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INITIAL VALUE PROBLEMS FOR FIRST ORDER IMPULSIVE INTEGRO-DIFFERENTIAL INCLUSIONS IN BANACH SPACES

Abdelkader Boucherif[1 §], Mouffak Benchohra[2]

[1]Dept. of Mathematical Sciences
King Fahd University of Petroleum and Minerals
P.O. Box 5046, Dhahran 31261, SAUDI ARABIA
e-mail: aboucher@kfupm.edu.sa

[2]Laboratoire de Mathématiques
Université de Sidi Bel Abbes
B.P. 89, Sidi Bel Abbes 22000, ALGÉRIE
e-mail: benchohra@yahoo.com

Abstract: We investigate the solutions for initial value problems for first order impulsive integro-differential inclusions in Banach spaces. Our approach is based on a fixed point theorem for condensing set-valued maps.

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1. Introduction

Differential equations involving impulsive effects occur in many applications: population dynamics, ecology, biological systems, optimal control, etc. The interested reader can consult the monographs [2], [6], [14] and [20] and the papers [8], [10], [16], [17] and [23]. Most of these works deal mainly with differential equations or integro-differential equations. However, there are few results for integro-differential inclusions or related topics (see, for instance, [5] and [19]).

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§Correspondence author

Our objective, in this paper, is to investigate the existence of solutions of first order initial value problems for impulsive integro-differential inclusions in Banach spaces. More specifically, let J denote the compact real interval $[0,T]$, $0 < T < +\infty$ and $t_0 = 0 < t_1 < t_2 < \cdots < t_m < t_{m+1} = T$. Let E be a real Banach space with norm $\|\cdot\|$. Consider the following initial value problem

$$y'(t) \in F(t, y(t), (Sy)(t)), \quad t \in J', \tag{1}$$

$$y(t_k^+) = y(t_k^-) + I_k(y(t_k^-)), \quad k = 1, 2, \ldots, m, \tag{2}$$

$$y(0) = 0, \tag{3}$$

where $J' = J \setminus \{t_1, .t_2, \ldots, t_m\}$, $F : J \times E \times E \to 2^E$ is a set-valued map, $I_k \in C(E; E)$, $k = 1, 2, \ldots, m$; $(Sy)(t) = \int_0^t K(t,s)y(s)ds$, $y(t_k^+) - y(t_k^-)$ represents the jump of $y(\cdot)$ at $t = t_k$. Here $y(t_k^-)$ and $y(t_k^+)$ are the left and right limits of $y(\cdot)$ at $t = t_k$, respectively.

To motivate our work, we consider an impulse control problem (see [6] for more details):

$$\begin{cases} y'(t) = f(t, y(t), u(t)), \quad t \in J, \\ y(0) = 0, \end{cases} \tag{4}$$

subject to the constraints $y(\cdot) \in E$ and $u(t) \in \mathcal{U}(t, y(t))$, where $\mathcal{U}(t, y(t))$ is a nonempty compact subset of a Banach space U. Let the control actions $u(t)$ be generated by a closed-loop law in the following form:

$$u(t) = S(t, y(t)), \quad t \in J, \tag{5}$$

where S is a Volterra operator with kernel $K(t,s)$, i.e.

$$u(t) = \int_0^t K(t,s)y(s)ds \quad \text{for} \quad t \in J.$$

Then $\mathcal{U}(t, y(t)) = \{(Sy)(t) = \int_0^t K(t,s)y(s)ds;\ y(\cdot) \in E\}$. It is well known that in impulse control it is assumed that at certain instants (called impulse instants or moments) the state undergoes jumps (see [6]). Finally, let $F(t, y(t), (Sy)(t)) := f(t, y(t), \mathcal{U}(t, y(t)))$, $t \in J$. We see that the impulse control problem (4) subjected to the above constraints can be formulated as a problem for impulsive differential inclusions in abstract spaces (see [4] for the case, where F is a subdifferential and more general control problems).

We shall provide sufficient conditions on the set-valued map F and on the operator S, which guarantee the existence of solutions of problem (1)-(3). Our approach is based on a fixed point theorem for condensing maps.

2. Preliminaries

In this section we introduce notations, definitions and preliminary results which will be used throughout the paper.

$C(J;E)$ is the Banach space of continuous functions $y : J \to E$, with norm

$$\|y\|_\infty := \sup\{\|y(t)\|;\ t \in J\},\quad y \in C(J;E).$$

$AC(J;E)$ is the Banach space of absolutely continuous functions on J with values in E.

Let $y : J \to E$ be a measurable function. By $\int_J y(s)ds$ we mean the Bochner integral of y, assuming it exists. A measurable function y is Bochner-integrable, if and only if $\|y\|$ is Lebesgue integrable.

$L^1(J;E)$ is the Banach space of Bochner-integrable functions y, with norm

$$\|y\|_{L^1} = \int_J \|y(t)\|dt.$$

For details and properties of the Bochner integral (see [22]).

A set-valued map $G : E \to 2^E$ has convex (closed) values, if $G(y)$ is convex (closed) for all $y \in E$. G is bounded on bounded sets, if $G(B)$ is bounded in E for each bounded subset B of E, i.e.

$$\sup\{\sup\{\|y\|;\ y \in G(x)\};\ x \in B\} < +\infty.$$

G is called upper semicontinuous (u.s.c for short) on E if: for each $x_0 \in E$ the set $G(x_0)$ is a nonempty closed subset of E, and for each open set $N \subset E$ containing $G(x_0)$, there exists an open neighborhood M of x_0 such that $G(M) \subset N$.

G is said to be completely continuous if $G(B) = \bigcup_{x\in B} G(x)$ is relatively compact for every bounded subset B of E. If the set-valued map G is completely continuous with nonempty compact-values, then G is u.s.c., if and only if G has a closed graph (i.e. $x_n \to x_0, y_n \to y_0$ with $y_n \in G(x_n)$ imply that $y_0 \in G(x_0)$).

An u.s.c. map $G : E \to E$ is said to be condensing, if for any bounded subset A of E we have $\alpha(G(A)) < \alpha(A)$, where α denotes the Kuratowski measure of noncompactness (see [3] and [13]) for definitions and properties of the measures of noncompactness). Notice that a compact map is the simplest example of a condensing map. G has a fixed point if there is $y \in E$ such that $y \in G(y)$. Also, we shall denote by $\mathrm{bcc}\,(E)$ the set of all bounded, closed, convex and nonempty subsets of E. A set-valued map $G : J \to \mathrm{bcc}\,(E)$ is said to be measurable if for each $x \in E$ the function $t \mapsto \mathrm{dist}(x, G(t))$ is measurable on J. We refer to the books of Aubin and Cellina [1], Deimling [7], Gorniewicz [9] and Hu and Papageorgiou [12] for more details on set-valued maps.

Let $\Omega := \{y \in C(J'; E);\ y(t_k^+)$ and $y(t_k^-)$ exist for $k = 1, 2, \dots, m\}$, and let

$$\Omega^1 := \Omega \cap AC(J'; E).$$

It is clear that Ω is a Banach space, when it is equipped with norm

$$\|y\|_\Omega := \sup\{\|y(t)\|; t \in J\} \quad \text{for } y \in \Omega.$$

Finally, by a solution of (1)-(3) we mean a function $y \in \Omega^1$ with the following property: there exists $x \in L^1(J'; E)$ such that

$$x(t) \in F(t, y(t), Sy((t))) \qquad \text{for} \quad t \in J', \quad y'(t) = x(t) \ \text{ a.e. on } J$$

and $y(0) = 0$, $y(t_k^+) = y(t_k^-) + I_k(y(t_k^-))$, $k = 1, 2, \dots, m$.

We shall write $y(t_k)$ for $y(t_k^-)$ for $k = 1, 2, \dots, m$.

The following lemmas are crucial in the proof of our main result.

Lemma 2.1. (see [15]) *Let I be a compact real interval and X be a Banach space. Let F be a multivalued map satisfying (H1) (see below) and let Γ be a linear continuous mapping from $L^1(I, X)$ to $C(I, X)$. Then the operator*

$$\Gamma \circ S_F : C(I, X) \longrightarrow bcc\,(C(I, X)),\ y \longmapsto (\Gamma \circ S_F)(y) := \Gamma(S_{F,y})$$

is a closed graph operator in $C(I, X) \times C(I, X)$.

Lemma 2.2. (see [11]) *If $q \in L^1(J; \mathbb{R}_+)$ and $\phi : \mathbb{R}_+ \to (0, \infty)$ is increasing with $\displaystyle\int_0^{+\infty} \frac{du}{\phi(u)} = +\infty$, then for each $z_0 \in \mathbb{R}_+$, the initial value problem*

$$z'(t) = q(t)\phi(z(t)), \quad z(0) = z_0 \tag{6}$$

has a unique solution. Moreover, if $u \in AC(J; E)$ *satisfies the integral inequality*

$$\|u(t)\| \leq z_0 + \int_0^t q(s)\phi(z(s))ds\,, \quad t \in J,$$

then $\|u\| \leq z$*, i.e.* $\|u(t)\| \leq z(t)$ *for all* $t \in J$*.*

Lemma 2.3. (see [18]) *Let* $G : E \to bcc(E)$ *be a condensing map. If the set*

$$V := \{y \in E; \lambda y \in G(y) \text{ for some } \lambda > 1\}$$

is bounded, then G *has a fixed point.*

3. The Main Result

Let us introduce the following assumptions:

(H1) $F : J \times E \times E \to \text{bcc}(E)$; $(t, y, z) \mapsto F(t, y, z)$

- i) is measurable, with respect to t, for each $y, z \in E$
- ii) u.s.c. with respect to $(y, z) \in E \times E$ for a.e. $t \in J$
- iii) for each $y \in C(J; E)$ the set

$$S_{F,y} := \{g \in L^1(J; E); g(t) \in F(t, y(t), (Sy)(t)) \text{ for a.e. } t \in J\}$$

is nonempty.

(H2) There exist $p \in L^1(J : \mathbb{R}_+)$ and a continuous function $\psi : [0, +\infty) \times [0, +\infty) \to (0, +\infty)$ nondecreasing in both of its arguments, with $\displaystyle\int_0^{+\infty} \frac{du}{\psi(u, u)} = +\infty$, such that

$$\|F(t, y, z)\| = \sup\{|v| : v \in F(t, y, z)\} \leq p(t)\psi(\|y\|, \|z\|)$$
$$\text{for a.e. } t \in J, \text{ all } y, z \in E.$$

(H3) $K : \{(t, s) \in J \times J; 0 \leq s \leq t \leq T\} \to \mathbb{R}$ is continuous in t, measurable in s and such that $\displaystyle\sup_{t\in J} \int_0^t |K(t, s)|ds \leq 1$.

(H4) For each bounded set $B \subset C(J,E)$, and $t \in J$ the set

$$\left\{\int_0^t g(s)ds + \sum_{0<t_k<t} I_k(y(t_k)) : g \in S_{F,B}\right\}$$

is relatively compact in E, where $S_{F,B} = \cup\{S_{F,y} : y \in B\}$.

Remark 3.1. (i) The set $S_{F,y}$ is the set of selections of F. In general, this set may be empty. However, for measurable $F(\cdot, y(\cdot), Sy(\cdot))$ $S_{F,y} \neq \phi$, if and only if

$$\inf\{\|v\|; v \in F(\cdot, y(\cdot), Sy(\cdot))\} \in L^1(J) \text{ (see [12])}.$$

(ii) Also, if $\dim E = \infty$, in order to get meausurable selections for the multifunction $t \mapsto F(t, y(t), z(t))$, we can suppose that F is measurable with respect to $\mathcal{L} \otimes \mathcal{B} \otimes \mathcal{B}$, where $\mathcal{L}$ and $\mathcal{B}$ are the Lebesgue and Borel σ-fields on J and E, respectively.

(iii) If $\dim E < +\infty$ then $S_{F,y} \neq \phi$ (see [15]).

(iv) Assumption (H2) is satisfied, for example, if F satisfies the standard condition

$$\|F(t,y,z)\| \leq p(t)(1 + \|y\| + \|z\|), \quad p \in L^1,\ t \in J,\ y,\ z \in E.$$

(v) If we assume that for each $t \in J$, the multivalued map $F(t,\cdot,\cdot)$ maps bounded sets into relatively compact sets in E, then condition (H4) is satisfied.

(vi) If the Banach space E is finite dimensional, then (H4) is automatically satisfied.

(vii) Assumption (H3) implies that $\|S\|_{op.} \leq 1$.

We have the following auxiliary result.

Lemma 3.1. *If $y \in \Omega^1$, then*

$$y(t) = y(0) + \int_0^t y'(s)ds + \sum_{0<t_k<t} \left[y(t_k^+) - y(t_k)\right] \text{ for } t \in J.$$

Proof. Recall that $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = T$. Suppose $t_k < t \leq t_{k+1}$. Then

$$y(t_1) - y(0) = \int_0^{t_1} y'(s)ds\,,$$
$$y(t_2) - y(t_1^+) = \int_{t_1}^{t_2} y'(s)ds\,,$$
$$\cdots\cdots$$
$$y(t_k) - y(t_{k-1}^+) = \int_{t_{k-1}}^{t_k} y'(s)ds\,,$$
$$y(t) - y(t_k^+) = \int_{t_k}^{t} y'(s)ds.$$

Adding these inequalities together, we get

$$y(t) - y(0) - \sum_{i=1}^{k} \left[y(t_i^+) - y(t_i)\right] = \int_0^t y'(s)ds.$$

Hence

$$y(t) = y(0) + \int_0^t y'(s)ds + \sum_{0<t_k<t} \left[y(t_k^+) - y(t_k)\right]. \qquad \square$$

We, now, state and prove our main result.

Theorem 3.1. *Assume that (H1)-(H4) are satisfied. Then the impulsive initial value problem (1)-(3) has at least one solution.*

Proof. It follows from Lemma 3.1 that y is a solution of problem (1)-(3), if $y \in \Omega$ and satisfies

$$y(t) \in \int_0^t F(s, y(s), (Sy)(s))ds + \sum_{0<t_k<t} \left[y(t_k^+) - y(t_k)\right].$$

By assumption (H1), for each $y \in \Omega$ the set

$$S_{F,y} := \{g \in L^1(J;E); g(t) \in F(t, y(t), (Sy)(t)) \text{ for a.e. } t \in J\}$$

is not empty.

For $g \in S_{F,y}$, let

$$h(t) = \int_0^t g(s)ds + \sum_{0<t_k<t} \left[y(t_k^+) - y(t_k)\right].$$

We can easily show that $h \in \Omega$.

Define a set-valued operator $G : \Omega \to 2^{\Omega}$ by

$$G(y) := \left\{ h \in \Omega; h(t) = \int_0^t g(s)ds + \sum_{0<t_k<t} \left[y(t_k^+) - y(t_k) \right], g \in S_{F.,y} \right\}.$$

It is clear, that a fixed point of G is a solution of (1)-(3) and vice-versa.

We shall show that G is a completely continuous set-valued operator, u.s.c. with convex, closed values. This proof will be given in several steps.

Step 1. $G(y)$ is convex for each $y \in \Omega$. In fact, if h_1 and h_2 belong to $G(y)$, then for $0 \leq \mu \leq 1$ and $t \in J$ we have

$$\mu h_1(t) + (1-\mu)h_2(t) = \int_0^t \left[\mu g_1(s) + (1-\mu) g_2(s) \right] ds + \sum_{0<t_k<t} I_k(y(t_k)).$$

Since F has convex values, then $S_{F,y}$ is convex, and so $\mu h_1 + (1-\mu)h_2 \in G(y)$.

Step 2. G maps bounded subsets of Ω into bounded subsets in Ω.

Let $B_\rho := \{ y \in \Omega; \|y\|_\Omega \leq \rho \}$ be a bounded subset of Ω. Let $y \in B_\rho$. Then for each $h \in G(y)$ and $t \in J$, we have (by (H2))

$$\begin{aligned} \|h(t)\|_\Omega &\leq \int_0^t \|g(s)\|_{L^1} ds + \sum_{0<t_k<t} \|I_k(y(t_k))\| \\ &\leq \int_0^t p(s)\psi(\|y(s)\|, \|Sy(s)\|)ds + \sum_{0<t_k<t} \|I_k(y(t_k))\|. \end{aligned}$$

Now

$$\|Sy(t)\| \leq \int_0^t |K(t,s)| \, \|y(s)\| ds \leq \left(\sup_{t \in J} \int_0^t |K(t,s)| ds \right) \|y\|_\Omega.$$

Hence

$$\|Sy(t)\| \leq \|y\|_\Omega \quad \text{(by (H3))}.$$

Also

$$\sum_{0<t_k<t} \|I_k(y(t_k))\| \leq \sum_{k=1}^{m} \max_{y \in B_\rho} \|I_k(y)\|.$$

This implies that

$$\begin{aligned}\|h(t)\|_{\Omega} &\leq \int_0^t p(s)\psi\left(\|y\|_{\Omega}, \|y\|_{\Omega}\right) ds + \sum_{k=1}^{m} \max_{y \in B_\rho} \|I_k(y)\| \\ &\leq \|p\|_{L^1} \max_{y \in B_\rho} \psi\left(\|y\|_{\Omega}, \|y\|_{\Omega}\right) + \sum_{k=1}^{m} \max_{y \in B_\rho} \|I_k(y)\|.\end{aligned}$$

Let

$$R := \|p\|_{L^1}\psi(\rho,\rho) + \sum_{k=1}^{m} \max\left\{\|I_k(y)\|;\ \|y\|_{\Omega} \leq \rho\right\}.$$

Then the above inequalities show that

$$\|G(y)\| = \sup\{\|h\|_{\Omega} : h \in G(y)\} \leq R,$$

which shows that $G(B_\rho)$ is bounded for each bounded subset B_ρ of Ω.

Step 3. $G(B_\rho)$ is equicontinuous for each bounded set B_ρ in Ω. Let $\sigma_1, \sigma_2 \in J$, $\sigma_1 < \sigma_2$ and let $B_\rho := \{y \in \Omega; \|y\|_{\Omega} \leq \rho\}$. Then, for $y \in B_\rho$ and $h \in G(y)$ we have

$$h(\sigma_2) - h(\sigma_1) = \int_{\sigma_1}^{\sigma_2} g(s)ds + \sum_{0<t_k<\sigma_2} I_k(y(t_k)) - \sum_{0<t_k<\sigma_1} I_k(y(t_k)).$$

This yields

$$\|h(\sigma_2) - h(\sigma_1)\| \leq (\sigma_2 - \sigma_1)\|p\|_{L^1} \cdot \psi(\rho,\rho) + \sum_{0<t_k<\sigma_2-\sigma_1} \|I_k(y(t_k))\|,$$

which implies that $G(y)$ is equicontinuous for each $y \in B_\rho$.

As a consequence of Step 2 and Step 3 and (H4) together with the Arzela-Ascoli theorem we can conclude to the compactness of the operator G. Therefore, G is a condensing map.

Step 4. G has a closed graph.

Let $y_n \longrightarrow y_0,\ h_n \in G(y_n)$ and $h_n \longrightarrow h_0$. We shall prove that $h_0 \in G(y_0)$.

$h_n \in G(y_n)$ means that there exist $g_n \in S_{F,y_n}$ such that

$$h_n(t) = \int_0^t g_n(s)ds + \sum_{0<t_k<t} I_k(y_n(t_k)).$$

We must prove that there exists $g_0 \in S_{F,y_0}$ such that

$$h_0(t) = \int_0^t g_0(s)ds + \sum_{0<t_k<t} I_k(y_0(t_k)).$$

Consider the linear continuous operator $\Gamma : L^1(J,E) \longrightarrow C(J,E)$ defined by

$$(\Gamma v)(t) = \int_0^t v(s)ds.$$

Since I_k, $k = 1, ..., m$, are continuous, then we have

$$\|(h_n - \sum_{0<t_k<t} I_k(y_n(t_k))) - (h_0 - \sum_{0<t_k<t} I_k(y_0(t_k)))\|_\infty \longrightarrow 0 \quad \text{as} \quad n \longrightarrow \infty.$$

From Lemma 2.1 it follows, that $\Gamma \circ S_F$ is a closed graph operator.
Also from the definition of Γ we have that

$$h_n(t) - \sum_{0<t_k<t} I_k(y_n(t_k)) \in \Gamma(S_{F,y_n}).$$

This, besides to $y_n \longrightarrow y_0$ and Lemma 2.1, furnishs

$$h_0(t) = \int_0^t g_0(s)ds + \sum_{0<t_k<t} I_k(y_0(t_k))$$

for some $g_0 \in S_{F,y_0}$.

To complete the proof of the theorem, it remains to show that the set

$$V := \{y \in \Omega; \lambda y \in G(y) \ \text{ for some } \ \lambda > 1\}$$

is bounded.

In fact, let $y \in V$. Then $\lambda y \in G(y)$ for some $\lambda > 1$. There exists $g \in S_{F,y}$ such that

$$y(t) = \lambda^{-1} \int_0^t g(s)ds + \lambda^{-1} \sum_{0<t_k<t} I_k(y(t_k)).$$

Since $\lambda^{-1} < 1$, it follows from (H2) and (H3) that

$$\|y(t)\| \leq \int_0^t p(s)\psi(\|y(s)\|, \|y(s)\|)ds + \sum_{k=1}^m \sup_{t\in J} \|I_k(y(t))\|.$$

As a consequence of Lemma 2.2, we can conclude that

$$\|y\|_\Omega \leq \|z\|_\infty ,$$

where z is the unique solution of the initial value problem

$$\begin{cases} z'(t) = p(t)\psi(z(t), z(t)) & \text{for a.e. } t \in J \\ z(0) = \sum_{k=1}^{m} \sup_{t \in J} \|I_k(y(t))\| . \end{cases} \tag{7}$$

We conclude from Lemma 2.3, that G has a fixed point, which is a solution of our problem (1)-(3). This completes the proof of Theorem 3.1. □

Remark 3.2. The existence and uniqueness of solutions of problem (7) follow from assumption (H2) combined with Lemma 2.2.

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