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THE APPLICATION OF LOG BARRIER ALGORITHM FOR MULTIUSER DETECTION IN CDMA

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Abstract: In this paper, a detection strategy based on a log barrier algorithm of the CDMA maximum likelihood (ML) multiuser detection problem is presented. The log barrier algorithm based on semidefinite programming is proposed to solve multiuser detection problem, which is global convergence. Simulations demonstrate that the log barrier algorithm have the similar BER performances of the multiuser detection problem to the interior point methods based on semidefinite programming design. But the average CPU time of this approach is significantly reduced.

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1. Introduction

In code division multiple access (CDMA) system, users are assigned unique signature waveforms that are used to modulate their transmitted symbols. It is,

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however, not possible to ensure orthogonality among received signature waveforms in a mobile environment, and thus, multiple access interference arises. Multiuser detection Verdu [1] plays an important role in suppressing the performance degrading effect of multiuser interference. Consider a K users synchronous CDMA system with additive white Gaussian noise (AWGN) of variance

$$\delta^2 = N_0/2.$$

Each user transmits data using BPSK signaling and spreading. Without loss of generality, we assume that all K signature waveforms have unit energy. A minimal set of sufficient statistics of dimension K is obtained through matched filtering of the received spreading code of the desired user $y = Rd + z$, where y is the matched filter output vector, d is the spreading code, R is the correlation matrix, z is the zero mean Gaussian noise vector with autocorrelation matrix $\delta^2 R$. The negative log-likelihood function with an AWGN channel based on $p(y|d)$ may be described as $F(d) = d^T Rd - 2y^T d$. Then the constrained maximum likelihood (ML) problem may be described as Peng et al [2]

$$\overline{d} = \mathrm{argmin}_{d \in \{-1,1\}^K} d^T Rd - 2y^T d\,. \tag{1}$$

The problem can be solved by an exhaustive search, however, the exhaustive search is prohibitive for large number of users because of its exponentially increasing computational complexity. It is known that the polynomial time algorithms of the problem (1) exist if the autocorrelation matrix exhibits some special structure. However, in general case, it is NP-hard problem Verdu [1].

Because of intrinsic difficulty in solving the detection problem (1), there has been much interest in the development of suboptimal but computationally efficient ML detector. A tree search method Wei et al [3] and the coordinate descent algorithm Sharfer et al [4] have been proposed to perform an incomplete search for a solution to the problem (1) with limited complexity, however the performances of these methods strongly depends on the initialization. The algorithms to this problem may be found in Luo et al [5], Damen et al [6] and Wang et al [7]. However, the convergence analysis cannot received. In Peng et al [2] and Ma et al [8], a detection strategy based on a semidefinite relaxation of the CDMA maximum likelihood (ML) problem is investigated. The semidefinite relaxation may solved in interior point methods polynomial time end. However, the semidefinite relaxation encounters difficulty in practice as the problem size increases because the cost of solving semidefinite programming by interior point algorithm goes up quickly as the size of the problem increases.

In this paper, we propose a method to seek a sub-optimal solution to the ML detection problem by using the log barrier algorithm based on semidefinite programming. The paper is organized as follows. A detection strategy based on log barrier algorithm is given in the following section. The simulation results and conclusions are found in Section 3 and Section 4, respectively.

2. Algorithms for ML Multiuser Detection Problem

2.1. Notation and Terminology

Some defines are introduced firstly. $\Re, \Re^n$,and $\Re^{n\times n}$ denote the space of real numbers, real n-dimensional column vectors, and real $n \times n$ matrices, respectively. By S^n we denote the space of real $n \times n$ symmetric matrices, and we definite S^n_+ and S^n_{++} to be the subsets of S^n consisting of the positive semidefinite and positive definite matrices respectively. We write $A \succeq 0$ and $A \succ 0$ to indicate that $A \in S^n_+$ and $A \in S^n_{++}$, respectively. Let $\pounds^n$ denote the set of real lower triangular $n \times n$ matrices, and let $\pounds^n_+$ and $\pounds^n_{++}$ denote the subsets of $\pounds^n$ whose elements have nonnegative diagonal entries and positive diagonal entries, respectively. We let $\mathrm{tr}(A)$ denote the trace of a matrices $A \in \Re^{n\times n}$, we denote $A \bullet B = \mathrm{tr}(A^T B)$, and the Frobenius norm of $A \in \Re^{n\times n}$ is denoted to be $\| A \|_F = (A \bullet A)^{1/2}$.

We adopt the convention of denoting matrices by capital letters and matrices entries by lowercase letters with double subscripts. For example, a matrix $A \in \Re^{n\times n}$ has entries a_{ij} for $i, j = 1, ..., n$. In addition, we denote the rows of a matrix by lowercase letters with single subscripts. For example, $A \in \Re^{n\times n}$ has rows a_i for $i = 1, ..., n$. In this paper, we will often find it necessary to compute the dot product of two row vectors a_i and b_j which arise as rows of the matrices A and B. Instead of denoting this dot product as $a_i b_j^T$, we will denote it as $\langle a_i, b_j \rangle$.

2.2. Semidefinite Programming Relaxation for ML Multiuser Detection Problem

Peng et al [2] has proposed a semidefinite programming relaxation for ML detection, we introduced the relaxation as follows.

Let $n = K + 1, x = [\overline{d}^T, \overline{d}_n]^T, (\overline{d}_n = 1)$ and

$$C_1 = \begin{pmatrix} R & -y \\ -y^T & 0 \end{pmatrix}.$$

Since the cost function is symmetric, $d_n = 1$ need not to be maintained explicitly. Problem (1) may be formulated as

$$x^* = \text{argmin}_x x^T C_1 x \quad \text{s.t.} \quad x \in \{-1,1\}^n \tag{2}$$

This optimization problem is well-known to be NP-hard and which can be solved by semidefinite programming, as explained the following.

For $x \in \{-1,1\}^n$, $X = xx^T$ is a matrix which satisfies that it is a positive semidefinite, its diagonal entries equal to 1, and it is a rank one matrix. Then (2) has another formulation as

$$\begin{array}{ll} \min & C_1 \bullet X\,, \\ \text{s.t.} & \text{diag}(X) = e\,, \\ & \text{rank}(X) = 1\,, \\ & X \succeq 0\,, \end{array} \tag{3}$$

where diag(X) is a vector consisting of the diagonal elements of X, e is an all-ones vector of length n.

Our initial problem is then equivalent to min $C_1 \bullet X$ under the constraints $X \succeq 0$, rank$(X) = 1$ and $X_{ii} = 1$. Ignoring the nonoconvex "rank one" constraint, we are left with convex optimization problem,

$$\begin{array}{ll} \min & C_1 \bullet X\,, \\ \text{s.t.} & \text{diag}(X) = e\,, \\ & X \succeq 0\,. \end{array} \tag{4}$$

Peng et al [2] have use interior point methods to solve this semidefinite programming. However, interior point methods are still quite time and memory intensive and are not adapted for our communication problem, due to the typical size N which is from hundreds to thousands in the CDMA considered herein.

2.3. A Log Barrier Method Detection Strategy for ML Multuser Detection Problem

Instead of solving (4) by interior point methods, we will solve it by log barrier algorithm. Let $C = -C_1$, the (4) may be rewritten as the following problem Helmberg et al [9]

$$\begin{array}{ll} \max & C \bullet X\,, \\ \text{s.t.} & \text{diag}(X) = e\,, \\ & X \succeq 0\,. \end{array} \tag{5}$$

Without loss of generality, we assume that C is a positive semidefinite matrix, because of the equivalence between max $C \bullet X$ and max $(C + \text{Diag}(y)) \bullet X$ for all $y \in \Re$ subject to $\text{diag}(X) = e, X \succeq 0$. In mathematical term, it means that $C \succeq 0$. Where $\text{Diag}(y)$ is a diagonal matrix with y on its main diagonal.

Now, we consider the following problem

$$\begin{array}{ll} \max & C \bullet X\,, \\ \text{s.t.} & \text{diag}(X) \le e\,, \\ & X \succeq 0\,. \end{array} \tag{6}$$

Obviously, any feasible solution to the problem (5) is a feasible solution to the problem (6), that is the problem (6) is a relaxation of the problem (5). Conversely, we assumed that X is an optimal solution to the problem (6) and let Y be a matrix which is satisfied that $y_{ii} = 1, \text{for } i = 1, \cdots, n$ and $y_{ij} = x_{ij}, i \ne j, \text{for } i, j = 1, \cdots, n$. By using the $C \succeq 0$ and the fact that $x_{ii} \le 1$ for $i = 1, \cdots, n$, we have $C \bullet X \le C \bullet Y$. Together with that Y is a feasible solution to the problem (5), the optimal values to the problem (5) and (6) coincide. This shows the problem (5) and (6) are equivalent.

We now present the nonlinear programming reformulation of the problem (6) which is the basis of our algorithm for finding an approximate solution of the quadratic $\{-1, 1\}$ programming (2). For every $X \in S^n_{++}$, there exists an unique matrix $L \in \pounds^n_{++}$ such that $X = LL^T$. In addition, for every $X \in S^n_{+}$, there exists a matrix $L \in \pounds^n_{+}$ such that $X = LL^T$, though L is not necessarily unique. Based on the fact, the problem (6) can be stated as the following problem,

$$\begin{array}{ll} \max & C \bullet (LL^T)\,, \\ \text{s.t.} & \text{diag}(LL^T) \le e\,, \\ & L \in \pounds^n_{+}\,. \end{array} \tag{7}$$

Notice that we have replaced the requirement $X \succeq 0$ with $X = LL^T$. So the objective function of the problem (7) is nonconvex, but the feasible set of the problem (7) is convex.

In the following sections, we always find it more useful to describe the problem (7) in terms of the rows of L. More precisely, let l_i be the $i-$th row of L, then the problem (7) can also be rewritten as

$$\begin{array}{ll} \max & \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \langle l_i, l_j \rangle\,, \\ \text{s.t.} & \langle l_i, l_i \rangle \le 1, \text{ for } i = 1, \cdots, n\,, \\ & l_{i(i+1)} = \ldots = l_{in} = 0, \text{ for } i = 1, \cdots, n\,, \\ & l_{ii} \ge 0, \text{ for } i = 1, \cdots, n. \end{array} \tag{8}$$

2.4. The Log Barrier Algorithm of the Relaxation Problem

Here, we give and discuss the log barrier algorithm to solve the problem (8). Before giving the basic steps of the algorithm, we introduce the barrier problem

$$\begin{array}{ll} \max & \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\langle l_i, l_j\rangle + 2\mu \log\det(L)\,, \\ \text{s.t.} & \langle l_i, l_i\rangle \le 1, \text{ for } i = 1, \cdots, n\,, \\ & l_{i(i+1)} = \ldots = l_{in} = 0, \text{ for } i = 1, \cdots, n\,, \\ & l_{ii} > 0, \text{ for } i = 1, \cdots, n. \end{array} \tag{9}$$

A few definitions are also introduced. We denote $f : \mathcal{L}^n_{++} \mapsto \Re$ by $f(L) = C \bullet (LL^T) + 2\mu \log\det(L)$, and let low $:\Re^{n\times n} \longrightarrow \mathcal{L}^n$ be the operator which maps $A \in \Re^{n\times n}$ into the matrix $L \in \mathcal{L}^n$ such that $l_{ij} = a_{ij}$ if , $i \ge j$, and $l_{ij} = 0$ if $i < j$.

The formula for the gradient G of the function $f(L)$ at a point L is

$$G = 2\text{low}(CL) + 2\mu\,(\text{Diag}(\text{diag}(L)))^{-1}\,, \tag{10}$$

where A^{-1} is the inverse of the matrix A.

Given a matrix L^k feasible for the problem (9), detecting the feasible ascent direction of the function $f(L)$ at a point L^k consist of the following steps:

1. Compute the gradient G^k for the function f at the point L^k.
2. Compute the $(k+1)$-st adjective point V^{k+1} using the following formula

$$v_i^{k+1} = \begin{cases} \frac{g_i^k}{\|g_i^k\|}, & \text{if } g_i^k \ne 0\,, \\ l_i^k, & \text{if } g_i^k = 0\,. \end{cases} \tag{11}$$

3. Let $\overline{G^k} = V^{k+1} - L^k$.

If $\overline{G^k} \ne 0$, $\overline{G^k}$ is a feasible direction of the problem (9) at the point L^k based on the fact that the feasible set of the problem (9) is convex. We define $\varphi : \Re \mapsto \Re$ by $\varphi(h) = f(L^k + h\overline{G^k})$, then the $\overline{G^k}$ is also an ascent direction for the function $f(L)$ at the point L^k, that is, $\varphi(h) = f(L^k + h\overline{G^k}) > \varphi(0) = f(L^k)$ for all sufficiently small $h > 0$, because of the fact that $\overline{G^k} \ne 0$, that is $\exists\ i_0$, such that $\frac{g_{i_0}^k}{\|g_{i_0}^k\|} \ne l_{i_0}^k$ and

$$\begin{aligned} \frac{d}{dh}(\varphi(h))\,|_{h=0} &= \triangledown f(L^k) \bullet \overline{G^k} = G^k \bullet \overline{G^k} \\ &= \sum_{i=1}^{n}(\|g_i^k\| - \langle g_i^k, l_i^k\rangle) \ge \|g_{i_0}^k\|(1 - \langle \frac{g_{i_0}^k}{\|g_{i_0}^k\|}, l_{i_0}^k\rangle) > 0\,, \end{aligned} \tag{12}$$

where the final inequality follows from $\|l^k_{i_0}\| \leq 1$.

When $\overline{G^k} = 0$, L^k is a stationary point of the problem (9), we have the following proposition.

Proposition 1. *$\overline{G^k} = 0$ if and only if $g^k_i = \|g^k_i\| l^k_i$, for $i = 1, \cdots, n$.*

Proof. When $\overline{G^k} = 0$, we have $g^k_i = 0$ or $l^k_i = \frac{g^k_i}{\|g^k_i\|}$ for every $i = 1, \cdots, n$ based on equality (11). So we have $g^k_i = \|g^k_i\| l^k_i$, for $i = 1, \cdots, n$. Conversely, if $g^k_i = 0$, then$v^{k+1}_i = l^k_i$, and $\overline{g^k_i} = 0$. If $g^k_i \neq 0, v^{k+1}_i = \frac{g^k_i}{\|g^k_i\|} = l^k_i$, and $\overline{g^k_i} = 0$, thus, $\overline{G^k} = 0$.

After computing the feasible ascent direction $\overline{G^k}$, the algorithm generally selects a step-size such that $f(L^k + h_k \overline{G^k})$ is sufficiently larger than $f(L^k)$. A line search along $\overline{G^k}$ can be performed to find such an h_k, for example, the Armijo line search technique can be used as S. Burer et al [10]. Hence, in the sake of reducing complexity, we propose the following method.

We easily have that

$$\begin{aligned}
\varphi(h) &= f(L^k + h\overline{G^k}) \\
&= C \bullet (L^k(L^k)^T) + h(2\mathrm{low}(CL^k)) \bullet \overline{G^k} + h^2 C \bullet (\overline{G^k}(\overline{G^k})^T) \\
&+ 2\mu \sum_{i=1}^{n} \log(1 + h\frac{\overline{g^k_{ii}}}{l^k_{ii}}) = C \bullet (L^k(L^k)^T) + hG^k \bullet \overline{G^k} + h^2 C \bullet (\overline{G^k}(\overline{G^k})^T) \\
&+2\mu \sum_{i=1}^{n} (\log(1 + h\frac{\overline{g^k_{ii}}}{l^k_{ii}}) - h(\frac{\overline{g^k_{ii}}}{l^k_{ii}})).
\end{aligned}$$

Let $h_0 = \min\{-\frac{l^k_{ii}}{\overline{g^k_{ii}}} \mid \overline{g^k_{ii}} < 0\}$. If and only if $0 \leq h < h_0$, $\varphi(h)$ is continuous. Thus we chooses h_k as the largest scalar h satisfying $0 < h_k < h_0$.

Using the fact $\log(1 + x) - x + x^2 \geq 0$, when $x \geq -0.6838$, let

$$\begin{aligned}
\varphi_1(h) &= f(L^k + h\overline{G^k}) \\
&= C \bullet (L^k(L^k)^T) + hG^k \bullet \overline{G^k} + h^2 C \bullet (\overline{G^k}(\overline{G^k})^T) - 2\mu h^2 \sum_{i=1}^{n} (\frac{\overline{g^k_{ii}}}{l^k_{ii}})^2,
\end{aligned}$$

we have

$$\varphi(h) \geq \varphi_1(h) \text{ when } 0 \leq h \leq 0.6838h_0. \tag{13}$$

Thus, if $C \bullet (\overline{G^k}(\overline{G^k})^T) - 2\mu \sum_{i=1}^{n} (\frac{\overline{g^k_{ii}}}{l^k_{ii}})^2 \geq 0, \varphi_1(h)$ is a strictly monotone increasing function when $h \geq 0$. Based on inequality (13) and the feasibility of $L^k + h\overline{G^k}$, we choose

$$h_k = \min\{0.6838 h_0, 1\}. \tag{14}$$

If $C \bullet (\overline{G^k}(\overline{G^k})^T) - 2\mu \sum_{i=1}^{n} (\frac{\overline{g_{ii}^k}}{l_{ii}^k})^2 < 0$, $h = -\frac{G^k \bullet \overline{G^k}}{2C \bullet (\overline{G^k}(\overline{G^k})^T) - 4\mu \sum_{i=1}^{n} (\frac{\overline{g_{ii}^k}}{l_{ii}^k})^2}$ is the maximum point of $\varphi_1(h)$, that is, $\varphi_1(h)$ is a strictly monotone increasing function when

$$0 \leq h \leq -\frac{G^k \bullet \overline{G^k}}{2C \bullet (\overline{G^k}(\overline{G^k})^T) - 4\mu \sum_{i=1}^{n} (\frac{\overline{g_{ii}^k}}{l_{ii}^k})^2}.$$

Therefore, we choose

$$h_k = \min\{0.6838 h_0, -\frac{G^k \bullet \overline{G^k}}{2C \bullet (\overline{G^k}(\overline{G^k})^T) - 4\mu \sum_{i=1}^{n} (\frac{\overline{g_{ii}^k}}{l_{ii}^k})^2}, 1\}. \tag{15}$$

It is obvious that $h_k > 0$ and $\varphi(h_k) > \varphi(0)$. Let $L^{k+1} = L^k + h_k \overline{G^k}$, it is easy to see that $L^{k+1} \in \pounds_{++}^n$.

Log Barrier Algorithm. Let μ be a fixed small positive real number, L_0 be a feasible point of (9), $k = 0$.

Step 1. Compute the gradient G^k for the function f at the point L^k.

Step 2. Compute the $(k+1)$-th adjective point V^{k+1} using the formula (11).

Step 3. Compute the ascent direction $\overline{G^k} = V^{k+1} - L^k$.

Step 4. Compute the step-size h_k using the formula (14) or (15).

Step 5. $L^{k+1} = L^k + h^k \overline{G^k}, k = k+1$.

The chosen stopping criterion that is as follows. For $k \geq 1$, Let

$$r_k = \frac{f(L^k) - f(L^{k-1})}{f(L^k)}.$$

For some prespecified constants $m \geq 1, \varepsilon > 0$, our algorithm terminates once some $k \geq m$ is found such

$$r_{k-m+1} + r_{k-m+2} + \cdots + r_k < \varepsilon.$$

The global convergence of the log barrier algorithm can be received easily. For each $\mu > 0$, the problem (9) has an unique maximum point, which is also its unique stationary point (for detail proof see Appendix A).

The randomized cut generation scheme was introduced by Goeman et al [11] to work on the semidefinite relaxation of Max-Cut problem. Here we use the scheme to obtain the feasible solution of detection problems. The scheme state as the follows. Let M be the number of randomized rounding, X to be positive definite, we find P satisfing $X = P^T P$.

The randomized cut generation scheme. (input P, output y^*)

Step 1. Let $k = 1, f^0 = -\infty$.

Step 2. If $k > M$, then stop output y^*. Otherwise, μ_k is a round vector with every element subject to standard normal distribution $N(0, 1)$, let $y = sign(P^T \mu_k)$.

Step 3. If $y^T Cy > f^0, f^0 = y^T Cy, y^* = y, k = k + 1$, otherwise $k = k + 1$. Goto Step 2.

2.5. The Complexity of Log Barrier Algorithm

Now, we derive the complexity of an iteration of our algorithm.

The first step of an iteration of our method is the computation of the gradient $G = 2\text{low}(CL^k) + 2\mu(\text{Diag}(\text{diag}(L^k)))^{-1}$ for the function f at the point L^k. This formula shows that the computation of the gradient amounts to a single matrix multiplication and the inverse of a diagonal matrix. The the complexity of computation of the gradient is $O(n^3)$.

The second step of an iteration of our method is the computation of the next adjective point using the formula (11). Since norm $\|g_i^k\|$ can be computed in $O(n)$, we conclude that the overall complexity to evaluate the next adjective point is $O(n^2)$.

The third step of an iteration of our method is the computation of the ascent direction using the formula $\overline{G^k} = V^{k+1} - L^k$. It is easily seen that the complexity is $O(n^2)$.

The fourth step of an iteration of our method is the determination of the step-size according formula (13) or (14). h_0 can be computed in $O(n)$. To detect $C \bullet (\overline{G^k}(\overline{G^k})^T) - 2\mu \sum_{i=1}^{n} (\frac{\overline{g_{ii}^k}}{l_{ii}^k})^2$ will cost $O(n^3)$. The complexity of computing $G^k \bullet \overline{G^k}$ is $O(n^2)$. We easy see that the overall forth can be carried out in $O(n^3)$.

The last step of an iteration of our method take $O(n^2)$.

Hence, we get that the overall complexity of an iteration of the our algorithm is $O(n^3)$.

3. Simulation Results

We report the numerical example in this section. As stated before, the purpose of the results presented here are to show that our log barrier algorithm is considerably faster than the interior-point method for multiuser detection problem.

All the algorithms are run in the *MATLAB 6.1* environment on a 1.6GHz *Pentium IV* personal computer with 256Mb of RAM. In all the test problems, we choose the initial iterate L^0 to be $n \times n$ identity matrix and $\mu = 0.001/n$. In interior-point method, we solve the SDP relaxation by using SDPpack software Nayakakuppam et al [12].

We get a feasible solution of (2) using the randomized cut generation scheme and choose $M = n$. In iteration stops technique, we choose $m = 1$ and $\varepsilon = 0.001$. In Figure 1 and Figure 2 (see Appendix B), we use "IPM" presents for interior point methods based on semidefinite programming, "LBA" for the log barrier algorithm, "BER" for bit error rate, "time" for the average time of two method.

We first report numerical results on the bit error rate (BER) performance of the detector based on log barrier algorithm and interior-point methods based on the semidefinite relaxation in Figure 1. A synchronous CDMA system with length-63 Golden codes is used. Two different scenarios with $K = 24, 40$ are considered. The simulation results show that the BER of detection strategy based on log barrier algorithm is approaching that of the single user for $K = 24, 40$ and no appreciable performance difference between the detection strategy based on log barrier algorithm and that on interior-point methods.

Secondly, we use simulations to evaluate the computational time of log barrier algorithm and interior-point methods based on the semidefinite relaxation. The time result is shown in Figure 2. Clearly, for large K, the CPU time of the detection strategy based on log barrier algorithm is significantly lower than that of interior-point methods based on the semidefinite relaxation.

4. Conclusion

In this paper, we apply the log barrier algorithm strategy to NP-hard multiuser detection problem. Simulation results demonstrate that the similar BER performances between the interior point methods and our algorithm. However, it is worthwhile to mention that, for large length, the CPU time of the our method is significantly lower than that of interior-point methods.

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Appendix A

In this appendix, we give the global convergence of the log barrier algorithm. To proof the results, we need the following proposition.

Proposition 1. *Let L be a stationary point and G be the gradient of f at the point L, and let $z_i = 0.5\|g_i\|$, for $i = 1, \cdots, n$, then* $(\mathrm{Diag}(z)-C)LL^T = \mu I$ *and* $\mathrm{diag}(LL^T) = e$, *where I is the $n \times n$ identity matrix.*

Proof. Based on Proposition 1 and (10), we have

$$\mathrm{low}(CL) + \mu\,(\mathrm{Diag}(\mathrm{diag}(L)))^{-1} = \mathrm{Diag}(z)L\,,$$

that is,

$$CL + \mu\,(\mathrm{Diag}(\mathrm{diag}(L)))^{-1} - \mathrm{Diag}(z)L$$

is a strictly upper triangular matrix. It implies that

$$L^TCL + \mu\,L^T(\mathrm{Diag}(\mathrm{diag}(L)))^{-1} - L^T\mathrm{Diag}(z)L$$

is also a strictly upper triangular matrix. Together with

$$\mu\,L^T(\mathrm{Diag}(\mathrm{diag}(L)))^{-1}$$

being upper triangular matrix and $L^TCL - L^T\mathrm{Diag}(z)L$ being a symmetric matrix, we have that $L^TCL - L^T\mathrm{Diag}(z)L$ is a diagonal matrix. In addition that all diagonal elements of $L^T(\mathrm{Diag}(\mathrm{diag}(L)))^{-1}$ are ones and

$$L^TCL + \mu\,L^T(\mathrm{Diag}(\mathrm{diag}(L)))^{-1} - L^T\mathrm{Diag}(z)L$$

is a strictly upper triangular matrix, so $L^TCL - L^T\mathrm{Diag}(z)L = -\mu I$. Therefore, $(\mathrm{Diag}(z) - C)LL^T = \mu I$. This imply that $(\mathrm{Diag}(z) - C) \succ 0$, together with $C \succeq 0$, we have $z_i > c_{ii} \geq 0, \mathrm{for}\, i = 1, \cdots, n$, so $\|g_i^k\| > 0, \mathrm{for}\ i = 1, \cdots, n$. Further, if there exist that $\|l_i\| < 1$, L is not a stationary point of the problem (9) by using (11), so $\mathrm{diag}(LL^T) = e$. □

Obviously, we have the following corollary.

Corollary 2. *Let L be the stationary point of problem (9), X be the optimal solution of problem (6), then* $C \bullet (LL^T) \leq C \bullet X < C \bullet (LL^T) + n\mu$.

It is known that the following problem

$$\begin{array}{ll} \max & C \bullet X + \mu \log \det(X)\,, \\ \text{s.t.} & \operatorname{diag}(X) = e\,, \\ & X \succ 0\,. \end{array}$$

has the unique optimal solution and its KKT-condition is

$$\begin{array}{ll} \operatorname{diag}(X) = e & X \succ 0\,, \\ \operatorname{Diag}(y) - Z = C & Z \succ 0\,, \\ XZ = \mu I. & \end{array}$$

Combining Proposition 1 with the fact that for every $X \in S^n_{++}$, there exists an unique matrix $L \in \pounds^n_{++}$ such that $X = LL^T$, we have the following theorem.

Theorem 3. *For each $\mu > 0$, the problem (9) has an unique maximum point, which is also its unique stationary point.*

Appendix B

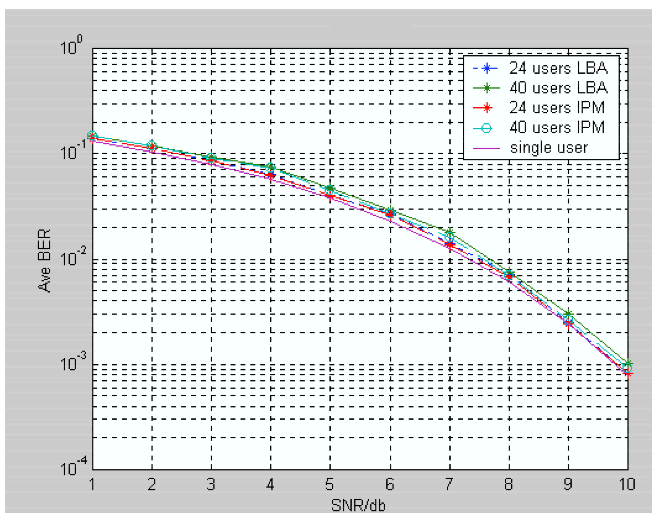

Figure 1: Average BER in different SNR

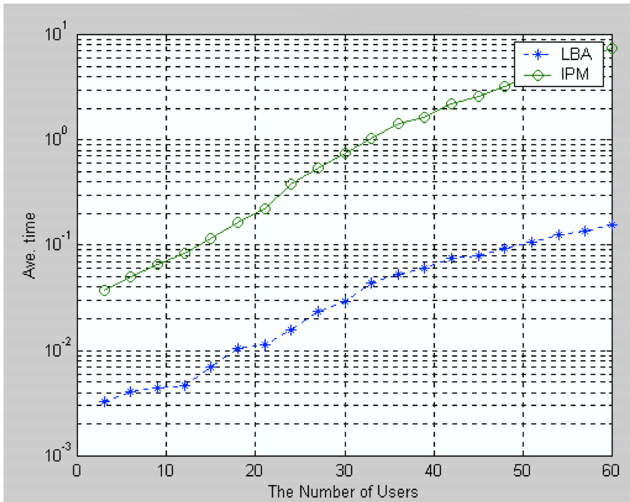

Figure 2: Average time in different users