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A LOOK AT THE MODIFIED SIRD MODELS WITH "INTERVENTION POLYNOMIAL FACTOR". METHODOLOGICAL ASPECTS. II

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ABSTRACT: Our research is a natural continuation of previous results in [1]. The ideas given in [1] can be extended for other models of SIRD-type. From a methodological point of view, we recommend the specialists working in this scientific field to study the possibility of using input functions $\lambda(t)$ and $k_d(t)$ of polynomial type (for the MSIRD model [2]). A similar modifications are proposed for the modified model proposed in [3]. The specialists working in the field of "reaction-kinetic mechanisms" have the word. Numerical examples, illustrating our results are given using *CAS Mathematica*.

AMS Subject Classification: 41A46
Key Words: reaction equations, mass action kinetics, MSIRD model, new MSIRD model with "intervention polynomial factor" (MSIRD-IPF), modified SIRD model, recovery rate, death rate

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1. INTRODUCTION

In the last 2 years several thousand publications have appeared, incl. and these articles in arXiv.org and bioRxiv on SIR, SIS, SEIR, SEIRD, GSEIR and other models.

In [1] we develop a novel modification of the classic Kermack–McKendrick SIR (Susceptible–Infectious–Recovered) model with new reproduction numbers: $\rho_0, \rho_1, \dots \rho_l$ in appropriate intervals (t_0, t_1), $(t_1, t_2), \dots (t_l, t_{l+1})$. The new SIR model with "intervention polynomial factor" (SIR-IPF) can be used successfully to model and play different scenarios for the infectious disease spread.

Our research is a natural continuation of previous results in approximating specific data of a strictly exponential nature (e.g., COVID-19 Bulgaria, Cuba, China, South Korea, etc.) using modified logistics and other models in which the typical reaction constants were replaced by "polynomial variable transfer" and showed good results in performing the regression analysis. Some modifications of the SIS, SEIR, generalized SEIR (G–SEIR) with intervention polynomial factor are considered in [1].

The ideas given in [1] can be extended for other models of SIRD-type.

We will explicitly note that the article contains the explorations only of a model nature and much more complex issues related to the resistance and stability of the solution of such stratified systems differential equations (especially for the SEIRD model) can be considered as open, until specialists working in the field of "reaction kinetics mechanisms and models" decide that there is reason to use our modest explorations.

Numerical examples, illustrating our results using *CAS MATHEMATICA* are given.

2. MAIN RESULTS

2.1. A LOOK AT THE MODIFIED SIRD MODEL (MSIRD) PROPOSED BY SEN AND SEN [?]

In [2] a modified (MSIRD) model is proposed by Sen and Sen (see, Fig. 1).

The authors consider the effect of exposure, quarantine, confinement, and asymptomatic population, which are the cases in reality.

Here we will make a brief presentation based on the above article.

The modified SIRD model can be formulated as a chemical reaction network by:

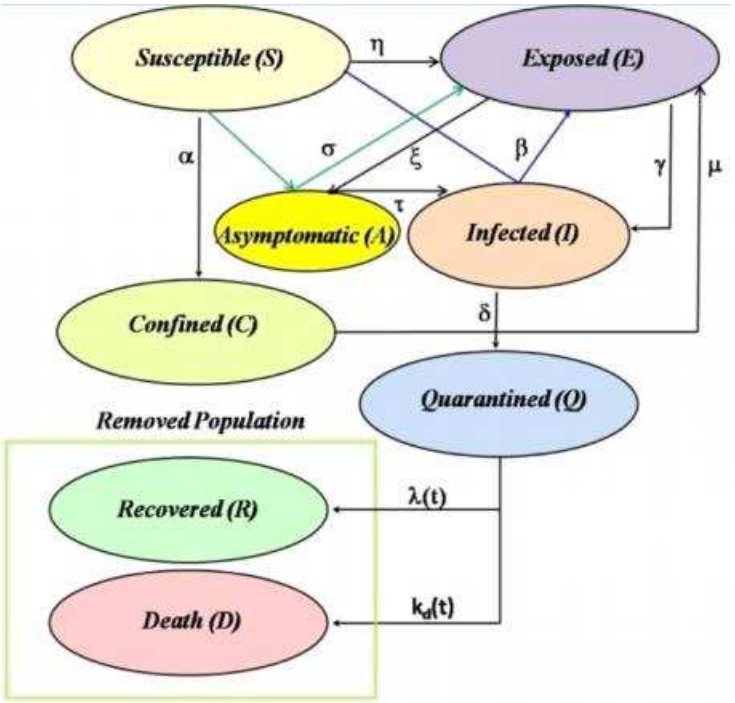

Figure 1: The modified (MSIRD) model [2].

$$
\begin{aligned}
S &\xrightarrow{\alpha} C \\
I+S &\xrightarrow{\beta} E \\
C &\xrightarrow{\mu} E \\
S &\xrightarrow{\eta} E \\
S+A &\xrightarrow{\sigma} E \\
E &\xrightarrow{\xi} A \\
E &\xrightarrow{\gamma} I \\
A &\xrightarrow{\tau} I \\
I &\xrightarrow{\delta} Q \\
Q &\xrightarrow{\lambda(t)} R \\
Q &\xrightarrow{k_d(t)} D,
\end{aligned}
\tag{1}
$$

where:

— N is the total population;

— $C(t)$ represents the confined population, who maintain social distancing norms, wear preventive face masks, and follow lockdown rules;

— $E(t)$ represents the exposed population, those people who have been exposed to the virus but have not been tested positive for infection yet;

— $A(t)$ represents the asymptomatic population, that is, those people who have been exposed to the virus but do not show any explicit symptom of infection;

— $Q(t)$ is the quarantined population at time t. It is considered that people who are infected are eventually quarantined and do not come in contact with susceptible people;

— $\lambda(t)$ represents the recovery rate;

— $k_d(t)$ represents the death rate due to infection/disease".

The model can be described by [2]:

$$\begin{cases} \frac{ds(t)}{dt} &= -\alpha s(t) - \frac{\beta}{N} s(t)i(t) - \frac{\sigma}{N} s(t)a(t) - \eta s(t) \\ \frac{da(t)}{dt} &= -\tau a(t) + \xi e(t) \\ \frac{dc(t)}{dt} &= \alpha s(t) - \mu c(t) \\ \frac{de(t)}{dt} &= -\gamma e(t) + \frac{\beta}{N} s(t)i(t) + \mu c(t) + \eta s(t) + \frac{\sigma}{N} s(t)a(t) - \xi e(t) \\ \frac{di(t)}{dt} &= \tau a(t) + \gamma e(t) - \delta i(t) \\ \frac{dr(t)}{dt} &= \lambda(t)q(t) \\ \frac{dq(t)}{dt} &= \delta i(t) - \lambda(t)q(t) - k_d(t)q(t) \\ \frac{dd(t)}{dt} &= k_d(t)q(t). \end{cases} \tag{2}$$

In [2] the authors consider the functions

$$\lambda(t) = \lambda_0(1 - e^{-\lambda_1 t})$$

$$k_d(t) = k_{d_0} e^{-k_{d_1} t}.$$

2.2. A NEW MODIFICATION OF THE (MSIRD) MODEL WITH "INTERVENTION POLYNOMIAL FACTOR" (MSIRD-IPF)

From a methodological point of view, we recommend the specialists working in this scientific field to study the possibility of using input functions $\lambda(t)$ and $k_d(t)$ of polynomial

type (in the light of the discussions in [1]):

$$\lambda(t) = \sum_{i=1}^{m} k_{2i-1} t^{i-1}$$

$$k_d(t) = \sum_{i=1}^{m} k_{2i} t^{i-1}.$$

We will explicitly note that this choice is not accidental and it opens the possibility for dynamic research and visualization of this good model while playing interesting scenarios by authorized bodies monitoring the spread of the epidemic.

We will call the new modification MSIRD model with "intervention polynomial factor" – (MSIRD-IPF).

Example 1. Consider the MSIRD–IPF model for fixed

$$K = 1000;\ \alpha = 0.15;\ \beta = 0.1;\ \sigma = 0.01;\ \eta = 0.06;$$

$$\tau = 0.2;\ \xi = 0.15;\ \mu = 0.05;\ \gamma = 0.08;\ \delta = 0.1;$$

$$s_0 = s(0) = 300;\ a_0 = a(0) = 250;\ c_0 = c(0) = 250;\ e_0 = e(0) = 100;$$

$$i_0 = i(0) = 30;\ r_0 = r(0) = 30;\ q_0 = q(0) = 20;\ d_0 = d(0) = 20;$$

$$\lambda(t) = 100 + 0.2t$$

$$k_d(t) = 100 - 0.05t.$$

The solutions are visualized on Fig. 2 in the interval $(0, 100)$.

Example 2. Consider the MSIRD–IPF model for fixed

$$K = 1000;\ \alpha = 0.15;\ \beta = 0.1;\ \sigma = 0.01;\ \eta = 0.06;$$

$$\tau = 0.2;\ \xi = 0.15;\ \mu = 0.05;\ \gamma = 0.08;\ \delta = 0.1;$$

$$s_0 = s(0) = 300;\ a_0 = a(0) = 250;\ c_0 = c(0) = 250;\ e_0 = e(0) = 100;$$

$$i_0 = i(0) = 30;\ r_0 = r(0) = 30;\ q_0 = q(0) = 20;\ d_0 = d(0) = 20;$$

$$\lambda(t) = 100 + 0.2t + 0.01t^2$$

$$k_d(t) = 100 - 0.05t - 0.001t^2.$$

The solutions are visualized on Fig. 3 in the interval $(0, 100)$.

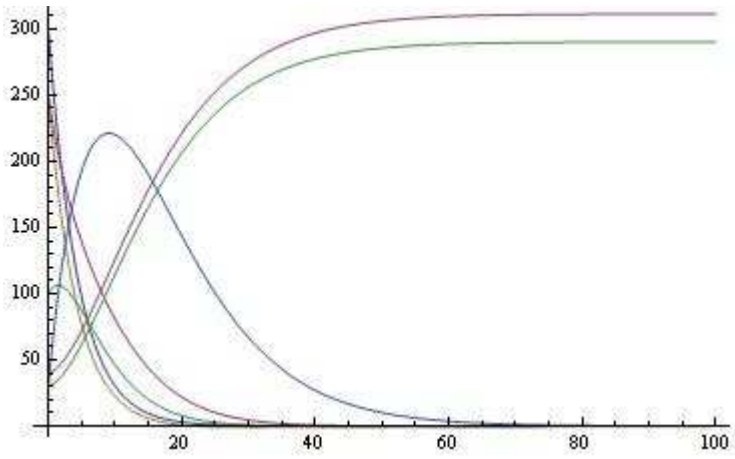

Figure 2: Simulations using new (MSIRD–IPF) model (Example 1).

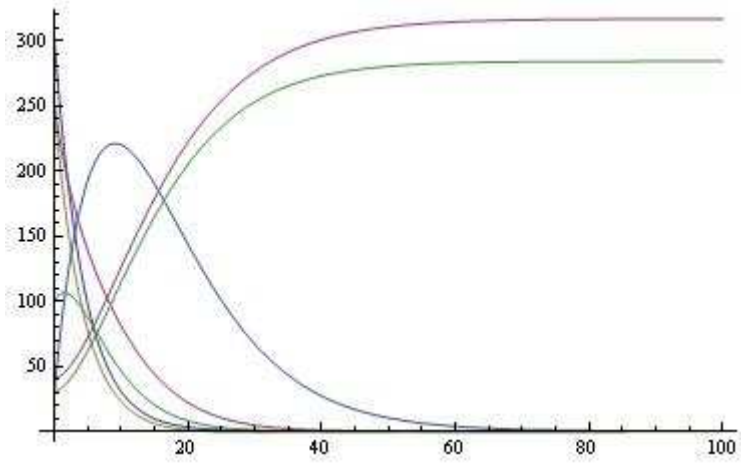

Figure 3: Simulations using new (MSIRD–IPF) model (Example 2).

2.3. A LOOK AT THE MODIFIED MODEL PROPOSED BY ANGUELOV, BANASIAK, BRIGHT, LUBUMA AND OUIFKI [?]

We consider the model proposed by Anguelov, Banasiak, Bright, Lubuma and Ouifki [3]:

$$\begin{cases} \frac{ds(t)}{dt} &= -\beta c(i(t)+a(t))s(t) - \lambda(t)s(t) \\ \frac{de(t)}{dt} &= \beta c(i(t)+a(t))s(t) + \lambda(t)s(t) - \sigma e(t) \\ \frac{da(t)}{dt} &= (1-\rho)\sigma e(t) - \gamma_a a(t) \\ \frac{di(t)}{dt} &= \rho\sigma e(t) - (\delta_i + \alpha_i + \gamma_i)i(t) \\ \frac{dh(t)}{dt} &= \delta_i i(t) - (\alpha_h + \gamma_h)h(t) \\ \frac{dr_a(t)}{dt} &= \gamma_a a(t) \\ \frac{dr_{ih}(t)}{dt} &= \gamma_i i(t) + \gamma_h h(t) \\ \frac{dd(t)}{dt} &= \alpha_i i(t) + \alpha_h h(t) \end{cases} \tag{3}$$

Here $\lambda(t)$ to be a smooth and monotone decreasing function [3] of the time t such that

$$\lambda(t) = \begin{cases} 2.18 \times 10^{-6};\ t \in [0,8] \\ 0;\ t \geq 9. \end{cases} \tag{4}$$

Consider the modification of the model (3) with "intervention polynomial factor– $\lambda^*(t)$".

Let

$$\lambda^*(t) = \begin{cases} \sum_{i=1}^{m} k_{2i-1} t^{i-1};\ t \in [t_0, t_1] \\ 0;\ t \geq t_1. \end{cases} \tag{5}$$

Some numerical examples.

For example, let

$$\lambda^*(t) = 0.5 - 0.024t - 0.0008t^2 + 0.00004t^3.$$

1. For fixed

$$K = 1,\ \beta = 0.03,\ c = 0.06,\ \sigma = 0.15,\ \rho = 0.07,\ \gamma_a = 0.08,\ \delta_i = 0.09,\ \alpha_i = 0.04,$$

$$\gamma_i = 0.01,\ \alpha_h = 0.04,\ \gamma_h = 0.09,\ s_0 = 0.8,\ e_0 = 0.05,\ a_0 = 0.05,\ i_0 = 0.02,\ h_0 = 0.02,$$

$$r_{a0} = 0.02,\ r_{ih0} = 0.02,\ d_0 = 0.02$$

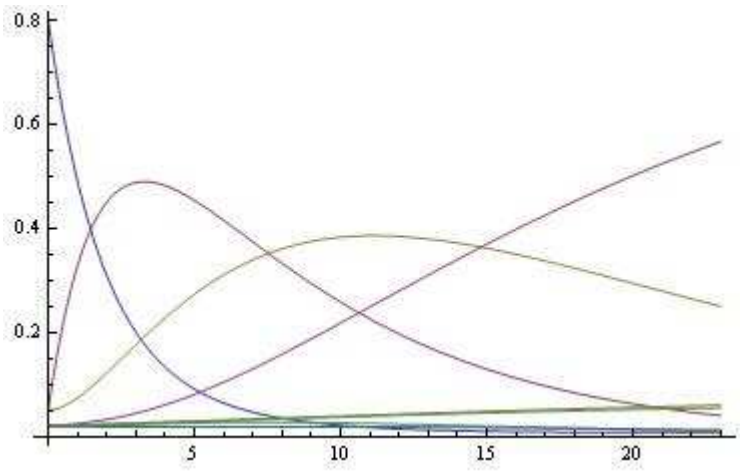

Figure 4: The simulation using model (3) with new $\lambda^*(t)$.

the simulation is depicted on Fig. 4.

2. For fixed

$$K = 1,\ \beta = 0.1,\ c = 0.05,\ \sigma = 0.07,\ \rho = 0.04,\ \gamma_a = 0.01,\ \delta_i = 0.09,\ \alpha_i = 0.06,$$

$$\gamma_i = 0.07,\ \alpha_h = 0.09,\ \gamma_h = 0.08,\ s_0 = 0.8,\ e_0 = 0.1,\ a_0 = 0.02,\ i_0 = 0.02,\ h_0 = 0.02,$$

$$r_{a0} = 0.02,\ r_{ih0} = 0.01,\ d_0 = 0.01$$

the simulation is depicted on Fig. 5.

3. CONCLUSIONS AND FUTURE WORK

We note that the choice of "input functions", especially for the MSIRD-IPF model, is quite specific and is almost subject to the requirement for these functions to be increasing and decreasing respectively in a fixed time interval.

Obviously, with the appropriate choice of the functions $\lambda_i^*(t)$ (for the model (3)) in appropriate intervals (t_i, t_{i+1}), it is possible to model "transmission risk" and play different scenarios for the infectious disease spread. In addition, the modified model has many free parameters that make it attractive to use.

Moreover, the methodological considerations presented in this article can be successfully applied to emerging dynamic models in the literature (but only those that are based on real reaction-kinetic schemes with many species). For example, see [4].

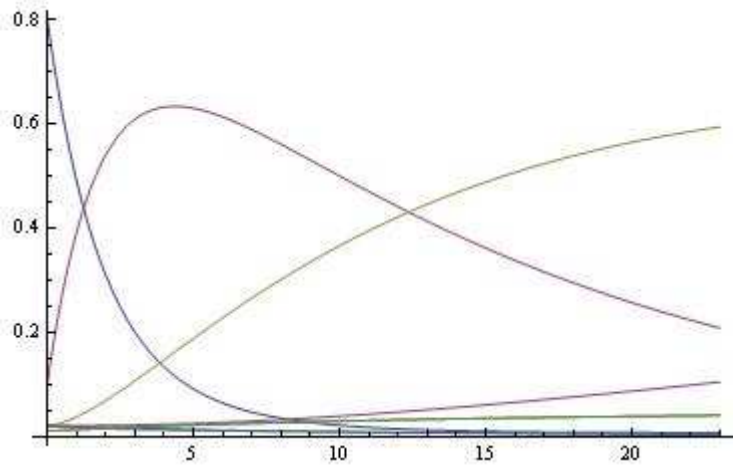

Figure 5: The simulation using model (3) with new $\lambda^*(t)$.

The compartments are:

S = Susceptibles;

E = Exposed (Latent Infection);

I_{Mild} = Mild Infections (Not Requiring Hospitalisation);

I_{Case} = Infections Requiring Hospitalisation;

$I_{Hospital}$ = Hospitalised (Requires Hospital Bed);

I_{ICU} = ICU (Requires ICU Bed);

I_{Rec} = Recovering from ICU Stay (Requires Hospital Bed);

R = Recovered;

D = Dead.

Evidently, our considerations can be extended for the model (with 9 species) using "new transmission factor of polynomial type".

We will not dwell on these studies here and present them to the reader for further reflection and future research.

It is planned to upgrade the Distributed Platform for e-Learning - DisPeL [5]–[6], including a specialized module for simulation of chemical kinetics, module for solving nonlinear differential equations and verifying the mass action balance and module for simulation by the introduced new reproduction number.

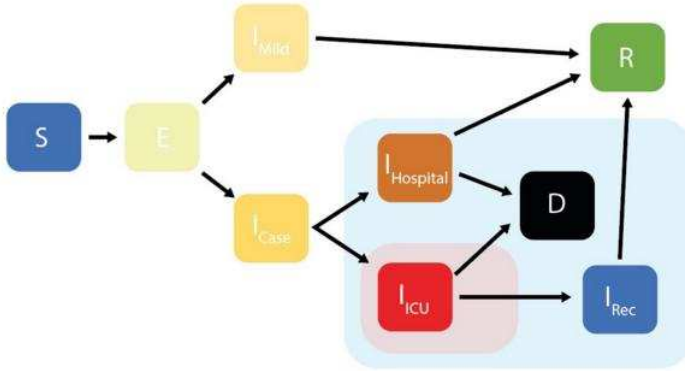

Figure 6: The extended model [4].

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REFERENCES

[1] N. Kyurkchiev, V. Kyurkchiev, A. Iliev, A. Rahnev, A new modifications of the SIR/SEIR models with "intervention polynomial factor". Methodological aspects, *International Journal of Differential Equations and Applications*, **20**, No. 1 (2021), 15–30.

[2] Devosmita Sen, Debasis Sen, Modified SIRD model to analyze COVID-19 data, *Ind. Eng. Chem. Res.*, **60** (2021), 4251–4260.

[3] R. Anguelov, J. Banasiak, Ch. Bright, J. Lubuma, R. Ouifki, The big unknown: The asymptomatic spread of COVID-19, *BIOMATH*, **9**, No. 1 (2020).

[4] https://github.com/mrc-ide/sqire

[5] A. Rahnev, N. Pavlov, V. Kyurkchiev, Distributed Platform for e-Learning - DisPeL, *European International Journal of Science and Technology*, **3**, No. 1 (2014), 95–109.

[6] V. Kyurkchiev, N. Pavlov, A. Rahnev, Cloud-based architecture of DisPeL, *International Journal of Pure and Applied Mathematics*, **120**, No. 4 (2018), 573–581.

[7] N. Kyurkchiev, Some new classes of growth functions generated by reaction networks and based on "correcting amendments" of Bateman-Gompertz and Bateman-Gompertz-Makeham-type. I., *Communications in Applied Analysis*, **24**, No. 1 (2020), 13–29.

[8] N. Kyurkchiev, On a Class of Growth Curves with Exponentially Variable Transfer Generated by Reaction Networks. II, *International Electronic Journal of Pure and Applied Mathematics*, **14**, No. 1 (2020), 21–29.

[9] N. Kyurkchiev, A New Class of "Growth Functions" with Polynomial Variable Transfer Generated by Real Reaction Networks, *Cybernetics and Information Technologies*, **20**, No. 6 (2020), 74–81.

[10] V. Kyurkchiev, A. Iliev, A. Rahnev, N. Kyurkchiev, Some New Logistic Differential Models: Properties and Applications, LAP LAMBERT Academic Publishing, (2019), ISBN: 978-620-0-43442-5.

[11] N. Kyurkchiev, A. Iliev, A. Rahnev, A look at the new logistic models with "polynomial variable transfer", LAP LAMBERT Academic Publishing, (2020), ISBN: 978-620-2-56595-0.

[12] N. Kyurkchiev, Selected Topics in Mathematical Modeling: Some New Trends (Dedicated to Academician Blagovest Sendov (1932-2020)), LAP LAMBERT Academic Publishing, (2020), ISBN: 978-620-2-51403-3.

[13] N. Kyurkchiev, A. Iliev, A. Rahnev, T. Terzieva, Another look at a good approximation of data for the distribution of COVID-19 in Cuba, *Revista Habanera de Ciencias Medicas*, **19**, No. 3 (2020).

[14] N. Kyurkchiev, A. Iliev, A. Rahnev, On the Verhulst growth model with "polynomial variable transfer". Some applications, *International Journal of Differential Equations and Applications*, **19**, No. 1 (2020), 15–32.

[15] N. Kyurkchiev, A. Iliev, A. Rahnev, On the half-logistic model with "polynomial variable transfer". Application to approximate the specific "DATA CORONA VIRUS", *International Journal of Differential Equations and Applications*, **19**, No. 1 (2020), 45–61.