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ASYMPTOTIC ANALYSIS OF THE GABITOV-TURITSYN EQUATION

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Abstract: The Gabitov-Turitsyn equation is the universal asymptotic equation that governs the evolution of amplitude of an optical pulse in a dispersion-managed soliton system. The nonlinear term of this equation is analysed asymptotically. The total spectral intensity, for a lossless system, is found to be an invariant of propagation, while for a lossy system it is dependent on the relative position of the amplifier in the dispersion map. We have considered both types of fibers namely polarization-preserving, as well as birefringent type. Finally, the case of multiple channels is also studied.

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1. Introduction

The propagation of solitons through optical fibers has been a major area of research given its potential applicability in all optical communication systems. The field of telecommunications has undergone a substantial evolution in the last couple of decades due to the impressive progress in the development of optical fibers, optical amplifiers, as well as transmitters and receivers. In a

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modern optical communication system, the transmission link is composed of optical fibers and amplifiers that replace the electrical regenerators. But the amplifiers introduce some noise and signal distortion that limit the system capacity. Presently the optical systems that show the best characterestics in terms of simplicity, cost and robustness against the degrading effects of a link are those based on intensity modulation with direct detection (IM-DD). Conventional IM-DD systems are based on non-return-to-zero (NRZ) format, but for transmission at higher data rate the return-to-zero (RZ) format is preferred. When the data rate is quite high, soliton transmission can be used. It allows the exploitation of the fiber capacity much more, but the NRZ signals offer very high potential especially in terms of simplicity.

In quasi-linear or low-powered systems, strong dispersion-management (DM), which uses multiple section of fibers that alternates positive and negative group-velocity dispersion, is used to compensate the fiber dispersion, manage fiber nonlinearity and suppress the inter-channel cross talk. The difference between DM soliton transmission and the quasi-linear pulses is that in the former, nonlinearity balances dispersion, while in the latter, nonlinearity is managed. In the past few years there has been quite a few theoretical, numerical and experimental studies that was conducted in regards to the quasi-linear pulse transmission. In this paper, we shall take a detailed look at the analysis of quasilinear pulse transmission through optical fibers. We have considered the polarization-preserving fibers, birefringent fibers, as well as multiple channels. The Gabitov-Turitsyn (GT) equation for the dispersion-managed nonlinear Schrödinger equation (NLSE) is considered in all these three cases and the asymptotic analysis is carried out. The significant reduction in the effective nonlinearity for large map strength is quantified.

2. Mathematical Formulation

The NLSE, with damping and periodic amplification, in the dimensionless form is [2, 16]:

$$iq_z + \frac{D(z)}{2} q_{tt} + |q|^2 q = -i\Gamma q + i\left[e^{\Gamma z_a} - 1\right] \sum_{n=1}^{N} \delta(z - nz_a) q \,. \tag{1}$$

Here, Γ is the normalized loss coefficient, z_a is the normalized characteristic amplifier spacing and z and t represent the normalized propagation distance and the normalized time, respectively, expressed in the usual nondimensional units.

Also, $D(z)$ is used to model strong dispersion-management. We decompose the fiber dispersion $D(z)$ into two components namely a path-averaged constant value δ_a and a term representing the large rapid variation due to large local values of the dispersion [2]. Thus, we write:

$$D(z) = \delta_a + \frac{1}{z_a}\Delta(\zeta)\,, \tag{2}$$

where $\zeta = z/z_a$. The function $\Delta(\zeta)$ is taken to have average zero (namely $\langle\Delta\rangle = 0$), so that the path-averaged dispersion $\langle D\rangle = \delta_a$. The proportionality factor in front of $\Delta(\zeta)$ is chosen so that both δ_a and $\Delta(\zeta)$ are quantities of order one. In practical situations, dispersion-management is often performed by concatenating together two or more sections of given length of a fiber with different values of fiber dispersion. In the special case of a two-step map it is convenient to write the dispersion map as a periodic extension of [2]

$$\Delta(\zeta) = \begin{cases} \Delta_1 & : \quad 0 \le |\zeta| < \frac{\theta}{2}\,, \\ \Delta_2 & : \quad \frac{\theta}{2} \le |\zeta| < \frac{1}{2}\,, \end{cases} \tag{3}$$

where Δ_1 and Δ_2 are given by

$$\Delta_1 = \frac{2s}{\theta} \tag{4}$$

and

$$\Delta_2 = -\frac{2s}{1-\theta}\,, \tag{5}$$

with the map strength s defined as

$$s = \frac{\theta\Delta_1 - (1-\theta)\Delta_2}{4}\,. \tag{6}$$

Conversely, we have

$$s = \frac{\Delta_1\Delta_2}{4(\Delta_2 - \Delta_1)} \tag{7}$$

and

$$\theta = \frac{\Delta_2}{\Delta_2 - \Delta_1}\,. \tag{8}$$

A typical dispersion map is shown in the Figure 1. We take into account

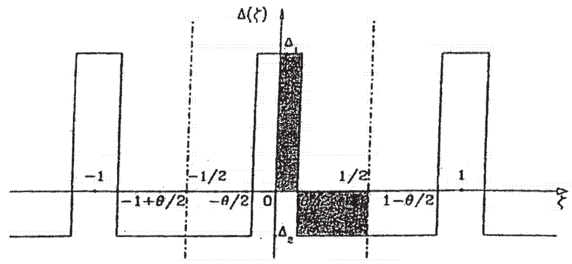

Figure 1: Schematic diagram of a two-step map

the loss and amplification cycles by looking for a solution of (1) of the form $q(z,t) = A(z)u(z,t)$ for real A. Taking A to satisfy

$$A_z + \Gamma A - \left[e^{\Gamma z_a} - 1\right] \sum_{n=1}^{N} \delta(z - nz_a) A = 0\,, \tag{9}$$

we can show that (1) transforms to

$$iu_z + \frac{D(z)}{2} u_{tt} + g(z)|u|^2 u = 0\,, \tag{10}$$

where we have

$$g(z) = A^2(z) = a_0^2 e^{-2\Gamma(z - nz_a)}\,, \tag{11}$$

for $z \in [nz_a, (n+1)z_a)$ and $n > 0$ and also

$$a_0 = \left[\frac{2\Gamma z_a}{1 - e^{-2\Gamma z_a}}\right]^{\frac{1}{2}}\,, \tag{12}$$

so that $\langle g(z) \rangle = 1$ over each amplification period. Equation (10) governs the propagation of a dispersion-managed soliton through a polarization preserved optical fiber with damping and periodic amplification and is known as the perturbed NLSE (PNLSE).

3. Polarization-Preserving Fibers

We note that, equation (10) is not integrable nor does it contain infinitely many conserved quantities. In fact, it has as few as two integrals of motion [3]. They are the energy (E) and the linear momentum (M) that are respectively given by

$$E = \int_{-\infty}^{\infty} |u|^2 \, dt \,, \tag{13}$$

$$M = i\frac{D(z)}{2} \int_{-\infty}^{\infty} (u^* u_t - u u_t^*) \, dt \,. \tag{14}$$

The Hamiltonian (H) that is given by

$$H = \frac{1}{2} \int_{-\infty}^{\infty} \left(D(z)|u_t|^2 - g(z)|u|^4 \right) \, dt \,, \tag{15}$$

is, in general, not a constant of motion unless $D(z)$ and $g(z)$ are constants in which case there are infinitely many conserved quantities. The existence of a Hamiltonian implies that we can also write the PNLSE in the Hamiltonian form as:

$$\frac{\partial u}{\partial z} = \{H, u\} = -i\frac{\delta H}{\delta u^*} \,, \tag{16}$$

$$\frac{\partial u^*}{\partial z} = \{H, u^*\} = i\frac{\delta H}{\delta u} \,, \tag{17}$$

where $\{A, B\}$ is the usual Poisson bracket between the two functionals that is defined as:

$$\{A, B\} = \int_{-\infty}^{\infty} \left(\frac{\delta A}{\delta u}\frac{\delta B}{\delta u^*} - \frac{\delta A}{\delta u^*}\frac{\delta B}{\delta u} \right) \, dt \,. \tag{18}$$

The Gabitov-Turitsyn (GT) equation for pulses in polarization preserving fibers in the Fourier domain is given by [12, 13]

$$i\frac{\partial \hat{U}}{\partial z} - \frac{\delta_a}{2}\omega^2 \hat{U} + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{U}(z, \omega + \omega_1)\hat{U}(z, \omega + \omega_2) \times \hat{U}^*(z, \omega + \omega_1 + \omega_2) r(\omega_1 \omega_2) \, d\omega_1 d\omega_2 = 0 \tag{19}$$

where the kernel $r(x)$ is given by

$$r(x; s) = \frac{1}{(2\pi)^2} \int_0^1 g(\zeta) e^{iC(\zeta)x} \, dx \,. \tag{20}$$

By taking the inverse Fourier transform of (19) we arrive at the following GT equation in the temporal domain.

$$i\frac{\partial U}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 U}{\partial t^2} + g(z)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(z,t+t_1)U(z,t+t_2) \times U^*(z,t+t_1+t_2)R(t_1,t_2)dt_1dt_2 = 0\,, \quad (21)$$

where we have the kernel $R(t_1, t_2)$ as

$$R(t_1,t_2;s) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(\omega_1 t_1 + \omega_2 t_2)} r(\omega_1\omega_2)\, d\omega_1 d\omega_2\,. \quad (22)$$

In (21), $U(z,t)$ is the slowly varying amplitude of $u(z,t)$ at the leading order. This equation is the universal asymptotic equation that governs the evolution of the amplitude of an optical pulse for a dispersion-managed system that is given by (10). Here, in (21) all fast and large variations are removed. It is necessary to note here that (21) is equally applicable to the case of pulse dynamics for a zero or normal value of average dispersion.

3.1. Quasi-Linear Pulses

In this section, we shall analyse and present the details of the nonlinear terms of the GT equations, for large s. The analysis will also explain the difference between the quasi-linear transmission and soliton propagation. We shall now split the study in two subsections that deals with the lossless and the lossy cases respectively.

Figure 2 is a schematic diagram of a two-step dispersion map for $\theta = 1/2$. It is a special case of Figure 1.

3.1.1. Lossless System

In a lossless system, namely $g(z) = 1$, we have the kernels given by (20) and (22) respectively modify to

$$r(x;s) = \frac{1}{(2\pi)^2}\frac{\sin(sx)}{sx} \quad (23)$$

and

$$R(t_1,t_2;s) = \frac{1}{2\pi}\frac{ci(\frac{t_1 t_2}{s})}{|s|}\,, \quad (24)$$

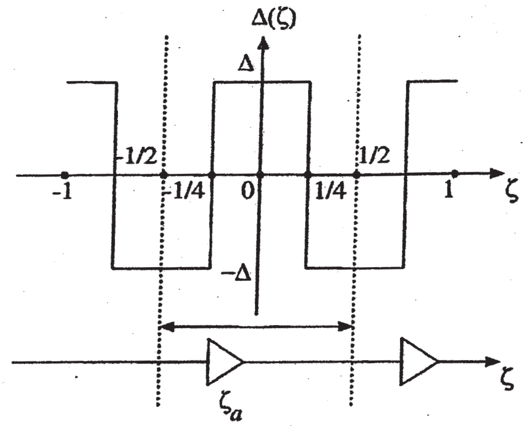

Figure 2: Schematic diagram of a two-step dispersion map

where the cosine integral $ci(x)$ is defined as

$$ci(x) = \int_1^\infty \frac{\cos(xy)}{y}\, dy\,. \tag{25}$$

We note that we have, from (23)

$$\lim_{s\to\infty} r(x;s) = 0\,, \tag{26}$$

thus showing that, for large map strength, we get the linear evolution equations. Assuming $\hat{U}(z,\omega)$ depends weakly on s, we have the following asymptotic expansions of the nonlinear terms from the GT equation [6, 10]

$$i\frac{\partial U}{\partial Z} + \frac{\delta_a}{2}\frac{\partial^2 U}{\partial t^2} + \frac{1}{2\pi s}\left[(\log s - \gamma)J_1(z,t) - J_2(z,t)\right] = 0\,, \tag{27}$$

where

$$J_1(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U\left(z,t+t_1\right)U\left(z,t+t_2\right) \times U^*\left(z,t+t_1+t_2\right)dt_1dt_2, \tag{28}$$

$$J_2(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \log|t_1t_2|\,U(z,t+t_1)U(z,t+t_2) \times U^*(z,t+t_1+t_2)dt_1dt_2 \tag{29}$$

and

$$\gamma = \lim_{n\to\infty}\left[1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots+\frac{1}{n}-\log n\right] = 0.57721\ldots \tag{30}$$

is the Euler constant. In the Fourier domain, (27) transforms to [6, 10]

$$i\frac{\partial\hat{U}}{\partial z} - \frac{\delta_a\omega^2}{2}\hat{U} + \frac{1}{2\pi s}\left[(\log s-\gamma)\hat{J}_1(z,\omega) - \hat{J}_2(z,\omega)\right] = 0\,, \tag{31}$$

where we have

$$\hat{J}_1(z,\omega) = \left|\hat{U}(z,\omega)\right|^2\hat{U}(z,\omega)\,, \tag{32}$$

$$\hat{J}_2(z,\omega) = \frac{1}{\pi}\hat{U}(z,\omega)\int_{-\infty}^{\infty}\left|\hat{U}(z,\omega')\right|^2 h\left(\omega'-\omega\right)d\omega' \tag{33}$$

and

$$h(\omega) = \int_{-\infty}^{\infty}\log|t|e^{-i\omega t}dt\,. \tag{34}$$

These asymptotic results can be obtained by the stationary phase method applied to (19) [6, 10]. They can also be derived directly from (21) and using the asymptotic expansion of the cosine integral namely

$$ci(x) \sim -\gamma - \ln x + O(x)\,, \tag{35}$$

as $x \to 0$. Now, neglecting the $O(1/s^2)$ term, one can see that the spectral intensity given by $|\hat{U}(z,\omega)|^2$ is preserved during the pulse propagation. Now, we have, from (31)

$$\frac{\partial}{\partial z}\left|\hat{U}\right|^2 = 0\,. \tag{36}$$

Also, the solution of (31) is

$$\hat{U}(z,\omega) = \hat{U}(z,0)\exp\left\{-i\frac{\delta_a\omega^2}{2}z + i\psi\left[\left|\hat{U}(0,\omega)\right|^2\right]z\right\}\,, \tag{37}$$

where

$$\psi\left[\left|\hat{U}(z,\omega)\right|^2\right] = \frac{1}{2\pi s}\left[(\log s-\gamma)\left|\hat{U}(z,\omega)\right|^2 - \frac{1}{\pi}\int_{-\infty}^{\infty}\left|\hat{U}(z,\omega)\right|^2 h(\omega'-\omega)d\omega'\right]. \tag{38}$$

The linear phase shift exp $\left(-i\delta_a\omega^2 z/2\right)$ can be corrected by the pre-transmission or the post-transmission compensation. After this, the linear phase is removed. However, if the system has a small value of the path averaged dispersion namely if $\delta_a \ll 1$ the average dynamics of the quasi-linear pulse transmission through an optical fiber is charactrerized only by the nonlinear phase shift $\phi_{NL}(z,\omega) = \psi\left[\left|\hat{U}(0,\omega)\right|^2\right] z$.

The nonlinear chirp, for small values of the map strength (s), can induce pulse broadening [6, 10]. However, we find a simple way to compensate for the nonlinear chirp by expanding $\phi_{NL}(z,\omega)$ in a Taylor series with respect to ω:

$$\phi_{NL}(z,\omega) = \phi_{NL}(z,0) + \frac{\omega^2}{2!}\phi''_{NL}(z,0) + \frac{\omega^2}{4!}\phi''''_{NL}(z,0) + \cdots ,$$

with $|\hat{U}(0,\omega)|^2$ assumed to be even.

We, also, note that large values of s reduces the effective nonlinearity [6]. Therefore, a pulse in a polarization preserved fiber will be able to propagate for much longer distances before being distorted by nonlinearity, as opposed to a pulses, with the same energy, in a system with constant dispersion. Thus, strong DM allows the propagation of stationary pulse in a quasi-linear regime, with energies comparable to that of classical NLS solitons but at the same time much lower than the energy required for the formation of a stable DM soliton at $\delta_a \sim 0$ for the same value of s owing to the energy enhancement of DM solitons.

3.1.2. Lossy System

For the lossy case, namely when $g(\zeta) \neq 1$, the kernel $r(x;s)$ depends on the relative position of the amplifier with respect to the dispersion map. We consider the two step map given by $\Delta(\zeta)$ in (3) and define ζ_a to represent the position of the amplifier within the dispersion map. So $|\zeta_a| < 1/2$ and $\zeta_a = 0$ means that the amplifier is placed at the mid point of the anomalous fiber segments. The function $g(\zeta)$ given by (11) can then be written as

$$g(\zeta) = \frac{2Ge^{2G}}{\sinh(2G)} e^{-4G(\zeta-n-\zeta_a)} , \tag{39}$$

for $\zeta_a + n \leq \zeta < \zeta_a + n + 1$, where we have defined $G = \Gamma z_a/2$. The kernel $r(x;s)$ in the lossy case is computed in a similar method as in the lossless case. If $|\zeta_a| < \theta/2$, namely the amplifier is located in the anomalous fiber segment, the resulting expression for kernel is

$$r(x;s) = \frac{1}{(2\pi)^2} \frac{Ge^{iC_0 x}}{(sx + 2iG\theta)(sx - 2iG(1-\theta))} \times \left[e^{G(4\zeta_a - 2\theta + 1)} sx \frac{\sin(sx - 2iG(1-\theta))}{\sinh(2G)} + i\theta e^{i(4\zeta_a - 2\theta+1)\frac{sx}{2\theta}} (sx - 2iG(1-\theta)) \right] \quad (40)$$

We note that in (40), unlike the lossless case, the kernel $r(x;s)$ is complex and is explicitly dependent on the parameters θ, Γ, z_a and ζ_a in a nontrivial way. However, we still have

$$\lim_{s \to 0} r(x;s) = \frac{1}{(2\pi)^2}\,, \quad (41)$$

and moreover,

$$\lim_{G \to 0} r(x;s) = \frac{1}{(2\pi)^2} \frac{\sin(sx)}{sx}\,, \quad (42)$$

which means that as $z_a \to 0$ we recover (23). The effect of nonlinearity on quasi-linear transmission with $g(\zeta) \neq 1$ is analysed by the asymptotic expansion of the nonlinear terms in the GT equations for large s. We shall now split the study into the following four cases, for $\theta = 1/2$, depending on the position of the amplifiers.

Case-I. ($\zeta_a = 0$)

This locates the amplifier in the middle of the anomalous GVD segments. In this case we have the kernels $r(x;s)$ and $R(t_1, t_2; s)$ as

$$r(x;s) = \frac{1}{(2\pi)^2} \frac{G}{x^2 s^2 + G^2} \left[sx \frac{\sin(sx)}{\sinh(G)} + isx \left\{ 1 - \frac{\cos(sx)}{\cosh(G)} \right\} + G \right] \quad (43)$$

and

$$R(t_1, t_2; s) = \frac{G}{(2\pi)^2} \left[\left(\frac{I_{2S}}{\sinh G} + I_1 G \right) + i \left(I_2 - \frac{I_{2C}}{\cosh G} \right) \right]\,, \quad (44)$$

where we have [6, 10]

$$I_1(t_1, t_2; s) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(\omega_1 t_1 + \omega_2 t_2)}}{G^2 + (s\omega_1\omega_2)^2} d\omega_1 d\omega_2\,, \quad (45)$$

$$I_2(t_1,t_2;s)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{i(\omega_1t_1+\omega_2t_2)}}{G^2+(s\omega_1\omega_2)^2}s\omega_1\omega_2d\omega_1d\omega_2\,, \tag{46}$$

$$I_{2S}(t_1,t_2;s)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{i(\omega_1t_1+\omega_2t_2)}}{G^2+(s\omega_1\omega_2)^2}s\omega_1\omega_2\sin\left(s\omega_1\omega_2\right)d\omega_1d\omega_2\,, \tag{47}$$

$$I_{2C}(t_1,t_2;s)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{e^{i(\omega_1t_1+\omega_2t_2)}}{G^2+(s\omega_1\omega_2)^2}s\omega_1\omega_2\cos\left(s\omega_1\omega_2\right)d\omega_1d\omega_2\,. \tag{48}$$

The asymptotic expansion of the integrals in the kernel $R(t_1,t_2;s)$ for large s gives [10]

$$R(t_1,t_2;s)\sim\frac{P_0}{s}\left(\log s-\log|t_1t_2|\right)-\frac{P_1}{s}-i\frac{P_2}{s}\mathrm{sgn}\,\left(t_1t_2\right)\,, \tag{49}$$

where $\mathrm{sgn}\,(x)$ is the signum function defined as

$$\mathrm{sgn}\,(t)=\frac{1}{i\pi}\int_{-\infty}^{\infty}\frac{e^{i\omega t}}{\omega}d\omega \tag{50}$$

and

$$P_0=\frac{G}{2\pi}\left(\frac{e^{-G}}{\sin G}+1\right)\,, \tag{51}$$

$$P_1=\frac{G}{4\pi}\frac{1}{\sin G}\left[e^GI_{G_1}+\left(3\gamma+\log G-I_{G_2}\right)e^{-G}\right]+\frac{G}{2\pi}\left(2\gamma+\log G\right)\,, \tag{52}$$

$$P_2=\frac{G}{4}\,, \tag{53}$$

with

$$I_{G_1}=\int_G^{\infty}\frac{e^{-x}}{x}dx\,, \tag{54}$$

$$I_{G_2}=\int_0^{G}\frac{e^{x}-1}{x}dx\,. \tag{55}$$

We note that, we have

$$\lim_{G\to 0} P_0(G) = \frac{1}{2\pi}\,, \tag{56}$$

$$\lim_{G\to 0} P_1(G) = \frac{\gamma}{2\pi}\,, \tag{57}$$

$$\lim_{G\to 0} P_2(G) = 0\,, \tag{58}$$

so that these reduce to the lossless case as Γ approaches zero. Thus, the GT equation, given by (21), for large s, reduces to [10]

$$i\frac{\partial U}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 U}{\partial t^2} + \frac{1}{s}\left[(P_0 \log s - P_1)J_1(z,t) - P_0 J_2(z,t) - iP_2 J_3(z,t)\right] = 0\,, \tag{59}$$

where we have

$$J_3(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn}(t_1 t_2) U(z,t+t_1)U(z,t+t_2) \times U^*(z,t+t_1+t_2)dt_1 dt_2\,. \tag{60}$$

In the Fourier domain, (59) transforms to [10]

$$i\frac{\partial \hat{U}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{U} + \frac{1}{s}\left[(P_0 \log s - P_1)\hat{J}_1(z,\omega) - P_0 \hat{J}_2(z,\omega) - iP_2 \hat{J}_3(z,\omega)\right] = 0\,, \tag{61}$$

where

$$\hat{J}_3(z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_1\omega_2}\hat{U}(z,\omega+\omega_1) \times \hat{U}(z,\omega+\omega_2)\hat{U}^*(z,\omega+\omega_1+\omega_2)d\omega_1 d\omega_2\,. \tag{62}$$

We note that in (62) integral represents the Cauchy principal value. Now, from (61) we observe that

$$\frac{\partial}{\partial z}\left|\hat{U}\right|^2 = \frac{P_2}{s}H(z,\omega)\,, \tag{63}$$

with

$$H(z,\omega) = \hat{J}_3(z,\omega)\hat{U}^*(z,\omega) + \hat{J}_3^*(z,\omega)\hat{U}(z,\omega)\,. \tag{64}$$

For large s, and moderate z, one can write

$$\left|\hat{U}(z,\omega)\right|^2 \approx \left|\hat{U}(0,\omega)\right|^2 + \frac{P_2 z}{s} H(0,\omega)\,, \tag{65}$$

and, thus, the total spectral intensity does not remain constant in this case.

Case-II. ($\zeta_a = -1/2$)

In this case, the amplifier is positioned in the middle of the normal GVD segment. The kernels of the GT equations, in this case, reduce to

$$r(x;s) = \frac{1}{(2\pi)^2}\frac{G}{x^2s^2+G^2}\left[sx\frac{\sin(sx)}{\sinh(G)} - isx\left\{1-\frac{\cos(sx)}{\cosh(G)}\right\} + G\right] \tag{66}$$

and

$$R(t_1,t_2;s) = \frac{G}{(2\pi)^2}\left[\left(\frac{I_{2S}}{\sinh G} + I_1 G\right) - i\left(I_2 - \frac{I_{2C}}{\cosh G}\right)\right]\,, \tag{67}$$

where the parameters are same as defined before. The only difference here is that we have negative imaginary part. Thus, we have in the asymptotic state

$$R(t_1,t_2;s) \sim \frac{P_0}{s}\left(\log s - \log|t_1t_2|\right) - \frac{P_1}{s} + i\frac{P_2}{s}\mathrm{sgn}\;(t_1t_2)\;. \tag{68}$$

The intensity satisfies

$$\frac{\partial}{\partial z}\left|\hat{U}\right|^2 = -\frac{P_2}{s}H(z,\omega)\,. \tag{69}$$

Finally, for $s \gg 1$ and moderate z we get

$$\left|\hat{U}(z,\omega)\right|^2 \approx \left|\hat{U}(0,\omega)\right|^2 - \frac{P_2 z}{s} H(0,\omega)\,. \tag{70}$$

So, the total spectral intensity does not stay conserved here, too.

Case-III. ($\zeta_a = -1/4$)

Here, the amplifier is placed at the boundary between the anomalous and normal GVD segment. In this case, the kernels are given by

$$\begin{aligned} r(x;s) &= \frac{1}{(2\pi)^2}\frac{G}{x^2s^2+G^2} \\ &\times\left[\left\{\left(\frac{e^G}{\sinh G}-1\right)sx\sin(sx) + G\cos(sx)\right\}\right. \\ &\left.-i\left\{\left(\frac{e^{-G}}{\cosh G}-1\right)sx\cos(sx) + G\sin(sx)\right\}\right] \end{aligned} \tag{71}$$

and

$$R(t_1, t_2; s) = \frac{G}{(2\pi)^2} \left[\left\{ \left(\frac{e^{-G}}{\sinh G} - 1 \right) I_{2S} + G I_{1C} \right\} \right.$$
$$\left. - i \left\{ \left(\frac{e^{-G}}{\cosh G} - 1 \right) I_{2C} + G I_{1S} \right\} \right], \quad (72)$$

where we have

$$I_{1S}(t_1, t_2; s) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(\omega_1 t_1 + \omega_2 t_2)}}{G^2 + (s\omega_1\omega_2)^2} \sin(s\omega_1\omega_2)\, d\omega_1 d\omega_2 \,, \quad (73)$$

$$I_{1C}(t_1, t_2; s) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{e^{i(\omega_1 t_1 + \omega_2 t_2)}}{G^2 + (s\omega_1\omega_2)^2} \cos(s\omega_1\omega_2)\, d\omega_1 d\omega_2 \,. \quad (74)$$

For large s we get,

$$R(t_1, t_2; s) \sim \frac{Q_0}{s} (\log s - \log |t_1 t_2|) - \frac{Q_1}{s} \,, \quad (75)$$

where

$$Q_0 = \frac{G}{2\pi} \frac{e^{-2G}}{\sinh G} \,, \quad (76)$$

$$Q_1 = \frac{G}{4\pi} \frac{e^{-G}}{\sinh G} \left[e^G I_{G_1} + (3\gamma + \log G - I_{G_2})\, e^{-G} \right] - \frac{G}{2\pi} e^G I_{G_1} \,. \quad (77)$$

Here, also, we have

$$\lim_{G \to 0} Q_0(G) = \frac{1}{2\pi} \,, \quad (78)$$

$$\lim_{G \to 0} Q_1(G) = \frac{\gamma}{2\pi} \,, \quad (79)$$

so that, once again, we get reduction to the lossless case as Γ approaches zero. The GT equations, in this case, reduce to

$$i \frac{\partial U}{\partial z} + \frac{\delta_a}{2} \frac{\partial^2 U}{\partial t^2} + \frac{1}{s} \left[(Q_0 \log s - Q_1) J_1(z, t) - Q_0 J_2(z, t) \right] = 0 \,, \quad (80)$$

while, in the Fourier domain, we have

$$i\frac{\partial \hat{U}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{U} + \frac{1}{s}\left[(Q_0 \log s - Q_1)\hat{J}_1(z,\omega) - Q_0 \hat{J}_2(z,\omega)\right] = 0\,. \qquad (81)$$

In this case, we note that the spectral intensity stays constant as we have from (81)

$$\frac{\partial}{\partial z}\left|\hat{U}\right|^2 = 0\,, \qquad (82)$$

and also from (81), we recover the solution (37) where we, now, have

$$\psi\left[\left|\hat{U}(z,\omega)\right|^2\right] = \frac{1}{s}\left[(Q_0 \log s - Q_1)\left|\hat{J}_1(z,\omega)\right|^2 - \frac{Q_0}{\pi}\int_{-\infty}^{\infty}\left|\hat{J}_2(z,\omega')\right|^2 h\left(\omega - \omega'\right)d\omega'\right]. \qquad (83)$$

Case-IV. ($\zeta_a = 1/4$)

Here, the amplifier is placed at the boundary between the normal and anomalous GVD segment. The kernels reduce to

$$\begin{aligned} r(x;s) = &\frac{1}{(2\pi)^2}\frac{G}{x^2 s^2 + G^2} \\ &\times\left[\left\{\left(\frac{e^G}{\sinh G} - 1\right)sx\sin(sx) + G\cos(sx)\right\}\right. \\ &\left.+i\left\{\left(\frac{e^{-G}}{\cosh G} - 1\right)sx\cos(sx) + G\sin(sx)\right\}\right] \end{aligned} \qquad (84)$$

and

$$R(t_1,t_2;s) = \frac{G}{(2\pi)^2}\left[\left\{\left(\frac{e^{-G}}{\sinh G} - 1\right)I_{2S} + GI_{1C}\right\} + i\left\{\left(\frac{e^{-G}}{\cosh G} - 1\right)I_{2C} + GI_{1S}\right\}\right]. \qquad (85)$$

The only difference in this case from that of the previous one is that here we have the imaginary part of the kernels with opposite sign. But, we have shown in the previous subsection that the imaginary part does not make any contribution to the dynamics of quasi-linear pulses, the sum of the spectral intensities is again conserved in this case during the pulse propagation.

4. Birefringent Fibers

A single mode fiber supports two degenerate modes that are polarized in two orthogonal directions. Under ideal conditions of perfect cylindrical geometry and isotropic material, a mode excited with its polarization in one direction would not couple with the mode in the orthogonal direction. However, small deviations from the cylindrical geometry or small fluctuations in material anisotropy result in a mixing of the two polarization states and the mode degeneracy is broken. Thus, the mode propagation constant becomes slightly different for the modes polarized in orthogonal directions. This property is refered to as *modal birefringence* [16].

The propagation of solitons in birefringent nonlinear fibers has attracted much attention in recent years. It has potential applications in optical communications and optical logic devices. The equations that describe the pulse propagation through these fibers was originally derived by Menyuk [10, 16]. They can be solved approximately in certain special cases only. The localized pulse evolution in a birefringent fiber has been studied analytically, numerically and experimentally [16] on the basis of a simplified chirp-free model without oscillating terms under the assumptions that the two polarizations exhibit different group velocities. In this paper we shall study the equations that describe the pulse propagation in birefringent fibers of the following dimensionless form:

$$iu_z + \frac{D(z)}{2}u_{tt} + g(z)\left(|u|^2 + \alpha|v|^2\right)u = 0\,, \tag{86}$$

$$iv_z + \frac{D(z)}{2}v_{tt} + g(z)\left(|v|^2 + \alpha|u|^2\right)v = 0\,. \tag{87}$$

Equations (86) and (87) are known as the Dispersion Managed Vector Nonlinear Schrodinger Equation (DM-VNLSE). Here, u and v are envelopes of the two linearly polarized components of the field along the x and y axis. Also, α is the cross-phase modulation (XPM) coefficient. These equations are, in general, not integrable. However, they can be solved analytically for certain specific cases [11, 17].

The two integrals of motion of (86) and (87) are the energy (E) and the linear momentum (M) of the pulses that are respectively given by:

$$E = \int_{-\infty}^{\infty}\left(|u|^2 + |v|^2\right)\,dt \tag{88}$$

and

$$M = \frac{i}{2}D(z)\int_{-\infty}^{\infty}\left(u^*u_t - uu_t^* + v^*v_t - vv_t^*\right)dt\,. \tag{89}$$

The Hamiltonian (H), given by

$$H = \frac{1}{2}\int_{-\infty}^{\infty} \left[D(z)\left(|u_t|^2 + |v_t|^2\right) - g(z)\left(|u|^4 + |v|^4\right) - 2\alpha|u|^2|v|^2 \right.$$
$$\left. -(1-\alpha)\left(u^2 v^{*2} + v^2 u^{*2}\right)\right] dt\,, \quad (90)$$

is however not a constant of motion, in general. The existence of a Hamiltonian implies that we can also write (86) and (87) in the Hamiltonian form as

$$iu_z = \frac{\delta H}{\delta u^*}\,, \quad (91)$$

and

$$iv_z = \frac{\delta H}{\delta v^*}\,. \quad (92)$$

This defines a Hamiltonian dynamical system on an infinite-dimensional phase space of two complex functions u and v that decrease to zero at infinity and can be analysed using the theory of Hamiltonian system.

The corresponding GT equations, in the Fourier domain, for the DM-VNLSE are [12, 13]

$$i\frac{\partial \hat{U}}{\partial z} - \frac{\delta_a}{2}\omega^2 \hat{U} + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(\omega_1\omega_2)\hat{U}(z,\omega+\omega_2)\left[\hat{U}(z,\omega+\omega_1)\hat{U}^*(z,\omega+\omega_1+\omega_2) \right.$$
$$\left. +\alpha\hat{V}(z,\omega+\omega_1)\hat{V}^*(z,\omega+\omega_1+\omega_2)\right] d\omega_1 d\omega_2 = 0\,, \quad (93)$$

$$i\frac{\partial \hat{V}}{\partial z} - \frac{\delta_a}{2}\omega^2 \hat{V} + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} r(\omega_1\omega_2)\hat{V}(z,\omega+\omega_2)\left[\hat{V}(z,\omega+\omega_1)\hat{V}^*(z,\omega+\omega_1+\omega_2) \right.$$
$$\left. +\alpha\hat{U}(z,\omega+\omega_1)\hat{U}^*(z,\omega+\omega_1+\omega_2)\right] d\omega_1 d\omega_2 = 0\,. \quad (94)$$

On computing the inverse Fourier transform of (93) and (94), the GT equations in the temporal domain are

$$i\frac{\partial U}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 U}{\partial t^2} + g(z)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(t_1,t_2)U(z,t_1+t_2)\left[U(z,t+t_1)U^*(z,t+t_1+t_2) \right.$$
$$\left. +\alpha V(z,t+t_1)V^*(z,t+t_1+t_2)\right] dt_1 dt_2 = 0\,, \quad (95)$$

$$
\begin{aligned}
& i\frac{\partial V}{\partial z}+\frac{\delta_a}{2}\frac{\partial^2 V}{\partial t^2} \\
& \quad +g(z)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R(t_1,t_2)V(z,t_1+t_2)\left[V(z,t+t_1)V^*(z,t+t_1+t_2)\right. \\
& \quad \left.+\alpha U(z,t+t_1)U^*(z,t+t_1+t_2)\right] dt_1 dt_2 = 0\,. \qquad (96)
\end{aligned}
$$

4.1. Quasi-Linear Pulses

In this section, we shall split the study in two subsections that deals with the lossless and the lossy cases, just as in the case of polarization preserving fibers.

4.1.1. Lossless System

Assuming that $\hat{U}(z,\omega)$ and $\hat{V}(z,\omega)$ depend weakly on s, we have the following asymptotic expansions of the nonlinear terms from the GT equation [6, 10]

$$
\begin{aligned}
& i\frac{\partial U}{\partial z}+\frac{\delta_a}{2}\frac{\partial^2 U}{\partial t^2} \\
& \quad +\frac{1}{2\pi s}\left[(\log s-\gamma)J_1^{(1)}(z,t)-J_2^{(1)}(z,t)\right] \\
& \quad +\frac{\alpha}{2\pi s}\left[(\log s-\gamma)K_1^{(1)}(z,t)-K_2^{(1)}(z,t)\right]=0\,, \qquad (97)
\end{aligned}
$$

$$
\begin{aligned}
& i\frac{\partial V}{\partial z}+\frac{\delta_a}{2}\frac{\partial^2 V}{\partial t^2} \\
& \quad +\frac{1}{2\pi s}\left[(\log s-\gamma)J_1^{(2)}(z,t)-J_2^{(2)}(z,t)\right] \\
& \quad +\frac{\alpha}{2\pi s}\left[(\log s-\gamma)K_1^{(2)}(z,t)-K_2^{(2)}(z,t)\right]=0\,, \qquad (98)
\end{aligned}
$$

where

$$
\begin{aligned}
J_1^{(1)}(z,t) = & \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U\left(z,t+t_1\right)U\left(z,t+t_2\right) \\
& \times U^*\left(z,t+t_1+t_2\right) dt_1 dt_2\,, \qquad (99)
\end{aligned}
$$

$$
\begin{aligned}
J_2^{(1)}(z,t) = & \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \log|t_1 t_2|\, U(z,t+t_1)U(z,t+t_2) \\
& \times U^*(z,t+t_1+t_2)dt_1 dt_2\,, \qquad (100)
\end{aligned}
$$

$$K_1^{(1)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(z,t+t_2)U(z,t+t_1) \times V^*(z,t+t_1+t_2)dt_1dt_2\,, \quad (101)$$

$$K_2^{(1)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \log|t_1t_2| V(z,t+t_2)U(z,t+t_1) \times V^*(z,t+t_1+t_2)dt_1dt_2\,, \quad (102)$$

$$J_1^{(2)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(z,t+t_1)V(z,t+t_2) \times V^*(z,t+t_1+t_2)dt_1dt_2\,, \quad (103)$$

$$J_2^{(2)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \log|t_1t_2| V(z,t+t_1)V(z,t+t_2) \times V^*(z,t+t_1+t_2)dt_1dt_2\,, \quad (104)$$

$$K_1^{(2)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} V(z,t+t_2)U(z,t+t_1) \times U^*(z,t+t_1+t_2)dt_1dt_2\,, \quad (105)$$

$$K_2^{(2)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \log|t_1t_2| V(z,t+t_2)U(z,t+t_1) \times U^*(z,t+t_1+t_2)dt_1dt_2\,. \quad (106)$$

In the Fourier domain, (97) and (98) respectively transform to [10]

$$i\frac{\partial \hat{U}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{U} + \frac{1}{2\pi s}\left[(\log s - \gamma)\hat{J}_1^{(1)}(z,\omega) - \hat{J}_2^{(1)}(z,\omega)\right] + \frac{\alpha}{2\pi s}\left[(\log s - \gamma)\hat{K}_1^{(1)}(z,\omega) - \hat{K}_2^{(1)}(z,\omega)\right] = 0\,, \quad (107)$$

$$i\frac{\partial \hat{V}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{V} + \frac{1}{2\pi s}\left[(\log s - \gamma)\hat{J}_1^{(2)}(z,\omega) - \hat{J}_2^{(2)}(z,\omega)\right] + \frac{\alpha}{2\pi s}\left[(\log s - \gamma)\hat{K}_1^{(2)}(z,\omega) - \hat{K}_2^{(2)}(z,\omega)\right] = 0\,, \quad (108)$$

where we have

$$\hat{J}_1^{(1)}(z,\omega) = \left|\hat{U}(z,\omega)\right|^2 \hat{U}(z,\omega)\,, \tag{109}$$

$$\hat{J}_2^{(1)}(z,\omega) = \frac{1}{\pi}\hat{U}(z,\omega)\int_{-\infty}^{\infty}\left|\hat{U}(z,\omega')\right|^2 h\left(\omega'-\omega\right)d\omega'\,, \tag{110}$$

$$\hat{K}_1^{(1)}(z,\omega) = \left|\hat{V}(z,\omega)\right|^2 \hat{U}(z,\omega)\,, \tag{111}$$

$$\hat{K}_2^{(1)}(z,\omega) = \frac{1}{\pi}\hat{U}(z,\omega)\int_{-\infty}^{\infty}\left|\hat{V}(z,\omega')\right|^2 h\left(\omega'-\omega\right)d\omega'\,, \tag{112}$$

$$\hat{J}_1^{(2)}(z,\omega) = \left|\hat{V}(z,\omega)\right|^2 \hat{V}(z,\omega)\,, \tag{113}$$

$$\hat{J}_2^{(2)}(z,\omega) = \frac{1}{\pi}\hat{V}(z,\omega)\int_{-\infty}^{\infty}\left|\hat{V}(z,\omega')\right|^2 h\left(\omega'-\omega\right)d\omega'\,, \tag{114}$$

$$\hat{K}_1^{(2)}(z,\omega) = \left|\hat{U}(z,\omega)\right|^2 \hat{V}(z,\omega)\,, \tag{115}$$

$$\hat{K}_2^{(2)}(z,\omega) = \frac{1}{\pi}\hat{V}(z,\omega)\int_{-\infty}^{\infty}\left|\hat{U}(z,\omega')\right|^2 h\left(\omega'-\omega\right)d\omega'\,. \tag{116}$$

One can see from (107) and (108) that the total spectral intensity given by $|\hat{U}(z,\omega)|^2 + |\hat{V}(z,\omega)|^2$ is preserved during the pulse propagation, namely

$$\frac{\partial}{\partial z}\left(\left|\hat{U}\right|^2 + \left|\hat{V}\right|^2\right) = 0\,. \tag{117}$$

Also, the solutions of (107) and (108) are

$$\hat{U}(z,\omega) = \hat{U}(z,0)\exp\left\{-i\frac{\delta_a\omega^2}{2}z + i\psi\left[\left|\hat{U}(0,\omega)\right|^2\right]z \right. \\ \left. + i\alpha\psi\left[\left|\hat{V}(0,\omega)\right|^2\right]z\right\}\,, \tag{118}$$

and

$$\hat{V}(z,\omega)=\hat{V}(z,0)\exp\left\{-i\frac{\delta_a\omega^2}{2}z+i\psi\left[\left|\hat{V}(0,\omega)\right|^2\right]z\right.$$
$$\left.+i\alpha\psi\left[\left|\hat{U}(0,\omega)\right|^2\right]z\right\}, \quad (119)$$

respectively, where

$$\psi\left[\left|\hat{U}(z,\omega)\right|^2\right]=\frac{1}{2\pi s}\left[(\log s-\gamma)\left|\hat{U}(z,\omega)\right|^2\right.$$
$$\left.-\frac{1}{\pi}\int_{-\infty}^{\infty}\left|\hat{U}(z,\omega)\right|^2 h(\omega'-\omega)d\omega'\right], \quad (120)$$

with a similar expression for $\psi\left[\left|\hat{V}(z,\omega)\right|^2\right]$.

4.1.2. Lossy System

We shall again split the study into the following four cases, for $\theta=1/2$, depending on the position of the amplifiers.

Case-I. ($\zeta_a=0$)

This locates the amplifier in the middle of the anomalous GVD segments. In this case we have the kernels $r(x;s)$ and $R(t_1,t_2;s)$ are as in (43) and (44) respectively. The GT equations, given by (58) and (59), for large s, reduces to [10]

$$i\frac{\partial U}{\partial z}+\frac{\delta_a}{2}\frac{\partial^2 U}{\partial t^2}$$
$$+\frac{1}{s}\left[(P_0\log s-P_1)J_1^{(1)}(z,t)-P_0J_2^{(1)}(z,t)-iP_2J_3^{(1)}(z,t)\right]$$
$$+\frac{\alpha}{s}\left[(P_0\log s-P_1)K_1^{(1)}(z,t)-P_0K_2^{(1)}(z,t)-iP_2K_3^{(1)}(z,t)\right]=0\,, \quad (121)$$

$$i\frac{\partial V}{\partial z}+\frac{\delta_a}{2}\frac{\partial^2 V}{\partial t^2}$$
$$+\frac{1}{s}\left[(P_0\log s-P_1)J_1^{(2)}(z,t)-P_0J_2^{(2)}(z,t)-iP_2J_3^{(2)}(z,t)\right]$$
$$+\frac{\alpha}{s}\left[(P_0\log s-P_1)K_1^{(2)}(z,t)-P_0K_2^{(2)}(z,t)-iP_2K_3^{(2)}(z,t)\right]=0\,, \quad (122)$$

where we have

$$J_3^{(1)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn}(t_1 t_2) U(z,t+t_1) U(z,t+t_2) \times U^*(z,t+t_1+t_2) dt_1 dt_2 \,, \quad (123)$$

$$J_3^{(2)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn}(t_1 t_2) V(z,t+t_1) V(z,t+t_2) \times V^*(z,t+t_1+t_2) dt_1 dt_2 \,, \quad (124)$$

$$K_3^{(1)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn}(t_1 t_2) V(z,t+t_1) U(z,t+t_2) \times V^*(z,t+t_1+t_2) dt_1 dt_2 \,, \quad (125)$$

$$K_3^{(2)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn}(t_1 t_2) U(z,t+t_1) V(z,t+t_2) \times U^*(z,t+t_1+t_2) dt_1 dt_2 \,. \quad (126)$$

In the Fourier domain, (121) and (122) respectively reduce to [10]

$$i\frac{\partial \hat{U}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{U} + \frac{1}{s}\left[(P_0 \log s - P_1)\hat{J}_1^{(1)}(z,\omega) - P_0 \hat{J}_2^{(1)}(z,\omega) - iP_2 \hat{J}_3^{(1)}(z,\omega)\right] + \frac{\alpha}{s}\left[(P_0 \log s - P_1)\hat{K}_1^{(1)}(z,\omega) - P_0 \hat{K}_2^{(1)}(z,\omega) - iP_2 \hat{K}_3^{(1)}(z,\omega)\right] = 0 \,, \quad (127)$$

$$i\frac{\partial \hat{V}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{V} + \frac{1}{s}\left[(P_0 \log s - P_1)\hat{J}_1^{(2)}(z,\omega) - P_0 \hat{J}_2^{(2)}(z,\omega) - iP_2 \hat{J}_3^{(2)}(z,\omega)\right] + \frac{\alpha}{s}\left[(P_0 \log s - P_1)\hat{K}_1^{(2)}(z,\omega) - P_0 \hat{K}_2^{(2)}(z,\omega) - iP_2 \hat{K}_3^{(2)}(z,\omega)\right] = 0 \,, \quad (128)$$

where

$$\hat{J}_3^{(1)}(z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_1\omega_2}\hat{U}(z,\omega+\omega_1)\hat{U}(z,\omega+\omega_2) \times \hat{U}^*(z,\omega+\omega_1+\omega_2)d\omega_1 d\omega_2\,, \quad (129)$$

$$\hat{J}_3^{(2)}(z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_1\omega_2}\hat{V}(z,\omega+\omega_1)\hat{V}(z,\omega+\omega_2) \times \hat{V}^*(z,\omega+\omega_1+\omega_2)d\omega_1 d\omega_2\,, \quad (130)$$

$$\hat{K}_3^{(1)}(z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_1\omega_2}\hat{V}(z,\omega+\omega_1)\hat{U}(z,\omega+\omega_2) \times \hat{V}^*(z,\omega+\omega_1+\omega_2)d\omega_1 d\omega_2\,, \quad (131)$$

$$\hat{K}_3^{(2)}(z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_1\omega_2}\hat{U}(z,\omega+\omega_1)\hat{V}(z,\omega+\omega_2) \times \hat{U}^*(z,\omega+\omega_1+\omega_2)d\omega_1 d\omega_2\,. \quad (132)$$

In (129)-(132), the integrals represent the Cauchy principal value. Now, from (127) and (128) we observe that

$$\frac{\partial}{\partial z}\left(\left|\hat{U}\right|^2 + \left|\hat{V}\right|^2\right) = \frac{P_2}{s}\left\{H^{(1)}(z,\omega) + \alpha H^{(2)}(z,\omega)\right\}\,, \quad (133)$$

with

$$H^{(1)}(z,\omega) = \hat{J}_3^{(1)}(z,\omega)\hat{U}^*(z,\omega) + \hat{J}_3^{(1)*}(z,\omega)\hat{U}(z,\omega) \quad (134)$$

and

$$H^{(2)}(z,\omega) = \hat{K}_3^{(1)}(z,\omega)\hat{V}^*(z,\omega) + \hat{K}_3^{(1)*}(z,\omega)\hat{V}(z,\omega)\,. \quad (135)$$

For large s, and moderate z, one can write

$$\left|\hat{U}(z,\omega)\right|^2 + \left|\hat{V}(z,\omega)\right|^2 \approx \left|\hat{U}(0,\omega)\right|^2 + \left|\hat{V}(0,\omega)\right|^2 + \frac{P_2 z}{s}\left\{H^{(1)}(0,\omega) + \alpha H^{(1)}(0,\omega)\right\}\,, \quad (136)$$

and, thus, the total spectral intensity does not remain constant in this case.

Case-II. ($\zeta_a = -1/2$)

In this case, the amplifier is positioned in the middle of the normal GVD segment. The kernels of the GT equations, in this case, are (66) and (67). The sum of the intensities satisfy

$$\frac{\partial}{\partial z}\left(\left|\hat{U}\right|^2 + \left|\hat{V}\right|^2\right) = -\frac{P_2}{s}\left\{H^{(1)}(z,\omega) + \alpha H^{(2)}(z,\omega)\right\} . \tag{137}$$

Finally, for $s \gg 1$ and moderate z we get

$$\left|\hat{U}(z,\omega)\right|^2 + \left|\hat{V}(z,\omega)\right|^2 \approx \left|\hat{U}(0,\omega)\right|^2 + \left|\hat{V}(0,\omega)\right|^2 - \frac{P_2 z}{s}\left\{H^{(1)}(0,\omega) + \alpha H^{(1)}(0,\omega)\right\} . \tag{138}$$

So, the total spectral intensity does not stay conserved here, too.

Case-III. ($\zeta_a = -1/4$)

Here, the amplifier is placed at the boundary between the anomalous and normal GVD segment. In this case, the kernels are given by (71) and (72) respectively. We get the GT equations, in this case, reduce to

$$i\frac{\partial U}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 U}{\partial t^2} + \frac{1}{s}\left[(Q_0 \log s - Q_1)J_1^{(1)}(z,t) - Q_0 J_2^{(1)}(z,t)\right] + \frac{\alpha}{s}\left[(Q_0 \log s - Q_1)K_1^{(1)}(z,t) - Q_0 K_2^{(1)}(z,t)\right] = 0\,, \tag{139}$$

$$i\frac{\partial V}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 V}{\partial t^2} + \frac{1}{s}\left[(Q_0 \log s - Q_1)J_1^{(2)}(z,t) - Q_0 J_2^{(2)}(z,t)\right] + \frac{\alpha}{s}\left[(Q_0 \log s - Q_1)K_1^{(2)}(z,t) - Q_0 K_2^{(2)}(z,t)\right] = 0\,, \tag{140}$$

while, in the Fourier domain, we have

$$i\frac{\partial \hat{U}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{U} + \frac{1}{s}\left[(Q_0 \log s - Q_1)\hat{J}_1^{(1)}(z,\omega) - Q_0 \hat{J}_2^{(1)}(z,\omega)\right] + \frac{\alpha}{s}\left[(Q_0 \log s - Q_1)\hat{K}_1^{(1)}(z,\omega) - Q_0 \hat{K}_2^{(1)}(z,\omega)\right] = 0\,, \tag{141}$$

$$
\begin{aligned}
&i\frac{\partial \hat{V}}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{V} \\
&\quad + \frac{1}{s}\left[(Q_0 \log s - Q_1)\hat{J}_1^{(2)}(z,\omega) - Q_0 \hat{J}_2^{(2)}(z,\omega)\right] \\
&\quad + \frac{\alpha}{s}\left[(Q_0 \log s - Q_1)\hat{K}_1^{(2)}(z,\omega) - Q_0 \hat{K}_2^{(2)}(z,\omega)\right] = 0\,. \qquad (142)
\end{aligned}
$$

In this case, we note that the total spectral intensity stays constant as we observe from (141) and (142) that

$$
\frac{\partial}{\partial z}\left(\left|\hat{U}\right|^2 + \left|\hat{V}\right|^2\right) = 0\,, \qquad (143)
$$

and, also from (141) and (142), we recover the solutions (118) and (119) respectively where we, now, have

$$
\begin{aligned}
&\psi\left[\left|\hat{U}(z,\omega)\right|^2\right] \\
&= \frac{1}{s}\left[(Q_0 \log s - Q_1)\left|\hat{J}_1^{(1)}(z,\omega)\right|^2 - \frac{Q_0}{\pi}\int_{-\infty}^{\infty}\left|\hat{J}_2^{(1)}(z,\omega')\right|^2 h\left(\omega-\omega'\right) d\omega'\right] \\
&+ \frac{\alpha}{s}\left[(Q_0 \log s - Q_1)\left|\hat{K}_1^{(1)}(z,\omega)\right|^2 - \frac{Q_0}{\pi}\int_{-\infty}^{\infty}\left|\hat{K}_2^{(1)}(z,\omega')\right|^2 h\left(\omega-\omega'\right) d\omega'\right] \qquad (144)
\end{aligned}
$$

and

$$
\begin{aligned}
&\psi\left[\left|\hat{V}(z,\omega)\right|^2\right] \\
&= \frac{1}{s}\left[(Q_0 \log s - Q_1)\left|\hat{J}_1^{(2)}(z,\omega)\right|^2 - \frac{Q_0}{\pi}\int_{-\infty}^{\infty}\left|\hat{J}_2^{(2)}(z,\omega')\right|^2 h\left(\omega-\omega'\right) d\omega'\right]
\end{aligned}
$$

$$
+\frac{\alpha}{s}\left[(Q_0 \log s - Q_1)\left|\hat{K}_1^{(2)}(z,\omega)\right|^2 - \frac{Q_0}{\pi}\int_{-\infty}^{\infty}\left|\hat{K}_2^{(2)}(z,\omega')\right|^2 h\left(\omega-\omega'\right)d\omega'\right]. \quad (145)
$$

Case-IV. ($\zeta_a = 1/4$)

Here, the amplifier is placed at the boundary between the normal and anomalous GVD segment. The kernels, here, are (84) and (85). The only difference in this case from that of the previous one is that here we have the imaginary part of the kernels with opposite sign. But, we have shown in the previous subsection that the imaginary part does not make any contribution to the dynamics of quasi-linear pulses, the sum of the spectral intensities is again conserved in this case during the pulse propagation.

5. Multiple Channels

The succesful design of low-loss dispersion-shifted and dispersion-flattened optical fibers [15] with low dispersion over relatively large wavelength range can be used to reduce or completely eliminate the group velocity mismatch for the multi-channel WDM systems resulting in the desirable simultaneous arrival of time aligned bit pulses, thus creating a new class of bit-parallel wavelength links that is used in high speed single fiber computer buses. In spite of the intrinsically small value of the nonlinearity-induced change in the refractivein-dex of fused silica, nonlinear effects in optical fibers cannot be ignored even at relatively low powers. In particular, in WDM systems with simultaneous transmission of pulses of different wavelengths, the cross-phase modulation (XPM) effects needs to be taken into account. Although the XPM will not cause the energy to be exchanged among the different wavelengths, it will lead to the interaction of pulses and thus the pulse positions and shapes gets altered significantly. The multi-channel WDM transmission of co-propagating wave envelopes in a nonlinear optical fiber, including the XPM effect, can be modeled [9] by the following N-coupled NLSE in the dimensionless form:

$$
iu_{l,z} + \frac{D(z)}{2}u_{l,tt} + g(z)\left\{|u_l|^2 + \sum_{m\neq l}^{N}\alpha_{lm}|u_m|^2\right\}u_l = 0\,, \quad (146)
$$

where we have $1 \leq l \leq N$. Equation (146) is the model for bit-parallel WDM soliton transmission. Here α_{lm} are known as the XPM coefficients. Equation

(146) is, in general, not integrable. However, it can be solved analytically for certain very specific cases [10, 16]. It does not have infinitely many conservation laws either. In fact it has at least two integrals of motion and they are the energy (E) and the linear momentum (M) that are respectively given by:

$$E = \sum_{l=1}^{N} \int_{-\infty}^{\infty} |q_l|^2 \, dt \,, \tag{147}$$

$$M = \frac{i}{2} D(z) \sum_{l=1}^{N} \int_{-\infty}^{\infty} \left(u_l^* u_{l,t} - u_l u_{l,t}^* \right) \, dt \,. \tag{148}$$

The Hamiltonian (H), given by

$$H = \frac{1}{2} \sum_{l=1}^{N} \int_{-\infty}^{\infty} \left\{ D(z) \left| u_{l,t} \right|^2 - g(z) \sum_{m=1}^{N} \alpha_{lm} \left| u_l \right|^2 \left| u_m \right|^2 \right\} dt \,, \tag{149}$$

is however not a conserved quantity unless the matrix of XPM coefficients $\Lambda = \left(\alpha_{ij}\right)_{N \times N}$ is a symmetric matrix namely $\alpha_{ij} = \alpha_{ji}$ for $1 \leq i, j \leq N$.

The corresponding GT equation for the case of multiple channels in the Fourier domain is given by [13]

$$\begin{aligned} & i \frac{\partial \hat{U}_l}{\partial z} - \frac{\delta_a}{2} \omega^2 \hat{U}_l \\ & \quad + \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} r(\omega_1 \omega_2) \hat{U}_l(z, \omega_1 + \omega_2) \Big[\hat{U}_l(z, \omega + \omega_1) \hat{U}_l^{\,*}(z, \omega + \omega_1 + \omega_2) \\ & \quad + \sum_{l \neq m}^{N} \alpha_{lm} \hat{U}_m(z, \omega + \omega_1) \hat{U}_m^{\,*}(z, \omega + \omega_1 + \omega_2) \Big] \, d\omega_1 d\omega_2 = 0 \,, \end{aligned} \tag{150}$$

while in in the time domain, the GT equation is

$$\begin{aligned} & i \frac{\partial U_l}{\partial z} + \frac{\delta_a}{2} \frac{\partial^2 U_l}{\partial t^2} \\ & \quad + g(z) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(t_1, t_2) U_l(z, t_1 + t_2) \left[U_l(z, t + t_1) U_l^*(z, t + t_1 + t_2) \right. \\ & \quad \left. + \sum_{l \neq m}^{N} \alpha_{lm} U_m(z, t + t_1) U_m^*(z, t + t_1 + t_2) \right] \, dt_1 dt_2 = 0 \,, \end{aligned} \tag{151}$$

where we have $1 \leq l \leq N$.

5.1. Quasi-Linear Pulses

We shall split the study of quasi-linear pulses into two subsections namely the lossless and the lossy cases.

5.1.1. Lossless System

In a lossless system, namely $g(z) = 1$, we have the kernels given by (23) and (24). We have the following asymptotic expansions of the nonlinear term from the GT equation [6, 10]

$$\begin{aligned} &i\frac{\partial U_l}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 U_l}{\partial t^2} \\ &\quad + \frac{1}{2\pi s}\left[(\log s - \gamma)J_1^{(l)}(z,t) - J_2^{(l)}(z,t)\right] \\ &\quad + \frac{1}{2\pi s}\sum_{m\neq l}\alpha_{lm}\left[(\log s - \gamma)K_1^{(m)}(z,t) - K_2^{(m)}(z,t)\right] = 0\,, \end{aligned} \tag{152}$$

where we have

$$\begin{aligned} J_1^{(l)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} & U_l(z,t+t_1)U_l(z,t+t_2) \\ & \times U_l^*(z,t+t_1+t_2)dt_1dt_2\,, \end{aligned} \tag{153}$$

$$\begin{aligned} J_2^{(l)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} & \log|t_1t_2|\, U_l(z,t+t_1)U_l(z,t+t_2) \\ & \times U_l^*(z,t+t_1+t_2)dt_1dt_2\,, \end{aligned} \tag{154}$$

$$\begin{aligned} K_1^{(l)}(z,t) = \sum_{m\neq l}\alpha_{lm}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} & U_l(z,t+t_2)U_m(z,t+t_1) \\ & \times U_m^*(z,t+t_1+t_2)dt_1dt_2\,, \end{aligned} \tag{155}$$

$$\begin{aligned} K_2^{(l)}(z,t) = \sum_{m\neq l}\alpha_{lm}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} & \log|t_1t_2| U_l(z,t+t_2)U_m(z,t+t_1) \\ & \times U_m^*(z,t+t_1+t_2)dt_1dt_2\,. \end{aligned} \tag{156}$$

In the Fourier domain, we have

$$
\begin{aligned}
& i\frac{\partial \hat{U}_l}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{U}_l \\
& \quad + \frac{1}{2\pi s}\left[(\log s - \gamma)\hat{J}_1^{(l)}(z,\omega) - \hat{J}_2^{(l)}(z,\omega)\right] \\
& \quad + \frac{\alpha}{2\pi s}\sum_{m\neq l}\alpha_{lm}\left[(\log s - \gamma)\hat{K}_1^{(m)}(z,\omega) - \hat{K}_2^{(m)}(z,\omega)\right] = 0\,, \qquad (157)
\end{aligned}
$$

where

$$
\hat{J}_1^{(l)}(z,\omega) = |\hat{U}_l(z,\omega)|^2\hat{U}_l(z,\omega)\,, \qquad (158)
$$

$$
\hat{J}_2^{(l)}(z,\omega) = \frac{1}{\pi}\hat{U}_l(z,\omega)\int_{-\infty}^{\infty}|\hat{U}_l(z,\omega')|^2 h(\omega'-\omega)d\omega'\,, \qquad (159)
$$

$$
\hat{K}_1^{(l)}(z,\omega) = \hat{U}_l(z,\omega)\sum_{m\neq l}\alpha_{lm}|\hat{U}_m(z,\omega)|^2\,, \qquad (160)
$$

$$
\hat{K}_2^{(l)}(z,\omega) = \frac{1}{\pi}\hat{U}_l(z,\omega)\sum_{m\neq l}\alpha_{lm}\int_{-\infty}^{\infty}|\hat{U}_m(z,\omega')|^2 h(\omega'-\omega)d\omega'\,. \qquad (161)
$$

Now, one can see from (157) that the total spectral intensity given by $\sum_{l=1}^{N}|\hat{U}_l(z,\omega)|^2$ is preserved during the pulse propagation. So, we have

$$
\sum_{l=1}^{N}\frac{\partial}{\partial z}\left(|\hat{U}_l|^2\right) = 0\,. \qquad (162)
$$

Thus, the solution of (157) is

$$
\begin{aligned}
\hat{U}_l(z,\omega) = \hat{U}_l(z,0)\exp\Bigg\{ & -i\frac{\delta_a\omega^2}{2}z + i\psi\left[|\hat{U}_l(0,\omega)|^2\right]z \\
& + i\sum_{m\neq l}\alpha_{lm}\psi\left[|\hat{U}_m(0,\omega)|^2\right]z\Bigg\}\,, \qquad (163)
\end{aligned}
$$

where

$$
\begin{aligned}
\psi\left[|\hat{U}_l(z,\omega)|^2\right] = \frac{1}{2\pi s}\Bigg[& (\log s - \gamma)|\hat{U}_l(z,\omega)|^2 \\
& - \frac{1}{\pi}\int_{-\infty}^{\infty}|\hat{U}_l(z,\omega)|^2 h(\omega'-\omega)d\omega'\Bigg]\,. \qquad (164)
\end{aligned}
$$

5.1.2. Lossy System

We shall now split the study into the following four cases depending on the position of the amplifiers.

Case-I. ($\zeta_a = 0$)

This locates the amplifier in the middle of the anomalous GVD segments. In this case we have the kernels $r(x;s)$ and $R(t_1,t_2;s)$ as in (43) and (44). The GT equations for large s reduce to

$$\begin{aligned} i\frac{\partial U_l}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 U_l}{\partial t^2} & \\ + \frac{1}{s}\Big[(P_0 \log s - P_1)J_1^{(l)}(z,t) - P_0 J_2^{(l)}(z,t) - iP_2 J_3^{(l)}(z,t)\Big] & \\ + \frac{1}{s}\sum_{m\neq l}\alpha_{lm}\Big[(P_0\log s - P_1)K_1^{(m)}(z,t) - P_0 K_2^{(m)}(z,t) & \\ -iP_2 K_3^{(m)}(z,t)\Big] = 0\,, & \quad (165) \end{aligned}$$

where we have

$$\begin{aligned} J_3^{(l)}(z,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn}(t_1 t_2) U_l(z,t+t_1)U_l(z,t+t_2) & \\ \times U_l^*(z,t+t_1+t_2)dt_1 dt_2\,, & \quad (166) \end{aligned}$$

$$\begin{aligned} K_3^{(m)}(z,t) = \sum_{m\neq l}\alpha_{lm}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \operatorname{sgn}(t_1 t_2) U_l(z,t+t_2)U_m(z,t+t_1) & \\ \times U_m^*(z,t+t_1+t_2)dt_1 dt_2\,. & \quad (167) \end{aligned}$$

In the Fourier domain, we have

$$\begin{aligned} i\frac{\partial \hat{U}_l}{\partial z} - \frac{\delta_a\omega^2}{2}\hat{U}_l & \\ + \frac{1}{s}\Big[(P_0 \log s - P_1)\hat{J}_1^{(l)}(z,\omega) - P_0 \hat{J}_2^{(l)}(z,\omega) - iP_2 \hat{J}_3^{(l)}(z,\omega)\Big] & \\ + \frac{1}{s}\sum_{m\neq l}\alpha_{lm}\Big[(P_0\log s - P_1)\hat{K}_1^{(m)}(z,\omega) - P_0 \hat{K}_2^{(m)}(z,\omega) & \\ -iP_2 \hat{K}_3^{(m)}(z,\omega)\Big] = 0\,, & \quad (168) \end{aligned}$$

where

$$\hat{J}_3^{(l)}(z,\omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_1\omega_2}\hat{U}_l(z,\omega+\omega_1)\hat{U}_l(z,\omega+\omega_2) \times \hat{U}_l^*(z,\omega+\omega_1+\omega_2)d\omega_1 d\omega_2\,, \quad (169)$$

$$\hat{K}_3^{(m)}(z,\omega) = \sum_{m\neq l}\alpha_{lm}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\omega_1\omega_2}\hat{U}_l(z,\omega+\omega_1)\hat{U}_m(z,\omega+\omega_2) \times \hat{U}_m^*(z,\omega+\omega_1+\omega_2)d\omega_1 d\omega_2\,. \quad (170)$$

We note in (169) and (170) that the integral represents the Cauchy principal value. Now, from (168) we have

$$\sum_{l=1}^{N}\frac{\partial}{\partial z}\left(|\hat{U}_l|^2\right) = \frac{P_2}{s}\left\{H^{(l)}(z,\omega) + \sum_{m\neq l}\alpha_{lm}H^{(m)}(z,\omega)\right\}\,, \quad (171)$$

with

$$H^{(l)}(z,\omega) = \hat{J}_3^{(l)}(z,\omega)\hat{U}_l^*(z,\omega) + \hat{J}_3^{(l)*}(z,\omega)\hat{U}_l(z,\omega) \quad (172)$$

and

$$H^{(m)}(z,\omega) = \hat{K}_3^{(m)}(z,\omega)\hat{U}_m^*(z,\omega) + \hat{K}_3^{(m)*}(z,\omega)\hat{U}_m(z,\omega)\,. \quad (173)$$

For large s, one can write for moderate z

$$\sum_{l=1}^{N}|\hat{U}_l(z,\omega)|^2 \approx \sum_{l=1}^{N}|\hat{U}(0,\omega)|^2 + \frac{P_2 z}{s}\left\{H^{(l)}(0,\omega) + \sum_{m\neq l}\alpha_{lm}H^{(1)}(0,\omega)\right\} \quad (174)$$

and, thus, the total spectral intensity does not remain constant in this case.

Case-II. ($\zeta_a = -1/2$)

In this case, the amplifier is positioned in the middle of the normal GVD segment. The kernels of the GT equations in this case are given by (66) and (67). The, the sum of the intensities satisfy

$$\sum_{l=1}^{N}\frac{\partial}{\partial z}\left(|\hat{U}_l|^2\right) = -\frac{P_2}{s}\left\{H^{(l)}(z,\omega) + \sum_{m\neq l}\alpha_{lm}H^{(m)}(z,\omega)\right\}\,. \quad (175)$$

Finally, for $s \gg 1$ and moderate z we get

$$\sum_{l=1}^{N} |\hat{U}_l(z,\omega)|^2 \approx \sum_{l=1}^{N} |\hat{U}_l(0,\omega)|^2 - \frac{P_2 z}{s} \left\{ H^{(l)}(0,\omega) + \sum_{m\neq l} \alpha_{lm} H^{(1)}(0,\omega) \right\} . \quad (176)$$

So, the total spectral intensity does not stay conserved here, too.

Case-III. ($\zeta_a = -1/4$)

Here, the amplifier is placed at the boundary between the anomalous and normal GVD segment. In this case, the kernels are given by (71) and (72). We get the GT equation, in this case, reduce to

$$i\frac{\partial U_l}{\partial z} + \frac{\delta_a}{2}\frac{\partial^2 U_l}{\partial t^2} + \frac{1}{s}\left[(Q_0 \log s - Q_1) J_1^{(l)}(z,t) - Q_0 J_2^{(l)}(z,t)\right] + \frac{1}{s}\sum_{m\neq l} \alpha_{lm} \left[(Q_0 \log s - Q_1) K_1^{(m)}(z,t) - Q_0 K_2^{(m)}(z,t)\right] = 0 , \quad (177)$$

so that, in the Fourier domain, we have

$$i\frac{\partial \hat{U}_l}{\partial z} - \frac{\delta_a \omega^2}{2}\hat{U}_l + \frac{1}{s}\left[(Q_0 \log s - Q_1) \hat{J}_1^{(l)}(z,\omega) - Q_0 \hat{J}_2^{(l)}(z,\omega)\right] + \frac{1}{s}\sum_{m\neq l} \alpha_{lm} \left[(Q_0 \log s - Q_1) \hat{K}_1^{(m)}(z,\omega) - Q_0 \hat{K}_2^{(m)}(z,\omega)\right] = 0 . \quad (178)$$

In this case, we note that the total spectral intensity from all the channels stays constant as we have

$$\sum_{l=1}^{N} \frac{\partial}{\partial z}\left(|\hat{U}_l|^2\right) = 0 , \quad (179)$$

so that, from (177), we obtain

$$\psi\left[|\hat{U}_l(z,\omega)|^2\right] = \frac{1}{s}\left[(Q_0 \log s - Q_1)|\hat{J}_1^{(l)}(z,\omega)|^2 - \frac{Q_0}{\pi}\int_{-\infty}^{\infty} |\hat{J}_2^{(l)}(z,\omega')|^2 h(\omega-\omega') d\omega'\right] + \frac{\alpha}{s}\left[(Q_0 \log s - Q_1)|\hat{K}_1^{(1)}(z,\omega)|^2 - \frac{Q_0}{\pi}\int_{-\infty}^{\infty} |\hat{K}_2^{(1)}(z,\omega')|^2 h(\omega-\omega') d\omega'\right] . \quad (180)$$

Case-IV. ($\zeta_a = 1/4$)

Here, the amplifier is placed at the boundary between the normal and anomalous GVD segment. The kernels are (84) and (85). The only differnce in this case from that of the previous one is that here we have the imaginary part of the kernels with opposite sign. But we have shown in the previous subsection that the imaginary part does not make any contribution to the dynamics of quasi-linear pulses, the sum of the spectral intensities is again conserved in this case during the pulse propagation.

6. Conclusions

In this paper, we have studied the dynamics of quasilinear pulses propagating through optical fibers with strong dispersion-management, by means of the asymptotic analysis of the GT equation. The GT equations for such fibers, that describe the average dynamics, are derived and analysed. We have analysed both the lossless, as well as the lossy case. For the lossless case and for the lossy cases, when the amplifier is placed at the boundary between the anomalous and normal GVD segments, the total spectral intensity stays a constant. Also, the nonlinearity is responsible for a phase shift in the frequency domain, resulting in some pulse broadening in the temporal domain.

In future, one may extend this analysis to study the effects of the perturbation terms on birefringent optical fibers with strong dispersion-management. Typically, one can study the effects of higher order dispersions, Raman scattering, filters, nonlinear damping and two photon absorption just to name a few.

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