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# NUMERICAL INVESTIGATION OF THE PENETRATION OF A RIGID BODY INTO THE SOIL

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**Abstract:** Conservation laws are used to model the penetration of a rigit body into a soil medium. For mathematical simplicity, an elastic-plastic model is assumed to represent the continum. Sample numerical solutions are presented for body penetration into a soil medium. Response quantities evaluated include the density variation within the penetrated medium, the depth and velocity are presented as function of the time during penetration.

**AMS Subject Classification:** 35H05
**Key Words:** rigid body, penetration, soil medium, elastic-plastic

## 1. Introduction

Problems dealing with the penetration of a rigid body into a soil medium have many applications in the real word. Example of such an application, which have been the subject of investigations for many years, is the penetration of a projectile into an elasto-plastic medium. These problems have dealt with the

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construction of docks and piers for ships and for making long term predictions about their viability, putting down pilings in soils for large buildings, design of projectiles for armor penetration as well as design of bunkers for passive protection against bombs and artillery shells.

Despite the importance or implications of the subject of soil penetration, little information exists in literature on the mechanisms involved during rigid body penetration into terrestrial materials. The reasons for the lack of information on the mechanisms penetration are obvious. The inherent mathematical complexities associated with defining constitutive equations and developing solutions to the conservation equations make the problem very hard to deal with.

Rigid body penetration into earth materials is characterized by wide variations in penetration. The process of body penetration may be separated into surface effects, which cause cratering, and penetration below the crater. During the cratering or impact phase, the surface of the soil breaks away and relieves compaction. This breaking away is caused by wave reflections from the surface near the rigid body.

Several analytical approaches to solve for example the projectile penetration problem have been used in the past. The penetration problem of a rigid projectile into a rigid-plastic target is based on postulating a general form of the force applied to the projectile with empirical constants. This force is assumed to consist of Bernoulli-like term which models inertial effects and is quadratic in velocity. However, this approach involves two empirical constants, which need to be measured for each combination of target. The same penetration problem is solved using a one-dimensional calculation of forces applied to the projectile by a work/energy balance. In [11] examples of this one-dimensional calculation of forces are given and in [1], [2], [9] a generalized equation for the force is proposed. Another group of analytic models (see [8], [12] and [10]) employs a two-dimensional calculation of the forces imposed on a rigid projectile by a rigid-plastic target. In this approach hydrodynamics of an inviscid fluid is used to propose an approximate velocity fields in the target. The composite velocity field is obtained by matching the on-dimensional field using certain geometric constrains.

Full numerical approaches to penetration have the most firm theoretical basis. With these both the target and the projectile are discretized and integrated numerically in both space and time. The work in uses the finite element method to solve for steady-penetration. But the three dimensional case is still extremely difficult to solve numerically. A computational model of viscoplasticity and ductile damage for projectile impact has been developed and implemented in by Borvik et al. Numerical simulations using this model give results

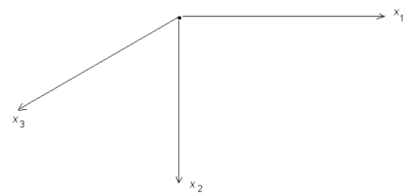


Figure 1: Coordinate system

in close agreement with experimental data for plugging failure caused by blunt projectiles (see [3] and [4]). What makes the numerical approach also difficult is the number of physical parameters involved. In [4] it has been identified nearly 30 possible and relevant input parameters in the general penetration problem of metal plates.

Unfortunately, most of the work done on projectile penetration has focus on steel, metal and aluminum targets with different thickness such as thick, intermediate and thin plates.

Only few papers dealing with rigid body penetration of elasto-plastic earth media have appeared in the open literature. As pointed out above, this is due to the inherent mathematical complexities associated with defining constitutive equations and developing solutions to the conservation equations. One may find more references in [6] and [7]. In these articles the general plasticity model was utilized with frozen soil properties. The general subject of projectile penetration of various media has been surveyed by [3], [7] and [5].

This paper presents the results of a numerical study concerning the penetration of a rigid body into a soil medium. The objective of this investigation is first to derive the equation of the model problem using a more up to date knowledge of continuum mechanics. A numerical solution is used to determine the response of the soil to the rigid body and how energy is transported into the soil. In the case of a piling, we want to know how much energy it takes to point the piling in. Knowing the velocity $v$, the kinetic energy would be $mv^2/2$.

## 2. Model Problem

The coordinate system is shown in Figure 1.

We assume that the gravity acts in the positive $x_3$ direction which we take to be down. The surface of the earth is assumed to be initially a plane, the $x_1x_2$-plane described by $x_3 = 0$.

After a very short time the configuration is shown in Figure 2.

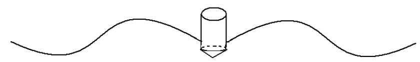

Figure 2: Strike of a rigid body into the earth

The rigid body strikes the earth from about the origin and penetrates into the soil. The surface is described by

$$x_3 = f(x_1, x_2, x_3), \quad f(x_1, x_2, 0) = 0\,.$$

We assume that the body is rigid and not rotating so as that any convenient point in the body can be chosen as a point from which to measure its motion.

## 3. Governing Equations

The governing equations are the differential equations derived from conservation laws that is the principle of mass, linear momentum, and energy in one-dimension space. Rectangular Cartesian coordinates are used. The unknowns are:

$\rho(x,t)$ = mass density of the soil,
$v(x,t)$ = velocity,
$E(x,t)$ = internal energy,
$p(x,t)$ = pressure.

The equations are to be solved in the one-dimension space for $x > 0$, $t > 0$, where $x$ is a spatial variable and $t$ is the time. The differential equations are as follows:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \quad \text{conservation of mass,} \tag{1}$$

$$\rho\left[\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right] = -\frac{\partial p}{\partial x} \quad \text{conservation of linear momentum,} \tag{2}$$

$$\rho\left[\frac{\partial E}{\partial t} + v\frac{\partial E}{\partial x}\right] = -p\frac{\partial v}{\partial x} \quad \text{conservation of energy.} \tag{3}$$

The pressure is given by the Mie-Gruneisen equation of state

$$p = \frac{\rho_0 c_0^2 \eta}{(1 - s\eta)^2} \left[ 1 - \frac{\gamma_0}{2}(1-\eta)(\frac{\rho_0}{\rho} - 1) \right] + \gamma_0 (1-\eta)\rho E \,, \tag{4}$$

where $\eta = 1 - \frac{\rho_0}{\rho}$, $\gamma_0$ is the Gruneisen constant, $c_0$ is the sonic velocity, and $s$ a constant. Here the 0 subscript refer to reference values.

We assume that the hyperbolic system (1-4) is to be solved in the domain $R = \{0 \le t \le a,\ x \ge 0\}$ in the absence of body forces, heat conduction, and energy sources. The initial and boundary conditions are:

$$\begin{aligned} v(x,0) &= 0, \quad \rho_0 = \rho(x,0) = 2.55\,\mathrm{gm/cm}^3 \,, \\ E(x,0) &= 2.5 \times 10^8 (\mathrm{cm/sec})^2 \,, \\ E(0,t) &= 1.25 \times 10^9, \quad v(0,t) = 5 \times 10^4 \ \mathrm{cm/sec}, \\ \rho(0,t) &= 4 \ \mathrm{gm/cm}^3 \end{aligned} \tag{5}$$

For $t > a$ we take $v(0,t) = \rho(0,t) = E(0,t) = 0$.

## 4. Finite Difference Scheme

There are many finite-difference schemes to the system (1-4), but the one which is very well suited for hyperbolic system is the Lax-Friedrichs 3-point scheme. To develop the explicit method, we consider a mesh of spacing $h$ for the space variable and a mesh of spacing $k$ for the time variable on the region $R$ with $h = L/n$ and $k = a/m$. We define approximations $v_i^j$, $\rho_i^j$, $E_i^j$, and $p_i^j$ of $v(x_i, t_j)$, $\rho(x_i, t_j)$, $E(x_i, t_j)$, and $p(x_i, t_j)$ respectively at the point $(x_i = ih, t_j = jk)$ by the finite difference equations (see [6] for more details):

$$v_i^{j+1} = -\frac{\lambda}{2} \left[ v_i^j (v_{i+1}^j - v_{i-1}^j) + \frac{1}{\rho_i^j}(p_{i+1}^j - p_{i-1}^j) \right] + \frac{v_{i-1}^j + v_{i+1}^j}{2} \,, \tag{6}$$

$$\rho_i^{j+1} = -\frac{\lambda}{2} \left[ \rho_i^j (v_{i+1}^j - v_{i-1}^j) + v_i^j (\rho_{i+1}^j - \rho_{i-1}^j) \right] + \frac{\rho_{i-1}^j + \rho_{i+1}^j}{2} \,, \tag{7}$$

$$E_i^{j+1} = -\frac{\lambda}{2} \left[ v_i^j (E_{i+1}^j - E_{i-1}^j) + \frac{p_i^j}{\rho_i^j}(v_{i+1}^j - v_{i-1}^j) \right] + \frac{E_{i-1}^j + E_{i+1}^j}{2} \,, \tag{8}$$

$$p_i^j = \frac{\rho_i^0 c_0^2 \eta_i^j}{(1 - s\eta_i^j)^2} \left[ 1 - \frac{\gamma_0}{2}(1 - \eta_i^j)(\frac{\rho_i^0}{\rho_i^j} - 1) \right] + \gamma_0 (1 - \eta_i^j) \rho_i^j E_i^j \,, \tag{9}$$

where $\lambda = \frac{k}{h}$ and $\eta_i^j = 1 - \frac{\rho_i^0}{\rho_i^j}$.

At $t = 0$, all quantities are specified at all meshes by initial data. The calculations necessary to advance all quantities by a time increment $k$ are performed at all meshes in order to advance to the next time $t + 2k$.

By linearizing $p(x)$ one can show that the system (6-9) is stable if the eigenvalues of the emplification matrix

$$H = \begin{bmatrix} A & B & C \\ D & A & 0 \\ E & 0 & A \end{bmatrix},$$

where

$$A = \cos\alpha h - i\, v\, \lambda \sin\alpha h\,, \quad B = -\, i\frac{\lambda}{\rho} c_0^2 \sin\alpha h\,,$$
$$C = -i\,\lambda\frac{\rho_0}{\rho}\gamma_0 \sin\alpha h\,, \quad D = -i\,\lambda\rho \sin\alpha h\,,$$
$$E = -i\;\lambda\frac{p}{\rho}\sin\alpha h$$

do not exceed unity in modulus. The eigenvalues of $H$ are

$$e_1 = A\,, \quad e_2 = A + \sqrt{BD + CE}\,, \quad e_3 = A - \sqrt{BD + CE}\,.$$

Hence the difference scheme is stable if

$$\lambda = \frac{k}{h} \le \frac{1}{|v|} \quad \text{and} \quad \lambda \le \frac{1}{\frac{|A\,|}{\sqrt{\rho}} - v}\,.$$

## 5. Results for Earth Media Penetration

In this section sample numerical solutions are presented for the penetration of rigid bodies into an elastic-plastic medium. Response quantities evaluated includes depth, velocity, kinetic energy and mass density variations within the penetrated medium. The parameters of the soil medium used in the numerical calculations are shown in Table 1.

The program was run with $h = 0.1$ and $k = 2 \times 10^{-9}$ in the time interval $[0, 10^{-6}]$.

Figures 3, 4, and 5 show the velocity, kinetic energy and mass density respectively at different time and space steps. It is interesting to observe from

| **Parameters** | **Soil** |
|---|---|
| Density $\rho_0$ | 2.55 |
| Sonic velocity $c_0$ | 47520 cm/sec. |
| Grunneisen constant $\gamma_0$ | 1.9 |
| Constant $s$ | 1.83 |

Table 1: Parameters used for the numerical solution

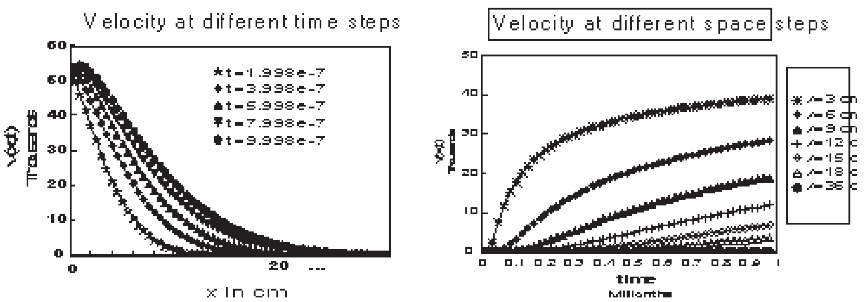


Figure 3: Velocity

Figure 3 that the depth of penetration for the rigid body in the elastic-plastic mediumis is 33 cm since at that point the velocity becomes zero for all $t$. The prediction is that the velocity should move like a hump which correspond to the numerical results shown in Figure 3.

## 6. Conclusion

An analysis has been presented to identify some of the significant factors affecting the penetration of a rigid body into a soil medium. Rigid body penetration is a very complex problem and some assumptions has to be made to simply the mathematical problem. Here we assumed that we have an elastic-plastic model.

Sample numerical solutions were presented for the penetration of a rigid body into a soil medium. Response quantity evaluated included the density variations within the penetrated medium. Also depth, velocity, and kinetic energy of the body were presented as a function of time and space variable. The numerical results were in good agreement with the physical expectations. Further study that includes the stess-strain relation should be conducted for better analysis of the soil penetration problem.

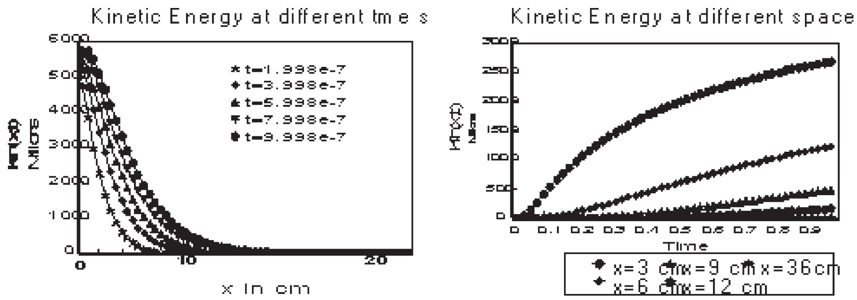


Figure 4: Kinetic energy

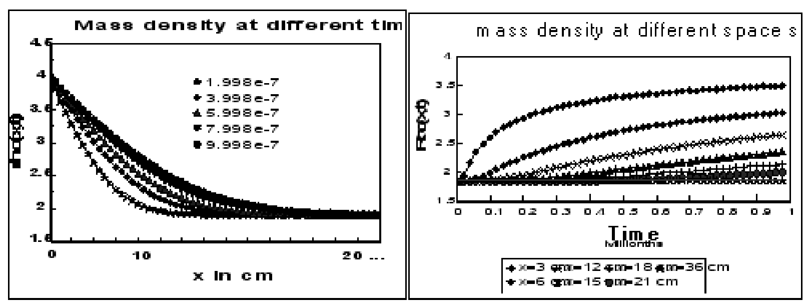


Figure 5: Mass density

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