International Journal of Differential Equations and Applications
Volume **22**, No. 1 (2023), pages: 123-136
ISSN (Print): 1311-2872; ISSN (Online): 1314-6084;
url: https://www.ijdea.eu
ijdea.eu

DYNAMICS OF A GENERALIZED HYPOTHETICAL RAYLEIGH–DUFFING–LIKE OSCILLATOR: INVESTIGATIONS IN THE LIGHT OF MELNIKOV'S APPROACH, SIMULATIONS (WEB–PLATFORM UPGRADE)

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ABSTRACT: The Rayleigh–Duffing oscillator models are widely used in physics, electronics and many other disciplines. Obviously, these studies can be successfully continued. In this paper, we propose a generalized hypothetical Rayleigh–Duffing–like oscillator model. Considerations in the light of Melnikov's approach are also given. Here we will focus on some interesting simulations with the proposed new model and demonstrate specialized modules for investigating the dynamics of these oscillators, an integral part of a planned much more general Web–based application for scientific computing.

Key Words: generalized Rayleigh–Duffing model, Melnikov's approach, generalized hypothetical Rayleigh–Duffing–like oscillator

Received: October 3, 2023 **Revised:** November 8, 2023
Published: November 12, 2023 **doi:** 10.12732/ijdea.v22i1.9
Academic Publications, Ltd. https://acadpubl.eu

1. INTRODUCTION

The Rayleigh–Duffing oscillator models are widely used in physics, electronics and many other disciplines. The following planar system has been the subject of research by a number of authors

$$\begin{cases} \dfrac{dx}{dt} &= y \\ \\ \dfrac{dy}{dt} &= a_1 x - a_2 x^3 + \epsilon((1-y^2)y + (1+x)\cos(gt)) \end{cases} \tag{1}$$

where $0 \le \epsilon \le 1$.

Obviously, these studies can be successfully continued.

In this article, we propose a generalized hypothetical differential model. Considerations in the light of Melnikov's approach are also given.

Here we will focus on some interesting simulations with the proposed new model and demonstrate specialized modules for investigating the dynamics of these oscillators, an integral part of a planned much more general Web–based application for scientific computing (for some details see [7]–[14] and [22]–[43]).

Specialists working in this scientific direction have the floor.

2. MAIN RESULTS

We consider the following new generalized hypothetical model

$$\begin{cases} \dfrac{dx}{dt} &= y \\ \\ \dfrac{dy}{dt} &= x - x^3 + \epsilon\left(y(1-y^{2n}) + (1+x^n)\cos(gt)\right) \end{cases} \tag{2}$$

where $0 \le \epsilon \le 1$ and $n = 1, 2, \ldots$.

2.1. INVESTIGATIONS IN THE LIGHT OF MELNIKOV'S APPROACH

Melnikov's method gives us an analytic tool for establiching the existence of transfer homoclinic points of the Poincare map for a periodic orbit of a perturbed dynamical system.

The case $n = 1$

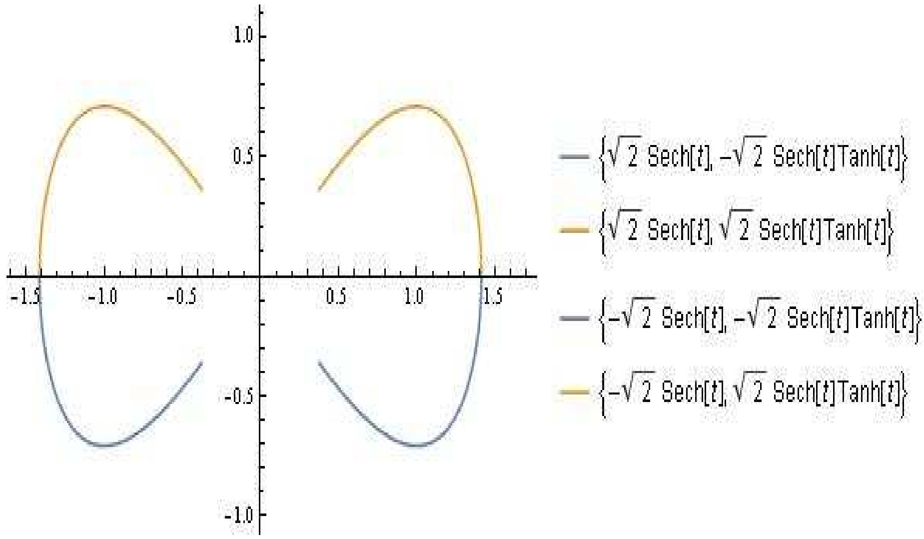


Figure 1: Double homoclinic orbit [8].

Obviously, for $a_1 = a_2 = 1$ and $n = 1$, model (2) coincides with model (1).

For $\epsilon = 0$, the resulting Hamiltonian of the system (2) is

$$H(x, y) = \frac{1}{2}y^2 - \frac{1}{2}x^2 + \frac{1}{4}x^4.$$

We nave double homoclinic orbit given by (see Fig.1)

$$x_0(t) = \pm\sqrt{2}sech\, t$$
$$y_0(t) = \mp\sqrt{2}sech\, t\, tanh\, t.$$

For example, we will compute only the first Melnikov function:

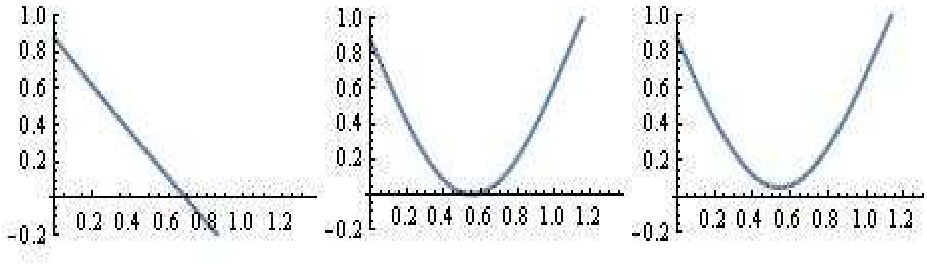


Figure 2: $M(t_0)$ (see Proposition 1).

$$\begin{aligned}
M(t_0) &= \int_{-\infty}^{\infty} y_0(t)\left(y_0(t)(1-y_0^2(t)) + (1+x_0(t))\cos(g(t+t_0))\right)\,dt \\
&= \int_{-\infty}^{\infty} y_0^2(t)\,dt - \int_{-\infty}^{\infty} y_0^4(t)\,dt + \int_{-\infty}^{\infty} y_0(t)(1+x_0(t))\cos(g(t+t_0))\,dt \\
&= \int_{-\infty}^{\infty} (\sqrt{2}sech\,t\,tanh\,t)^2\,dt - \int_{-\infty}^{\infty} (\sqrt{2}sech\,t\,tanh\,t)^4\,dt + \\
&\quad + \int_{-\infty}^{\infty} \sqrt{2}sech\,t\,tanh\,t(1+\sqrt{2}sech\,t)\cos(g(t+t_0))\,dt
\end{aligned}$$

The integrals can be evaluated (the third by the method of residuals) to yield

$$M(t_0) = \frac{92}{105} - \pi g\left(g\,csch\left(\frac{\pi g}{2}\right) + \sqrt{2}sech\left(\frac{\pi g}{2}\right)\right)\sin(gt_0). \tag{3}$$

From a numerical point of view, the task of finding a multiple root is mote interesting.

The following is valid (see also Fig. 2).

Proposition 1. If $g > 2.84$, the Melnikov function $M(t_0)$ has no zero; if $g = 2.84$ then $M(t_0)$ has zero $t_0 = 0.539498$ with multiplicity two; if $g < 2.84$, $M(t_0)$ has a simple zero.

The case $n = 2$

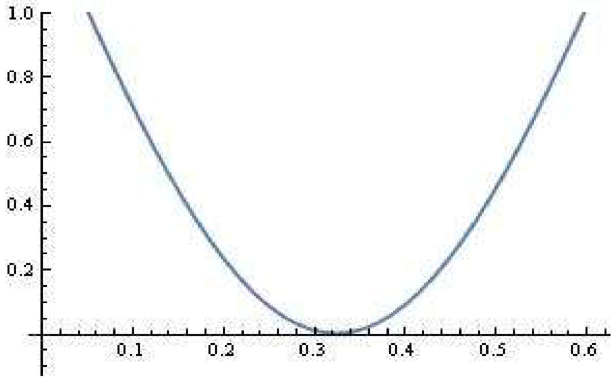


Figure 3: The root t_0 of multiplicity two.

For the first Melnikov function we have

$$M(t_0) = \frac{796}{693} - \frac{1}{3}\sqrt{2}g(4+g^2)\pi sech\left(\frac{g\pi}{2}\right)\sin(gt_0). \tag{4}$$

Using (4) we formulate the following

Proposition 2. If $g > 2.87$, the Melnikov function $M(t_0)$ has no zero; if $g = 2.87$ then $M(t_0)$ has zero $t_0 = 0.547316$ with multiplicity two; if $g < 2.84$, $M(t_0)$ has a simple zero.

The case $n = 6$

For the first Melnikov function we have

$$M(t_0) = \frac{46540556}{35102025} - \frac{g(855+259g^2+35g^4+g^6)}{315\sqrt{2}}\pi sech\left(\frac{g\pi}{2}\right)\sin(gt_0). \tag{5}$$

Using (5) (see also Fig.3) we formulate the following

Proposition 3. If $g > 4.85$, the Melnikov function $M(t_0)$ has no zero; if $g = 4.85$ then $M(t_0)$ has zero $t_0 = 0.323875$ with multiplicity two; if $g < 4.85$, $M(t_0)$ has a simple zero.

Remark. For calculating the Melnikov integrals we use the assumption $-1 < Im(g) \leq 1$.

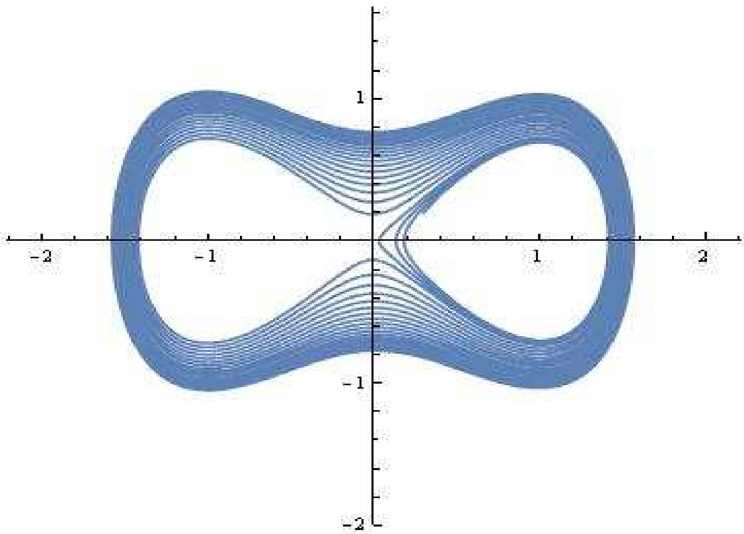
3. SOME SIMULATIONS

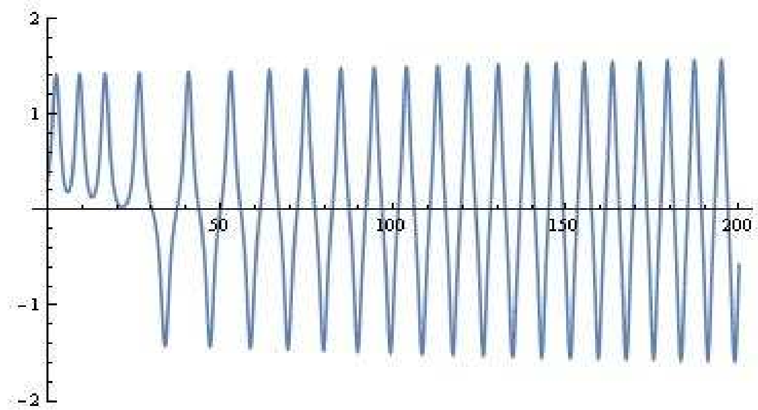
Here we will focus on some interesting simulations with the proposed new model and demonstrate specialized modules for investigating the dynamics of these oscillators.

1. For given $n = 1$, $g = 0.02$, $\epsilon = 0.01$ the simulations on the system (2) for $x_0 = 0.3$; $y_0 = 0.2$ are depicted on Fig. 4.

2. For given $n = 2$, $g = 0.05$, $\epsilon = 0.001$ the simulations on the system (2) for $x_0 = 0.3$; $y_0 = 0.2$ are depicted on Fig. 5.

3. For given $n = 4$, $g = 2.75$, $\epsilon = 0.001$ the simulations on the system (2) for $x_0 = 0.3$; $y_0 = 0.2$ are depicted on Fig. 6.





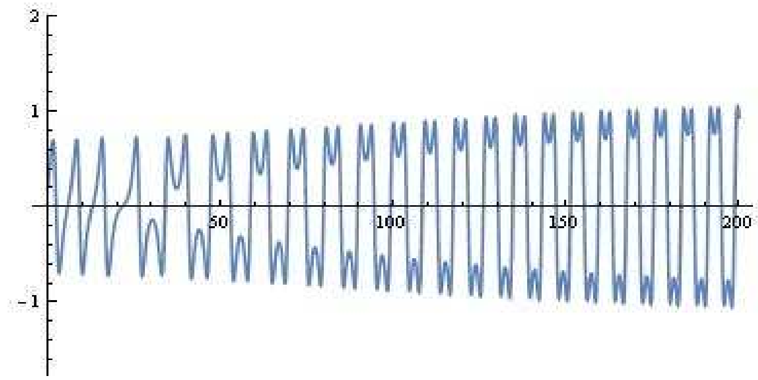


Figure 4: a) phase space; b) x-time series; c) y-time series; (example 1).

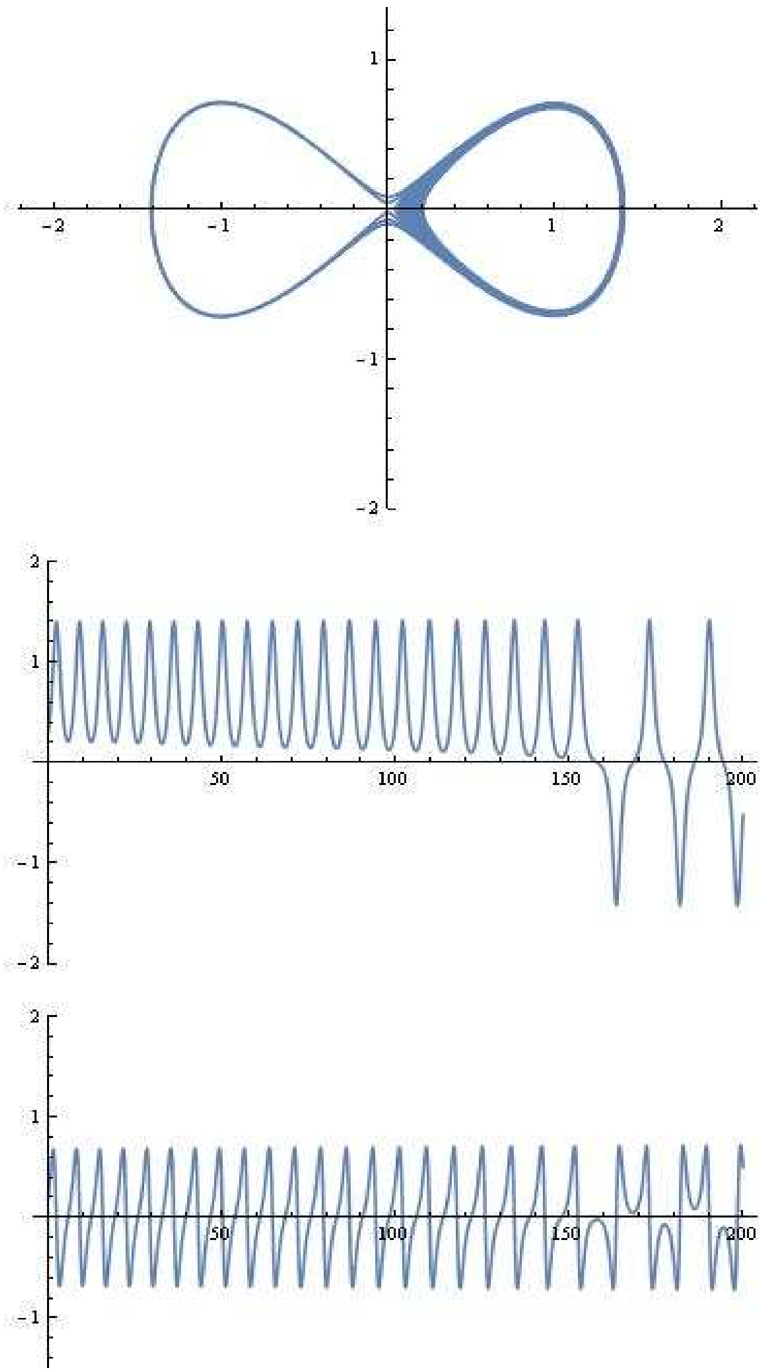


Figure 5: a) phase space; b) x-time series; c) y-time series; (example 2).

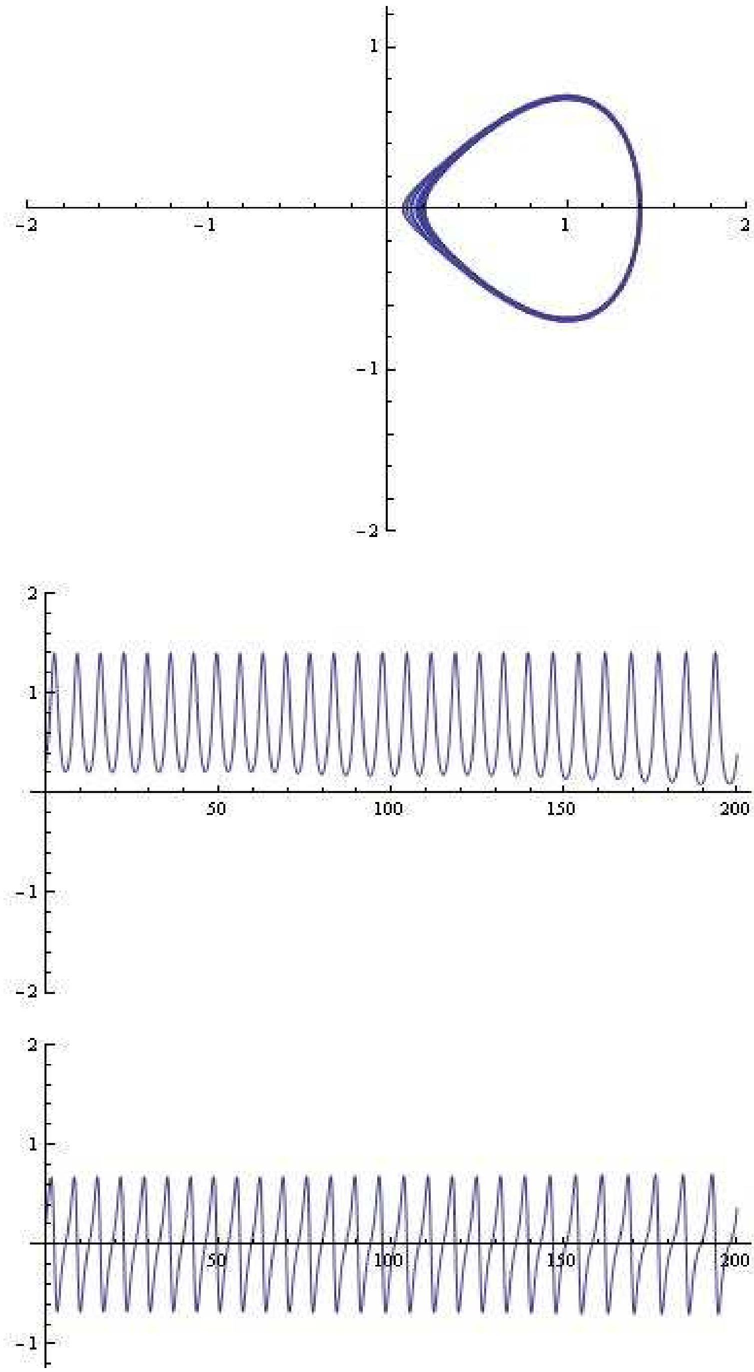


Figure 6: a) phase space; b) x-time series; c) y-time series; (example 3).

4. CONCLUDING REMARKS

The results can be used to estimate the associated total energy potential of the considered differential system. We have already mentioned that the method of residues is used when calculating the Melnikov integrals.

Modern computer algebraic systems for scientific calculations provide this opportunity for users.

The parameters appearing in the proposed generalized hypothetical oscillator are subject to a number of restrictions (of a physical and practical nature).

In this regard, we would like to note that direct reference to proposals, for example, in the CAS Mathematica module does not give satisfactory results.

The user does not have to be a mathematician to specify as an input parameter - a serious limitation for calculating the above mentioned integrals.

The upgrade of the Web Application planned by us foresees the use of an algorithm (hidden to the user) to define the limit, for example $|Im(\omega)| \leq Const.$

In this paper we demonstrate some specialized modules for investigating the dynamics of some new hypothetical oscillators, an integral part of a planned much more general Web–based application for scientific computing.

A number of technical and programming difficulties related to the large number of free parameters of this type of hypothetical oscillator have been overcome.

Remark. The study of corresponding critical levels of $H(x, y) = \frac{1}{2}y^2 - P(x)$ is very complicated.

In this regard, we recommend the study by Gavrilov and Iliev [21].

Specialists working in this scientific direction have the floor.

This study of the first author is financed by the project No FP23-FMI-002 ”Intelligent software tools and applications in research in Mathematics, Informatics, and Pedagogy of Education” of the Scientific Fund of the Paisii Hilendarski University of Plovdiv, Bulgaria.

The study of the second author was carried out with the financial support of the Fund ”Scientific Research” of Plovdiv University Paisii Hilendarski, within the project: ”A study of natural, demographic and cultural resources in the Western Rhodopes: potential for sustainable development of the region”,contract No FP23-FS 003.

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