = \frac{2e^a}{(e^a-1)^2}\left(cosh\left(a\left(1-e^{-\lambda t^\beta}\right)\right)-1\right) \tag{1}$$

where $a > 0$, $\beta > 0$, $\lambda > 0$.

Definition 2. The shifted Heaviside step function is defined by

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{2}$$

Definition 3. [2] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{\sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A-B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

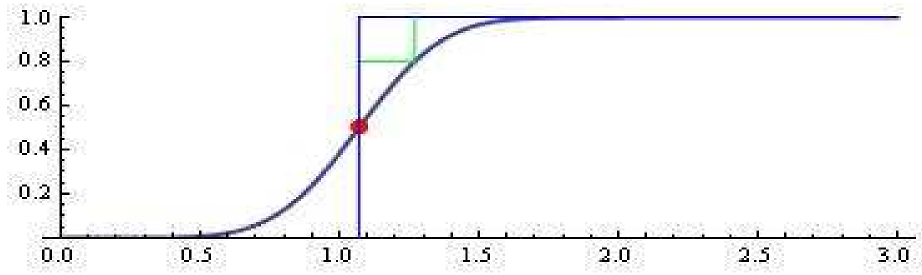

Figure 1: The cdf (1) for $a = 0.9$, $\beta = 3.5$, $\lambda = 0.9999$ and $t_0 = 1.07024$; H–distance $d = 0.199482$.

2. MAIN RESULTS

2.1. A NOTE ON THE NEW (HSW)

The investigation of the characteristic "supersaturation" of the cdf (1) to the horizontal asymptote is important.

Let t_0 is the value for which $M(t_0) = \frac{1}{2}$.

The one–sided Hausdorff distance d between the function $h_{t_0}(t)$ and the cdf (1) satisfies the relation

$$M(t_0 + d) = 1 - d. \quad (3)$$

For given a, β, λ and t_0, the nonlinear equation $M(t_0 + d) - 1 + d = 0$ has unique positive root – d.

The cdf (1) for $a = 0.9$, $\beta = 3.5$, $\lambda = 0.9999$ and $t_0 = 1.07024$ is visualized on Fig. 1.

From the nonlinear equation (3) we have: $d = 0.199482$.

The cdf (1) for $a = 0.98$, $\beta = 11.1$, $\lambda = 0.9999$ and $t_0 = 1.02216$ is visualized on Fig. 2.

From the nonlinear equation (3) we have: $d = 0.0875964$.

The cdf (1) for $a = 0.08$, $\beta = 21.1$, $\lambda = 0.9$ and $t_0 = 1.01485$ is visualized on Fig. 3.

From the nonlinear equation (3) we have: $d = 0.0532402$.

From the above examples, it can be seen that the "supersaturation" by the (cdf) $M(t)$ is faster.

Obviously, this "advantage" can actually be used to approximate some specific data from the field of analysis of Computer Viruses Propagation.

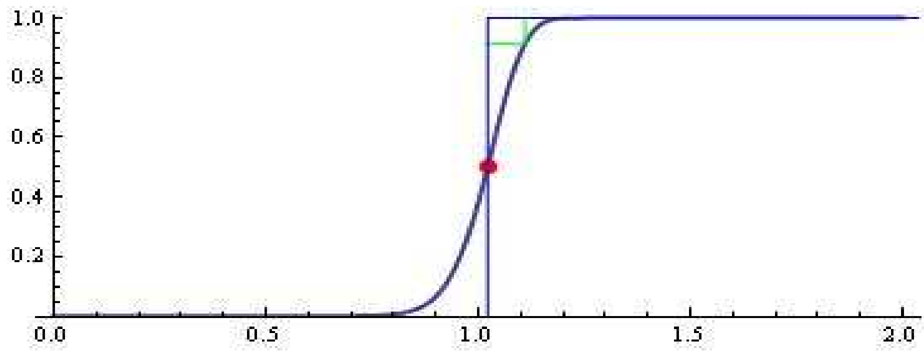

Figure 2: The cdf (1) for $a = 0.98$, $\beta = 11.1$, $\lambda = 0.9999$ and $t_0 = 1.02216$; H–distance $d = 0.0875964$.

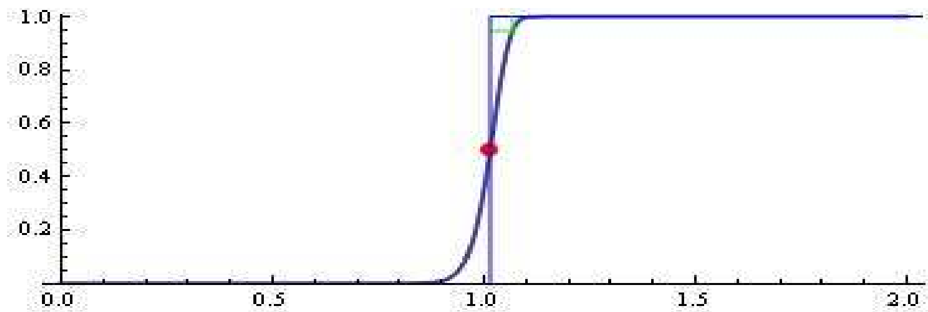

Figure 3: he cdf (1) for $a = 0.08$, $\beta = 21.1$, $\lambda = 0.9$ and $t_0 = 1.01485$; H–distance $d = 0.0532402$.

In the next Section, we will support what is said by analyzing real datasets.

2.2. APPLICATIONS

Example 1. Here we will present a new analysis of Conficker propagation in 2008 and we explore the Network Telescope project's daily dataset [3], [4] collected on November 21, 2008.

We analyze the following data

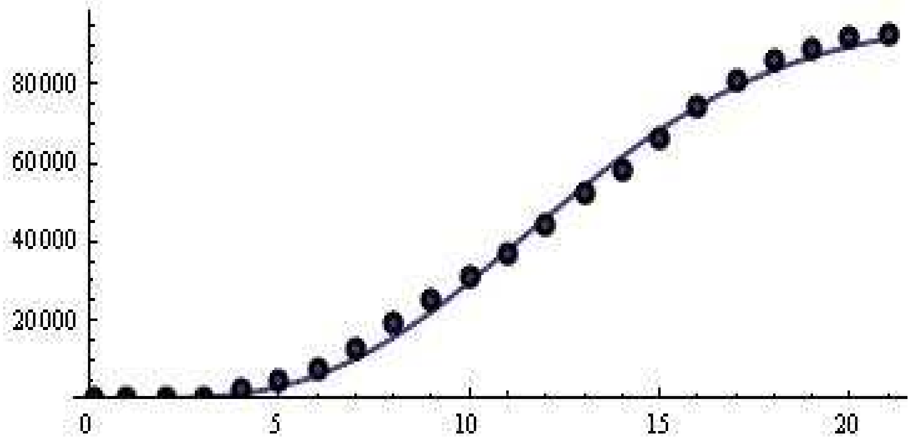

Figure 4: The fitted model $M^*(t)$.

$data\_Conficker :=$

$\{\{0.1, 10\}, \{1, 150\}, \{2, 300\}, \{3, 600\}, \{4, 2500\}, \{5, 5000\},$

$\{6, 7500\}, \{7, 13000\}, \{8, 19000\}, \{9, 25000\}, \{10, 31000\},$

$\{11, 37000\}, \{12, 44000\}, \{13, 52000\}, \{14, 58000\}, \{15, 66000\},$

$\{16, 74000\}, \{17, 81000\}, \{18, 86000\}, \{19, 89000\}, \{20, 92000\},$

$\{21, 92500\}\}$

The model

$$M^*(t) = \omega \frac{2e^a}{(e^a - 1)^2} \left(cosh \left(a \left(1 - e^{-\lambda t^\beta} \right) \right) - 1 \right) \tag{4}$$

for $\omega = 96650$; $\lambda = 0.0077009$; $a = 0.329302$; $\beta = 2.0234$ is visualized on Fig. 4.

Example 2. Blaster is a "sequential scan worm" [5].

We analyze the following "data Blaster worm"

$data\_Blaster :=$

$\{\{0, 10\}, \{100, 410\}, \{200, 4103\}, \{300, 25517\}, \{400, 95345\},$

$\{500, 472414\}, \{600, 565517\}, \{700, 581034\}, \{800, 590345\}\}$

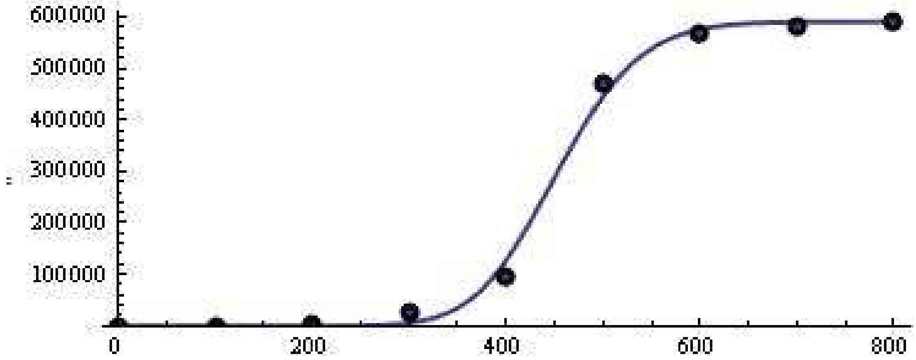

Figure 5: The fitted model $M^*(t)$.

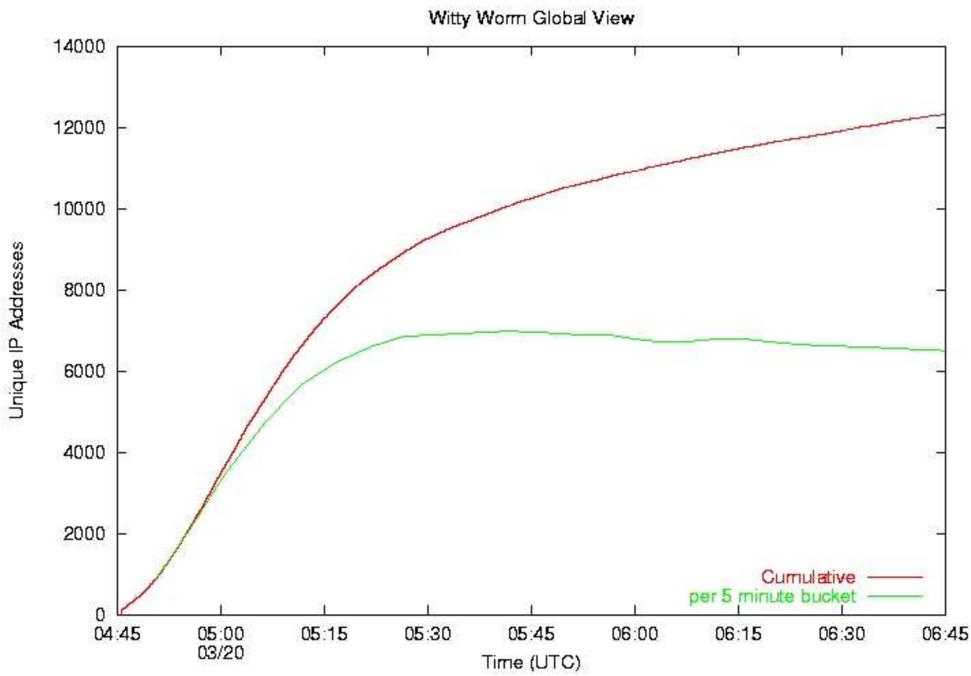

Figure 6: Epidemic data for entire world of Witty worm [9]

The fitted model $M^*(t) = \omega M(t)$ for $\omega = 590345$; $\lambda = 0.0000001$; $a = 1078$; $\beta = 2.80169$ is visualized on Fig. 5.

Example 3. Analysis of Witty worm infection behavior.

Here we will give an application of model (4) when provide analysis of this real "data" [9], see Fig. 6.

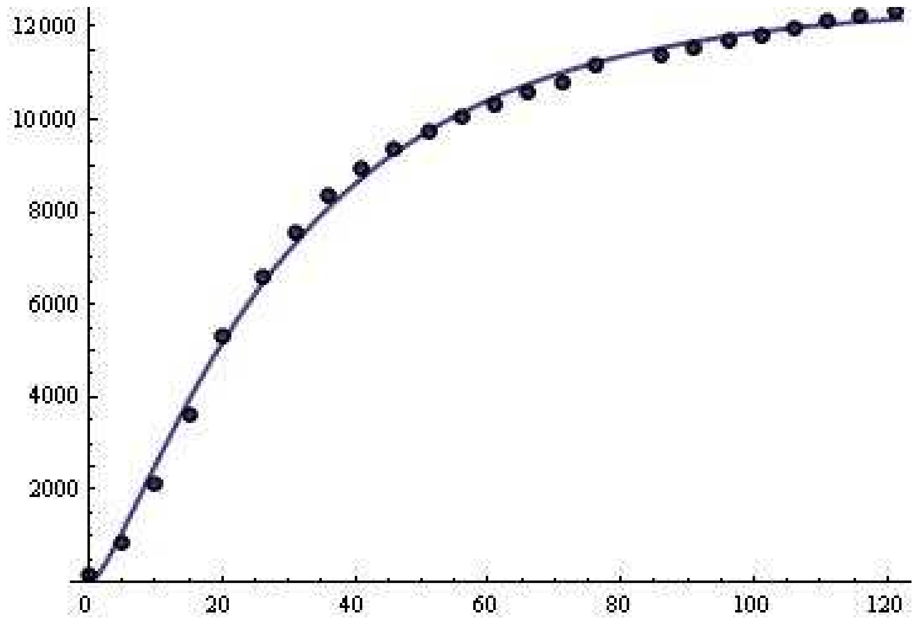

Figure 7: The fitted model $M^*(t)$.

$data\_Witty\_World =$

$\{\{0.1, 150\}, \{5, 869\}, \{10, 2141\}, \{15, 3637\}, \{20, 5312\},$

$\{26, 6602\}, \{31, 7562\}, \{36, 8340\}, \{41, 8941\}, \{46, 9389\}, \{51, 9734\},$

$\{56, 10060\}, \{61, 10349\}, \{66, 10586\}, \{71, 10800\}, \{76, 11169\},$

$\{86, 11362\}, \{91, 11532\}, \{96, 11684\}, \{101, 11823\}, \{106, 11972\},$

$\{111, 12118\}, \{116, 12256\}, \{121, 12372\}\}$

The fitted model $M^*(t) = \omega M(t)$ for $\omega = 12550$; $\lambda = 0.1$; $a = 0.3$; $\beta = 0.778789$ is visualized on Fig. 7.

3. CONCLUDING REMARKS

Finally, we note that the studied model produces extremely good results, generally when approximating specific "cumulative data" from Computer Viruses Propagation.

For other approximation and modelling results, see [6]–[23].

We hope that the results will be useful for specialists in this scientific area.

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