SOLVING MANPOWER PLANNING PROBLEM
WITH TWO TYPES OF JOBS UNDER
UNCERTAINTY DEMAND

Xiaoqiang Cai¹, Yongjian Li²§, Fengsheng Tu³

¹Department of System Engineering and Engineering Management
The Chinese University of Hong Kong
Shatin N.T., HONG KONG

²School of Economics and Management
Tsinghua University
Beijing, 100084, P.R. CHINA
e-mail: liyongjian@em.tsinghua.edu.cn

³Department of Automation
Nankai University
Tianjin, 300071, P.R. CHINA

Abstract: We consider a manpower planning problem (MPP) with two types of jobs under uncertainty demands over a long planning horizon where dynamic demands for manpower must be fulfilled by allocating enough number of employees of qualified skills. The jobs are ranked such that an employee, being qualified to take a higher ranked job, can be assigned to take a lower ranked job, but not vice versa. Taking into account the recruitment/dismissal factors, setup cost and staff substitution, we formulate the problem as a multi-period stochastic decision model. First, a mean value model approach is developed to solve the stochastic problem through a feedback control scheme. Then a sampling approach is introduced to solve the problem. Further, the sampling approach is applied in solving mixed model of the problem. These approaches
can be extended to the general case with any $n (n \geq 2)$ types of jobs.

**AMS Subject Classification:** 91B70, 93B40, 37N40

**Key Words:** manpower planning, mean value model approach, sampling model approach, uncertainty demands

---

### 1. Introduction

In today’s business world, capacities are predominately determined by the available manpower resources. To meet the changing business requirements and to keep a low operational cost, a timely adjustment of manpower is necessary. In the past decade, the rapid development of computing technology has enabled an effective planning of manpower, which helped improve the resource usage for government organizations and companies. Manpower planning techniques have become an essential tool for human resource managers, especially in a climate of economic recession. Due to its importance, considerable effort has been devoted to tackling the problem.

There has been some published work describing the use of optimization techniques to tackle the planning problem. Some of the previous results have been reviewed by Bowey [3], Purkiss [18], Edwards [9] and [1]. Many of the manpower planning problems have been formulated with goal programming models (e.g., Price and Piskor [17]). Moreover, Zanakis and Maret [23] have formulated a Markovian goal programming model with pre-emptive priorities, while Mehlmann [14] and Rao [19] have developed optimal recruitment and transition strategies for manpower systems using dynamic programming. The models for manpower planning can be classified into two categories (Purkiss [18]). The first is exploratory models which can give the managers insights into the way his/her manpower system works and the way it would respond to different stimuli. See, for example, Zanakis and Maret [23], McClean [13], Georgiou and Vassilion [11]). The second is the very powerful normative models, which can compute an optimal set of personnel decisions (on recruitment, promotion, training, etc.) against goals stated in some forms of objective function. A number of early attempts using linear programming have been described in Smith [22]. One class of problems is the recruitment or accession planning (see Bres et al [4] and Rao [19]) with the selection of a recruitment schedule over multiple time periods, which best meets goal pertaining to promotion opportunity, salary expenditure, desired levels of experience in the workforce and requirements for manpower in each planning period.

In this paper, we will propose a manpower planning model to address the
need of decision making in labor-intensive organizations facing dynamic fluctuations in their manpower demands. Our model has been motivated by the operational requirements in a local major air-cargo loading/unloading company. There are different types of staff employed by the company, who are defined in different groups according to their ranks, the normal duties and the regular working locations. Nevertheless, from time to time some staff may be assigned to work for a lower level job, or at a different location, for a temporary duration. Besides, there are needs to hire casual workers, to meet the manpower demands in some peak seasons, or to meet the sudden increase in manpower demands after some disruptive events (such as a typhoon or some unexpected events that severely affect the operations of the airport). A primary objective in the development of the computerized manpower planning and scheduling system of the company is to take into account all the factors of recruitment with setup cost, dismissal, substitution, and the information on the stochastic fluctuations in the staff demands over the planning horizon, to generate the near-optimal staffing strategy/decisions to minimize the total staffing cost for the company.

It is clear that similar problems arise in other labor-intensive organizations where staff substitution or casual staff recruitment/dismissal are often required. Examples including freight-forwarding, container terminals, manufacturing of seasonal products, etc.

Our manpower planning problem (MPP) will be modelled as a multi-stage decision process with constraints. For the case where the demands are deterministic, optimal approaches are proposed for the problem with single- and two-employee types respectively in [6]. These approaches are devised based on analysis on the optimal decisions corresponding to different shapes of demand patterns, which are not only computationally very efficient, but also can reveal some useful insights on the desirable solutions with respect to the demand data. Furthermore, for the case where the demands are deterministic, the problems with any multiple employee types are solved through a Dantzig-Wolfe approach and the near-optimal solution can be obtained.

In this paper, the problem with consideration of setup cost for recruitment under the stochastic demands will be considered. This kind of problem belongs to a special class known as Markov decision problems with constraints that have a large spectrum of practical applications (Ross [20]). Theoretically, the problem can be solved by applying stochastic dynamic programming (DP). Nevertheless, as we will show later, the common difficulty with DP, that is, the “curse of dimensionality”, makes it almost infeasible to use such a DP algorithm to solve MPP in practice.

The rest of this paper is organized as follows. We first develop and discuss
the models in Section 2. In Section 3, we propose a recursive approach to solve the problem through stochastic dynamic programming and discuss the computational complexity of the problem in general case. In Section 4, we propose a mean value approach to solve the MPP problem with a feedback control policy. In Section 5, we adopt sampling model approach to solve the MPP problem. In Section 6, we extend the sampling model approach to solve a mixed model of the manpower planning problem where the demands are deterministic during the first several periods and are stochastic after those. In Section 7, we illustrate the applications of the model. Section 8 concludes the study with a summary, extensions, and directions for future research.

2. Problem Description

Assume that the number of planning periods under consideration is $T$ and $T$ is finite. The length of each period may be a week or a year, depending on the application. There are two types of jobs, requiring manpower resources $D_{1t}$, $D_{2t}$ respectively during period $t$, $t = 1, 2, \cdots, T$. Without loss of generality, we let initial demands $D_{i0} = 0$, $i = 1, 2$ (a problem with nonzero initial demands can be tackled similarly). At the same time, there are two types of employees available in the organization. Type-1 employees have skill 1 and can be assigned to type-1 job, while type-2 employees hold better skills, and can be assigned not only to type-2 job, but also to type-1 job.

In the manpower planning context, the behavior of demand for employees can be described by a random component. This is an important characteristic because over a long-term planning horizon the demand is not known exactly and therefore must be estimated. Exact demands can only be determined for several near future periods but not for all future planning periods. In our discussion, we assume that only demand information in the current period is determined. For the remaining periods, we assume in the context, that the demands $D_{it}$ are mutually independent and stationary random variables with the Gaussian distribution $N(a_{it}, \sigma_{it}) = \Phi\left(\frac{x - a_{it}}{\sigma_{it}}\right)$.

We start by introducing some notation. For period $t$ ($t = 1, 2, \cdots, T$), let $b_{it}$ be setup cost when an activity of recruiting type-$i$ employees occurs and $u_{i}[t]$ be the quantity of type-$i$ employees to be recruited/dismissed at the end of period $t$. The cost of dismissing/recruiting type-$i$ employees with a quantity of $u_{i}[t]$ is given by $p_{it}(u_{i}[t])$ with $p_{it}(0) = 0$ and:

$$p_{it}(u_{i}[t]) = \begin{cases} b_{it}\delta_{i}(t) + p'_{it}(u_{i}[t]), & u_{i}[t] > 0, \\ p''_{it}(u_{i}[t]), & u_{i}[t] \leq 0, \end{cases} \quad (1)$$
where $p_i'(.)$ and $p''_i(.)$ are continuous and nondecreasing functions with $p_i'(0) = 0$, $p''_i(0) = 0$ respectively. The salary cost for $X_i[t]$ type-$i$ employees in period $t$ is given by $h_{it}(X_i[t])$, where $h_{it}(.)$ is a continuous and nondecreasing function with $h_{it}(0) \geq 0$. Because of the stochastic properties of demands in future periods, the total full-time (FT) employees may not cover the demands of every job in practice, part-time (PT) type-$i$ employees should be employed to take the unassigned type-$i$ jobs. No recruiting/dismissal costs, but a higher salary is paid for a PT type-$i$ employee. We assume that salary cost for $Z_i[t]$ PT type-$i$ employees is given by $c_{it}(Z_i[t])$ which satisfies that $c_{it}(x) > h_{it}(x)$ for $x > 0$ and $c_{it}(x) = 0$ for $x \leq 0$.

The manpower planning problem is that of determining amounts $u_i[0], u_i[1], \ldots, u_i[T - 1]$ that minimize the manpower-related total cost

$$\sum_{i=1}^{2} \left\{ \sum_{t=0}^{T-1} p_{it}(u_i[t]) + \sum_{t=1}^{T} \left[ h_{it}(X_i[t]) + c_{it}(Z_i[t]) \right] \right\}. \quad (2)$$

The basic constraint in the manpower planning problem is that the total available employees in the system must be able to cover the demands of all types of jobs. In period $t$, only $X_2[t]$ FT employees and $Z_2[t]$ PT employees can be assigned to do type-2 job. We get,

$$X_2[t] + Z_2[t] \geq D_{2t}, \quad t = 1, 2, \ldots, T. \quad (3)$$

If there are, however, unassigned FT type-2 employees at any time period $t$, they can be used to substitute FT type-1 employees to take type-1 job. Then, there may be FT type-1 employees, PT type-1 employees and FT type-2 employees that are available to meet the demand of type-1 job. So, we get,

$$X_1[t] + X_2[t] + Z_1[t] \geq D_{1t} + D_{2t}, \quad t = 1, 2, \ldots, T. \quad (4)$$

Because demand $D_{it}$ is a random variable, the constraints (3) and (4) may be violated if there are no enough PT employees to be employed in some period $t$. In order to reduce the risk and also to maintain explicitly the constraints in the model, we introduce new constraints to guarantee the proportion of the FT employees being assigned to meet the demands in every period.

$$Prob.(X_2[t] \geq D_{2t}) \geq \eta_t, \quad \eta_t \in [0, 1), \quad (5)$$

$$Prob.(X_1[t] + X_2[t] \geq D_{1t} + D_{2t}) \geq \eta_t, \quad \eta_t \in [0, 1). \quad (6)$$

The quantities $X_i[t]$ should satisfy a recursive state equation

$$X_i[t + 1] = X_i[t] + u_i[t], \quad i = 1, 2; \quad t = 1, 2, \ldots, T - 1. \quad (7)$$
Managerial rule will be used to select an appropriate value for $\eta_t$, which can be interpreted as a satisfactory level to be provided by the manager. If $\eta_t \in \left( \frac{1}{2}, 1 \right)$, it means that the manager desires to satisfy at least $100 \times \eta_t$ percent of the demand $D_{it}$ by FT employees in period $t$. Otherwise, $\eta_t \in [0, \frac{1}{2})$ means a high risk that the demands of type-$i$ job can not be fulfilled using the FT employees.

Therefore, the stochastic model of manpower planning problem with two types of jobs can be described as follows,

$$\text{SMP: } \min J = \mathbb{E} \left\{ \sum_{i=1}^{2} \sum_{t=1}^{T} \left( p_{it}(u_i[t]) + h_{it}(X_i[t]) + c_{it}(Z_i[t]) \right) \right\} \quad (8)$$

s.t.

$$X_{i}[t+1] = X_{i}[t] + u_{i}[t], \quad i = 1, 2; \quad t = 0, 1, 2, \ldots, T - 1, \quad (9)$$

$$X_1[t] + X_2[t] + Z_1[t] \geq D_{1t} + D_{2t}, \quad t = 1, 2, \ldots, T, \quad (10)$$

$$X_2[t] + Z_2[t] \geq D_{2t}, \quad t = 1, 2, \ldots, T, \quad (11)$$

$$\text{Prob. } (X_1[t] + X_2[t] \geq D_{1t} + D_{2t}) \geq \eta_t, \quad \eta_t \in [0, 1), \quad t = 1, \ldots, T, \quad (12)$$

$$\text{Prob. } (X_2[t] \geq D_{2t}) \geq \eta_t, \quad \eta_t \in [0, 1), \quad t = 1, 2, \ldots, T, \quad (13)$$

$$\max \{ u_i[t], 0 \} \leq M_i \delta_i(t), \quad i = 1, 2; \quad t = 0, 1, 2, \ldots, T - 1, \quad (14)$$

$$X_i[0] = 0, \quad i = 1, 2,$$

where $D_{it}, X_i[t], Z_i[t]$ are nonnegative integers, $i = 1, 2; \quad t = 1, 2, \ldots, T$.

3. Computational Complexity

The standard way to solve the stochastic manpower planning problem (SMP) is by means of dynamic programming approach. The higher salary that pays for the PT employee can be regarded as a penalty cost when the total FT employees do not cover the demands of all jobs.

Since $c_{it}(x)$ satisfies that $c_{it}(x) > h_{it}(x)$ for $x > 0$ and $c_{it}(x) = 0$ for $x \leq 0$, the objective function (8) can be converted into the following form.

$$\min J = \mathbb{E} \left\{ \sum_{i=1}^{2} \sum_{t=1}^{T} \left[ p_{it}(u_i[t]) + h_{it}(X_i[t]) \right] + c_{it}(D_{1t} + D_{2t}) \right\}$$
Let $f_t(x_1, x_2)$ be the cost of an optimal plan through the period 0 to $t$ subject to $X_i[t] = x_i, i = 1, 2$. $\mathbb{N}_t = \{Pr(X_1[t] + X_2[t] \geq D_{1t} + D_{2t}) \geq \eta_t, Pr(X_2[t] \geq D_{2t}) \geq \eta_t\}$ is a set of feasible states in period $t$ and $h_t(x_1, x_2) = \{x_i - \max D_i \leq u_i[t] \leq \max D_i - x_i, i = 1, 2\}$ is a set of feasible controls at the end of period $t$ subject to $X_i[t] = x_i (i = 1, 2)$, where $\max D_i$ is the expected maximal demand of type-$i$ employees over the entire planning horizon ($i = 1, 2$). We get the dynamic programming recursion as follows,

\begin{equation}
  f_0(0, 0) = 0, 
\end{equation}

\begin{equation}
  f_t(x_1, x_2) = \begin{cases}
    \min_{(u_1, u_2) \in h_t(x_1, x_2)} \left\{ \sum_{i=1}^{2} [h_{it}(x_i) + p_{i(t-1)}(u_i)] + c_{it}(D_{1t} + \right. \\
    \left. D_{2t} - x_1 - x_2) + c_{2t}(D_{2t} - x_2) + f_{t-1}(x_1 - u_1, x_2 - u_2) \right\}
  \end{cases}
\end{equation}

for $(x_1, x_2) \in \mathbb{N}_t, (x_1 - u_1, x_2 - u_2) \in \mathbb{N}_{t-1}$ and $t = 1, 2, \ldots, T$.

The cost of an optimal plan over the entire planning horizon is equal to $\min \{f_T(x_1, x_2), (x_1, x_2) \in \mathbb{N}_T\}$. The corresponding values of $u_i[0], u_i[1], \ldots, u_i[T - 1]$ are obtained by the recursion $X_i[t + 1] = X_i[t] + u_i[t] (i = 1, 2)$.

We consider the deterministic case to estimate the running time of the dynamic programming algorithm. It can be easily obtained that the recursion requires $O(T\max D_1 \max D_2)$ time, which provides a pseudopolynomial algorithm in the deterministic case. Thus dynamic program provides at least a pseudopolynomial algorithm in the stochastic case. We shall now present evidence that the manpower planning problem (SMP) is NP-hard in some case.

**Theorem 1.** Manpower planning problem can be transformed into production planning problem in which demand and production quantity may be negative.

**Proof.** In the deterministic case, the problem (8)-(14) can be described as follows:

\begin{equation}
  \min J = \left\{ \sum_{i=1}^{2} \left[ \sum_{t=0}^{T-1} p_{it}(u_i[t]) + \sum_{t=0}^{T} h_{it}(X_i[t]) \right] \right\},
\end{equation}
s.t. $X_i[t + 1] = X_i[t] + u_i[t], \quad t = 0, 1, \cdots, T - 1,$  
$X_1[t] + X_2[t] \geq D_{1t} + D_{2t}, \quad t = 1, 2, \cdots, T,$  
$X_2[t] \geq D_{2t}, \quad t = 1, 2, \cdots, T,$  
$max\{u_i[t], 0\} \leq M_i\delta_i(t), \quad t = 0, 1, \cdots, T - 1,$  
$D_{it}, X_i[t]$ are nonnegative integers, $i = 1, 2; t = 1, 2, \cdots, T.$

The constraints (20) and (21) can be replaced by the following constraints:

\[ w_2[t] \geq D_{2t}, \quad t = 1, 2, \cdots, T, \quad (23) \]
\[ w_2[t] \leq X_2[t], \quad t = 1, 2, \cdots, T, \quad (24) \]
\[ w_1[t] + w_2[t] \geq D_{1t} + D_{2t}, \quad t = 1, 2, \cdots, T, \quad (25) \]
\[ w_1[t] \leq X_1[t], \quad t = 1, 2, \cdots, T. \quad (26) \]

Let $D'_{1t} = 0$ and $D'_{it} = D_{it} - D_{i(t-1)}, t \geq 2; w'_i[1] = 0, w'_i[t] = w[t] - w[t - 1], t \geq 2$ and $X'_i[t] = X_i[t] - \sum_{l=1}^{t} w_i[l]$, the problem (18)-(22) is transformed into the following form.

\[ \text{DPP:} \quad \min J = \left\{ \sum_{i=1}^{2} \left[ \sum_{t=0}^{T-1} p_{it}(u_i[t]) + \sum_{t=1}^{T} h_{it}(X'_i[t] + \sum_{l=1}^{t} w'_i[l]) \right] \right\}, \quad (27) \]

s.t. $X'_i[t + 1] + w'_i[t + 1] = X'_i[t] + u_i[t], \quad t = 0, 1, \cdots, T - 1,$  
$w_1[t] + w_2[t] \geq D_{1t} + D_{2t}, \quad t = 1, 2, \cdots, T,$  
$w_2[t] \geq D_{2t}, \quad t = 1, 2, \cdots, T,$  
$max\{u_i[t], 0\} \leq M_i\delta_i(t), \quad t = 0, 1, \cdots, T - 1.$  

Since $D'_{it} = D_{it} - D_{i(t-1)}, t \geq 2$, demands are perhaps negative in some period, so are the values $w'_i[t]$. Moreover, since $X'[t] = X[t] - \sum_{l=1}^{t} w[l]$, the state $X'[t]$ also may be negative.

**Theorem 2.** The manpower planning problem (SMP) remains NP-hard in each of the following cases:

(a) Arbitrary setup costs, nondecreasing cost functions.
(b) Arbitrary setup costs, convex cost functions.
(c) Arbitrary setup costs, concave cost functions.
Proof. From the Theorem 1, the problem (SMP) is converted into a production planning problem (DPP) in which demands may be negative.

In the case (a), the remaining discussion resembles that of Proposition 1 in (Florian et al [10]), we can show that the known NP-complete KANPSACK problem can be reducible to a simplified problem (DPP).

In the case (b), the result is verified from the Proposition 2 in (Florian, et al [10]).

In the case (c), the discussion resembles that in Theorem 1 in (Cai et al [6]).

4. Mean Value Model Approach

4.1. Building a Deterministic Model

The stochastic problem (8)-(14) is hard to solve. A stochastic dynamic programming algorithm can be used to provide an optimal solution but computational difficulties reserve its application only to a low dimension problem. To overcome this difficulty, an approximate scheme is adopted to reduce the complexity of the original model. Deterministic equivalent approximations are preferred because they are easier to solve. Besides, the sub-optimal solutions often can be good approximations for true optimal solutions. There are many appropriate algorithms for deterministic mathematical programming that can be used to solve it (Minoux [15]). A more sophisticated version of the mean problem takes into account the second statistic moment of the variables (Lassere [12]). This alternative is focused on in this paper.

First, a deterministic approximation to the model (SMP) is now presented. The aim is to obtain an equivalent model that preserves the basic stochastic properties of the original problem. Thus, the first step in this analysis is to investigate the statistic moments associated with demand distribution functions, assumed here to be known during each period $t$.

The statistical properties of the stochastic process can be used for an approximation. The main advantage of the Gaussian assumption is that all the information required is entirely enclosed in the first two statistic moments. $D_{2t} \sim N(a_{2t}, \sigma_{2t})$ and $D_{1t} + D_{2t} \sim N(a_{1t} + a_{2t}, \sqrt{\sigma_{1t}^2 + \sigma_{2t}^2})$. On the other hand, the control $u_i[t]$ is a deterministic variable, i.e. $\text{Var}(u_i[t]) = 0, \forall t \geq 0$. Thus the recursive state equation

$$X_i[t+1] = X_i[t] + u_i[t], \quad i = 1, 2; \quad t = 0, 1, \cdots, T - 1,$$

(32)
is still satisfied. (32) describes the mean evolution of the employee’s number.

Taking the satisfactory chance constraints in (12) and (13) and using (9) and (32), it is possible to handle the probabilistic operator as follows

\[
\text{Prob}(X_2[t] \geq D_{2t}) \geq \eta_t \leftrightarrow \Phi\left(\frac{X_2[t] - a_{2t}}{\sigma_{2t}}\right) \geq \eta_t
\]

\[
\leftrightarrow X_2[t] \geq a_{2t} + \sigma_{2t} \Phi^{-1}_t(\eta_t), \quad (33)
\]

\[
\text{Prob}(X_1[t] + X_2[t] \geq D_{1t} + D_{2t}) \geq \eta_t \leftrightarrow \Phi\left(\frac{X_1[t] + X_2[t] - a_{1t} - a_{2t}}{\sqrt{\sigma_{1t}^2 + \sigma_{2t}^2}}\right) \geq \eta_t
\]

\[
\leftrightarrow X_1[t] + X_2[t] \geq a_{1t} + a_{2t} + \sqrt{\sigma_{1t}^2 + \sigma_{2t}^2} \Phi^{-1}_t(\eta_t), \quad (34)
\]

where \(\Phi(.)\) is the standard Gaussian distribution function, which depends only on the evolution of the variance \(\sigma_{it}(i = 1, 2)\) and satisfactory chance \(\eta_t\).

When \(u_i[t]\) are determined through a method, the states \(X_i[t]\) should be deterministic since \(X_i[t + 1] = X_i[t] + u_i[t]\) and \(X_i[0] = 0\). So in the objective function, the items \(p_{it}(u_i[t])\) and \(h_{it}(X_i[t])\) are deterministic when \(u_i[t]\) and \(X_i[t]\) are deterministic. However, the items \(c_{1t}(D_{1t} + D_{2t} - X_1[t] - X_2[t])\) and \(c_{2t}(D_{2t} - X_2[t])\) are stochastic since the demands \(D_{it}(i = 1, 2)\) are stochastic, so we should compute the expected penalty cost \(\mathbb{E}\left\{ \sum_{t=1}^{T} c_{1t}(D_{1t} + D_{2t} - X_1[t] - X_2[t]) \\right\} \) and \(\mathbb{E}\left\{ \sum_{t=1}^{T} c_{2t}(D_{2t} - X_2[t]) \right\}\).

In the paper (Papoulis [16]), the justification for interchanging expectation and summation operators, and integration of the function \(c_{it}(.)\) can be found. Thus we can get

\[
\mathbb{E}\left\{ \sum_{t=1}^{T} c_{2t}(D_{2t} - X_2[t]) \right\} = \sum_{t=1}^{T} \mathbb{E} c_{2t}(D_{2t} - X_2[t])
\]

\[
= \sum_{t=1}^{T} \mathbb{E} c_{2t}(\hat{D}_{2t} - X_2[t] + \epsilon_{2t}) = \sum_{t=1}^{T} \int_{-\infty}^{+\infty} c_{2t}(\hat{D}_{2t} - X_2[t] + \tau) \rho_{2}(\tau) d\tau
\]

\[
= \sum_{t=1}^{T} \int_{-\infty}^{+\infty} c_{2t}(\hat{D}_{2t} - X_2[t] + \tau) d\Phi_{\tau}(\tau)
\]

\[
= \Omega_2 + \sum_{t=1}^{T} C_{2t}(\hat{D}_{2t} - X_2[t]), \quad (35)
\]
where $\rho_{2t}(\tau) = \frac{\partial}{\partial \tau} \Phi_t(\tau)$ denotes the density function of random variable $e_{2t} = D_{2t} - \hat{D}_{2t}$. The term $\Omega_2$ is a constant of integration.

Similarly,

$$E\left\{ \sum_{t=1}^{T} c_{1t}(D_{1t} + D_{2t} - X_1[t] - X_2[t]) \right\}$$

$$= \Omega_1 + \sum_{t=1}^{T} C_{1t}(\hat{D}_{1t} + \hat{D}_{2t} - X_1[t] - X_2[t]). \quad (36)$$

In our discussion, $\hat{D}_{it} = E(D_{it}) = a_{it}(i = 1, 2)$. Thus the equivalent deterministic problem is formulated as follows,

$$\text{DMP: } \min J = \Omega_1 + \Omega_2 + \sum_{i=1}^{2} \left\{ \sum_{t=0}^{T-1} p_{it}(u_i[t]) + \sum_{t=1}^{T} h_{it}(X_i[t]) \right\}$$

$$+ \sum_{t=1}^{T} \left\{ C_{1t}(a_{1t} + a_{2t} - X_1[t] - X_2[t]) + C_{2t}(a_{2t} - X_2[t]) \right\}, \quad (37)$$

s.t.

$$X_i[t+1] = X_i[t] + u_i[t], \quad t = 0, 1, 2, \cdots, T - 1, \quad (38)$$

$$X_1[t] + X_2[t] \geq a_{1t} + a_{2t} + \sqrt{\sigma_{1t}^2 + \sigma_{2t}^2 \Phi_t^{-1}(\eta_t)}, \quad (39)$$

$$X_2[t] \geq a_{2t} + \sigma_{2t} \Phi_t^{-1}(\eta_t), \quad (40)$$

$$\max\{u_i[t], 0\} \leq \delta_i(t)M_i, \quad i = 1, 2; \quad t = 0, 1, 2, \cdots, T - 1. \quad (41)$$

An important characteristic of expressions (37)-(41) is to preserve the structure properties of (8)-(14) as, e.g., the convexity (or concavity) of costs and the linearity of the constraint (Lassere [12]). These properties are very important because they allow solving of the equivalent problem in spite of its dimension.

### 4.2. Feedback Control Scheme

In the model (DMP), the quantity of employees may be overestimated for the far future periods of the planning horizon, so is the total cost. This difficulty can be explained by the essentially deterministic nature of the optimal mean solution. Because the state constraints (10), (39) and (40) depend strongly on the variance of demands, the lower bounds of states are to increase proportionally with $\sigma \in [1/2, 1)$. To overcome the difficulty, a feedback mechanism is applied in the model to control the evolution of state variance.
Based on concepts from the optimal control theory (Bryson [5]), a feedback scheme can be used to adjust the control variables to improve the system performance. For the problem (DMP), a feedback scheme with linear feedback rule (Silva [21]) is given as follows,

\[ u_1[t] = \hat{u}_1[t] + \kappa_{1t}(X_1[t] - \hat{X}_1[t]) + \kappa_{2t}(D_{1(t+1)} - \hat{D}_{1(t+1)}), \]  
\[ u_2[t] = \hat{u}_2[t] + \kappa_{1t}(X_2[t] - \hat{X}_2[t]) + \kappa_{2t}(D_{1(t+1)} + D_{2(t+1)} - \hat{D}_{1(t+1)} - \hat{D}_{2(t+1)}), \]  

(42)  
(43)

where \( \hat{\cdot} \) denotes the desirable demand, state or control determined from the mean problem, \( \kappa_{1t} \) and \( \kappa_{2t} \) denote the optimal linear gains and the terms \( \kappa_{1t}(\cdot) \) and \( \kappa_{2t}(\cdot) \) are adjustments according to the changes of states and demands respectively.

In (42), \( u_1[t] \) is now a function of demand \( D_{1t} \), so it is a stochastic variable with a similar distribution function as the demand \( D_{1t} \). In (43), the control \( u_2[t] \) is now a function of the demands \( D_{1t} \) and \( D_{2t} \), so it is a stochastic variable with a similar distribution function as the demands \( D_{1t} \) and \( D_{2t} \) since \( D_{1t} \) and \( D_{2t} \) are independent with each other and hold a same probability distribution. With the feedback controls (42) and (43), we get

\[ X_1[t] = \sum_{s=0}^{t-1} (1 + k_{1s})X_1[s] + \sum_{s=1}^{t} \left\{ \hat{u}_1[s] - \kappa_{1s}\hat{X}_1[s] + \kappa_{2s}(D_{1s} - \hat{D}_{1s}) \right\}, \]  
\[ X_2[t] = \sum_{s=0}^{t-1} (1 + k_{2s})X_2[s] + \sum_{s=1}^{t} \left\{ \hat{u}_2[s] - \kappa_{1s}\hat{X}_2[s] + \kappa_{2s}(D_{1s} + D_{2s} - \hat{D}_{1s} - \hat{D}_{2s}) \right\}. \]  

(44)  
(45)

Thus, states \( X_i[t] \) are independent of the demands \( D_{is} \) for \( s > t \). In (42) and (43), \( X_1[t] \) is independent of \( D_{1(t+1)} \) and \( X_2[t] \) is independent of \( D_{1(t+1)} \) and \( D_{2(t+1)} \), so we get the mean values \( E(u_i[t]) = \hat{u}_i[t] \) for \( i = 1, 2 \) and the variances

\[ \text{Var}(u_1[t]) = \kappa_{1t}^2\text{Var}(X_1[t]) + \kappa_{2t}^2\sigma_{1t}^2, \]  
\[ \text{Var}(u_2[t]) = \kappa_{1t}^2\text{Var}(X_2[t]) + \kappa_{2t}^2(\sigma_{1t}^2 + \sigma_{2t}^2). \]  

(46)  
(47)
From (44) and (45), we get the mean values and variances of states $X_i[t]$ as follows,

$$E(X_i[t+1]) = E(X_i[t]) + \hat{u}_i[t], \quad i = 1, 2,$$  \hspace{1cm} (48)

$$\text{Var}(X_1[t+1]) = (1 + \kappa_{1t})^2 \text{Var}(X_1[t]) + \kappa_{2t}^2 \sigma_{1t}^2,$$  \hspace{1cm} (49)

$$\text{Var}(X_2[t+1]) = (1 + \kappa_{1t})^2 \text{Var}(X_2[t]) + \kappa_{2t}^2(\sigma_{1t}^2 + \sigma_{2t}^2).$$  \hspace{1cm} (50)

Next we discuss how to determine the values of $\kappa_{1t}$ and $\kappa_{2t}$. The objective we expect is to minimize the variances of state $X_i[t]$ and control $u_i[t]$ simultaneously. At the same time, we should also minimize the gap between the state $X_i[t]$ and its corresponding demand $D_i[t]$ that is measured by $\text{Var}(X_i[t] - D_i[t])$.

The Chebyshev inequality is applied to determine an interval, where the maximal possibility of finding the true value of the random variable occurs. For the state and control variables, the Chebyshev inequalities can be written respectively as follows:

$$\text{Prob}\left\{ |X_i[t+1] - \hat{X}_i[t+1]| \geq \epsilon_t \right\} \leq \frac{\text{Var}(X_i[t+1])}{\epsilon_t^2}, \quad i = 1, 2$$  \hspace{1cm} (51)

and

$$\text{Prob}\left\{ |u_i[t] - \hat{u}_i[t]| \geq \zeta_t \right\} \leq \frac{\text{Var}(u_i[t])}{\zeta_t^2}, \quad i = 1, 2.$$  \hspace{1cm} (52)

Bearing in mind that the main objective is to minimize, simultaneously, the probability $\text{Prob}\left\{ |X_i[t+1] - \hat{X}_i[t+1]| \geq \epsilon_t \right\}$, $\text{Prob}\left\{ |u_i[t] - \hat{u}_i[t]| \geq \zeta_t \right\}$ and the variance $\text{Var}(X_i[t] - D_i[t])$. Then from (51) and (52), for $i = 1, 2$ we get

$$\min\left\{ \text{Prob}\left\{ |X_i[t+1] - \hat{X}_i[t+1]| \geq \epsilon_t \right\} + \kappa_{3t} \text{Var}(X_i[t] - D_i[t]) \right\} \iff \min\left\{ \frac{\text{Var}(X_i[t+1])}{\epsilon_t^2} + \frac{\text{Var}(u_i[t])}{\zeta_t^2} + \kappa_{3t} \text{Var}(X_i[t] - D_i[t]) \right\},$$  \hspace{1cm} (53)

where $\kappa_{3t}$ is a weight factor that indicates the importance of the term $\text{Var}(X_i[t] - D_i[t])$.

Note that the minimization on the right-hand side of (53) is equivalent to

$$\min\{ \text{Var}(X_i[t+1]) + \lambda_t \text{Var}(u_i[t]) + \kappa_{3t} \text{Var}(X_i[t] - D_i[t]) \} \lambda_t$$  \hspace{1cm} (54)
which represents a minimum variable problem (Åstrom [2]), whose solution provides the optimal linear gain $\kappa_{it}^*$. It can be computed to obtain the solution as follows,

$$
\kappa_{1t} = -\frac{1}{1 + \lambda_t + (\sigma_{1t}^2 + \sigma_{2t}^2)},
$$

(55)

$$
\kappa_{2t} = \frac{1}{1 + \lambda_t + (\sigma_{2t}^2 + \sigma_{2t}^2)},
$$

(56)

$$
\kappa_{3t} = \sigma_{1t}^2 + \sigma_{2t}^2,
$$

(57)

where $\lambda_t$ denotes the trade-off parameter between the state and control variables. This parameter is determined, for each period $t$, by the confidence interval of states $X_i[t]$ and controls $u_i[t]$ for $i = 1, 2$.

Due to the stochastic nature of control variable $u_i[t]$, the deterministic model should be modified. The costs $p_{it}(u_i[t])$ and $h_{it}(X_i[t])$ are also expected costs in the objective function.

$$
E \sum_{t=1}^{T} p_{it}(u_i[t]) = \Psi_i + \sum_{t=1}^{T} P_{it}(\hat{u}_i[t]), \quad i = 1, 2,
$$

(58)

$$
E \sum_{t=1}^{T} h_{it}(X_i[t]) = \Xi_i + \sum_{t=1}^{T} h_{it}(\hat{X}_i[t]), \quad i = 1, 2.
$$

(59)

Then the objective function becomes

$$
\min J = \sum_{i=1}^{2} \left\{ \sum_{t=0}^{T-1} P_{it}(\hat{u}_i[t]) \sum_{t=1}^{T} H_{it}(\hat{X}_i[t]) + C_{1t}(a_{1t} + a_{2t} - \hat{X}_1[t] - \hat{X}_2[t]) + C_{2t}(a_{2t} - \hat{X}_2[t]) + \Sigma_i \right\},
$$

(60)

where $\Sigma_i = \Omega_i + \Psi_i + \Xi_i (i = 1, 2)$ are integration constants which depend strongly on the corresponding second moments respectively.

The constraints need not to be modified. We label the mean value problem with a feedback control by IMP.

Because the state variances are weighted by the gain $\kappa_{it}$, they can be reduced with an appropriate value of $\kappa_{it}$.

The mean value problem (IMP) must provide a better solution than the problem (DMP). Note also that the problem (DMP) is a particular case of the mean value problem with the objective function (60).
4.3. Solving the Deterministic Model

For the general objective function, the deterministic problems (DMP) and (IMP) can be solved by dynamic programming approach. Especially, for the objective function is decomposable concerning the index of employees, to reduce the computational complexity, we can solve the deterministic problems through Lagrangean relaxation or Dwantzig-Wolfe approaches, which were discussed in (Cai et al [7] and [8]).

5. Sampling Model Approach

In order to model random demands, we can formulate a stochastic integer program wherein the demand distributions are represented by a collection of random scenarios. The technique of generating random scenarios is equivalent to Monte Carlo sampling and is often used in stochastic programming. The objective of the stochastic program is then to minimize the expected total cost over all the scenarios. We will let $L$ denote the number of scenarios and superscript each of the following variables by the scenario index $l$: $u^t_i$, $D^t_{it}$, $X^t_{1i}$, $X^t_{2i}$, and $\delta^t_i$. Each scenario may be given a probability weight $w_l$ such that $w_l > 0$ and $\sum_{l=1}^L w_l = 1$. In this case, we can set the confidence degrees $\eta_t = 1$, and the item $c_t(\cdot)$ of the objective function can be neglected. Thus we now get the following formulation for the problem that models the demand distributions using the $L$ scenarios:

$$
\min J = \sum_{l=1}^L w_l \left\{ \sum_{i=1}^2 \left[ \sum_{t=0}^{T-1} p_{it}(u^t_i) + \sum_{t=1}^T h_{it}(X^t_{1i}) \right] \right\}
$$

s.t.

$$
X^t_{1i}[t + 1] = X^t_{1i}[t] + u^t_i[t], \quad i = 1, 2, \quad t = 0, 1, \ldots, T - 1, \quad l = 1, 2, \ldots, L,
$$

$$
X^t_{1i}[t] + X^t_{2i}[t] \geq D^t_{1i} + D^t_{2i}, \quad t = 1, 2, \ldots, T, \quad l = 1, 2, \ldots, L,
$$

$$
X^t_{2i}[t] \geq D^t_{2i}, \quad t = 1, 2, \ldots, T, \quad l = 1, 2, \ldots, L,
$$

$$
\max \{ u^t_i[t], 0 \} \leq M_i \delta^t_i, \quad i = 1, 2, \quad t = 1, 2, \ldots, T, \quad l = 1, 2, \ldots, L,
$$

$$
\sum_{l=1}^L w_l = 1,
$$

$$
D^t_{it}, X^t_{1i}[t] \text{ are nonnegative integers, } i = 1, 2, \quad t = 1, 2, \ldots, T,
$$
$l = 1, 2, \cdots, L$.

This problem can be decomposed into $L$ subproblems which can be solved independently. As assumed above, demands are normally distributed with the given mean $a_{it}$ and standard deviation $\sigma_{it}^2$. The left tail of the distribution is truncated below 0, which slightly increases the mean above $a_{it}$ and slightly decreases the standard deviation $\sigma_{it}^2$. We can generate the yield coefficients $w_l$ for every scenario according to a beta distribution, correlated, and summed to 1. In fact, the approach can model any demand structure according to any distribution, whether correlated or not.

The choice of the number of scenarios $L$ is important when the scenarios in the model can only approximate the demand distributions. As the number $L$ of scenarios is increased, there is a trade-off between the increased computation time to solve the model and the improved accuracy as a result of a better approximation of the model. As has been studied in the theory of Monte Carlo sampling, statistical tests are required to determine if sufficient accuracy is obtained within the desired confidence interval.

6. Mixed Model

The optimal plan, found by assuming the stochastic behavior of the demands, can serve as a guideline for how the system should react to the demands. When there is no information about future demands, it is the best planning the system can follow. But when there is available information about the future, utilizing that information should yield a better plan. This is a very realistic situation where demand information about the future is partially known. The most natural situation is that for a certain period of time, say from period 1 to $S$, the demands are known exactly. Then after the period $S$, the best knowledge about the demands is the stochastic behavior. If we abruptly take the optimal contingency plan which was developed based on the stochastic assumptions, we would have wasted the information about the $S$ future periods. A better modelling strategy must come from the combination of the deterministic model and stochastic model. Thus we can describe the stochastic problem with a mixed model, where in the first $S$ periods, $D_{it}, t = 1, 2, \cdots S$ is deterministic, we describe it with a deterministic model. After the $S$-th period, $D_{it}, t > S$ is stochastic, we describe it as stochastic model described in (8)-(14).

This mixed model can be described as follows.
SOLVING MANPOWER PLANNING PROBLEM...

\[
\begin{align*}
\min J &= \sum_{i=1}^{2} \left\{ \sum_{t=0}^{S-1} p_{it}(u_i[t]) + \sum_{t=1}^{S} h_{it}(X_i[t]) \right\} \\
&\quad + \sum_{i=1}^{2} \mathbb{E} \left\{ \sum_{t=S+1}^{T} \left[ p_{it}(u_i[t]) + h_{it}(X_i[t]) + c_{it}(Z_i[t]) \right] \right\},
\end{align*}
\]

\( \text{s.t.} \)

\( X_i[t+1] = X_i[t] + u_i[t], \quad t = 0, 1, \cdots, T - 1, \) \( (68) \)

\( X_1[t] + X_2[t] \geq D_{1t} + D_{2t}, \quad t = 1, 2, \cdots, S, \) \( (69) \)

\( X_2[t] \geq D_{2t}, \quad t = 1, 2, \cdots, S, \) \( (70) \)

\( X_1[t] + X_2[t] + Z_1[t] \geq D_{1t} + D_{2t}, \quad t = S + 1, S + 2, \cdots, T, \) \( (71) \)

\( X_2[t] + Z_2[t] \geq D_{2t}, \quad t = S + 1, S + 2, \cdots, T, \) \( (72) \)

\( \text{Prob} \ (X_1[t] + X_2[t] \geq D_{1t} + D_{2t}) \geq \eta_t, \quad t = S + 1, S + 2, \cdots, T, \) \( (73) \)

\( \text{Prob} \ (X_2[t] \geq D_{2t}) \geq \eta_t, \quad t = S + 1, S + 2, \cdots, T, \) \( (74) \)

\( \max \{u_i[t], 0\} \leq M_i \delta_i(t), \quad t = 0, 1, \cdots, T - 1, \) \( (75) \)

\( D_{it}, X_i[t], Z_i[t] \) are nonnegative integers, \( t = 1, 2, \cdots, T. \)

The mixed model can be solved by a dynamic programming approach. We have the following recursive relations.

\[ f(0, 0, 0) = 0, \] \( (76) \)

\[ f(t, X_1[t], X_2[t]) = \sum_{i=1}^{2} \left[ \alpha_i X_i[t] + \min_{(u_1, u_2) \in U[t]} \left\{ \beta_i^+ \max \{u_i, 0\} + \beta_i^- \max \{-u_i, 0\} + f(t-1, X_1[t] - u_1, X_2[t] - u_2) \right\} \right], \] \( (77) \)

\( U[t] = \left\{(u_1, u_2)|D_{2(t-1)} \leq X_2[t] - u_2 \leq \max D_2, 0 \leq X_1[t] - u_1 \leq \max D_1, D_{1(t-1)} + D_{2(t-1)} \leq X_1[t] - u_1 + X_2[t] - u_2 \right\}, \) \( (78) \)

for \( t = 1, 2, \cdots, S - 1, \) and

\[ f_{S+1}(x_1, x_2) \]
The model is changed into the following deterministic one.

\[
\begin{align*}
&\{f_{S}(x_1 - u_1, x_2 - u_2) 
\quad \min_{(u_1, u_2) \in B_{S+1}(x_1, x_2)} \{f_{S}(x_1 - u_1, x_2 - u_2) \} 
\quad + \sum_{i=1}^{2} [h_i(D_{S+1})]_i + \sum_{i=1}^{2} (D_{S+1})_i 
\quad + D_{2(S+1)} - x_1 - x_2 + c_2(S+1) (D_{2(S+1)} - x_2),
\quad + \infty, \text{ otherwise},
\end{align*}
\]

for \((x_1, x_2) \in \mathbb{N}_{S+1}, (x_1 - u_1, x_2 - u_2) \in \mathbb{N}_{S},

\[
\begin{align*}
&f_t(x_1, x_2) = \left\{ \begin{array}{ll}
\min_{(u_1, u_2) \in B_{t}(x_1, x_2)} \{f_{t-1}(x_1 - u_1, x_2 - u_2) \} 
\quad + \sum_{i=1}^{2} [h_i(x_i)]_i + \sum_{i=1}^{2} (D_{i})_i 
\quad + D_{2t} - x_1 - x_2 + c_2(D_{2t} - x_2),
\quad + \infty, \text{ otherwise},
\end{array} \right.
\end{align*}
\]

for \((x_1, x_2) \in \mathbb{N}_{t}, (x_1 - u_1, x_2 - u_2) \in \mathbb{N}_{t-1} \text{ and } t = S + 2, S + 3, \cdots, T.

We can also adopt the above sample model approach to solve the mixed model. The model is changed into the following deterministic one.

\[
\begin{align*}
\min J = \sum_{i=1}^{2} \left\{ \sum_{t=0}^{S-1} \left\{ p_{it}(u_i[t]) + \sum_{t=1}^{S} h_i(D_{i}[t]) \right\} 
\quad + \sum_{t=0}^{L} w_l \left\{ \sum_{t=S+1}^{T} \left[ p_{it}(u_i[t]) + h_i(D_{i}[t]) \right] \right\} \right\},
\end{align*}
\]

s.t.

\[
\begin{align*}
X_i[t + 1] &= X_i[t] + u_i[t], \ t = 0, 1, \cdots, T - 1, 
X_1[t] + X_2[t] &\geq D_{1t} + D_{2t}, \ t = 1, 2, \cdots, S, 
X_2[t] &\geq D_{2t}, \ t = 1, 2, \cdots, S, 
X_1[l][t] + X_2[l][t] &\geq D_{1l}[t] + D_{2l}[t], \ t = S + 1, \cdots, T; \ l = 1, 2, \cdots, L, 
X_2[l][t] &\geq D_{2l}[t], \ t = S + 1, S + 2, \cdots, S; \ l = 1, 2, \cdots, L, 
\max\{u_i[t], 0\} &\leq M_i \delta(t), \ t = 0, 1, \cdots, S - 1, 
\max\{u_i[t], 0\} &\leq M_i \delta_i(t), \ t = S, S + 1, \cdots, T - 1, 
\sum_{l=1}^{L} w_l = 1, 
D_{is}, X_i[s] \text{ are nonnegative integers, } s = 1, 2, \cdots, S, 
D_{is}, X_i[l][t] \text{ are nonnegative integers, } t = S + 1, 2, \cdots, T.
\end{align*}
\]

Then we solve \(L\) deterministic problems independently in which the demands are same in the first \(S\) periods for all problems.
7. Applications of the Model

7.1. Applications

The optimal solution obtained for the stochastic manpower planning problem can be used in a two-fold way: one way is to provide aggregate manpower planning and the other way is to create manpower scenarios.

The manpower planning provides aggregate targets to be followed by detailed scheduling in future periods. Difficulties, e.g. low flexibility and lead time delays of the planning makes it very difficult for the manager to implement a consistent long-term planning. This long-term decision control cannot be applied completely in the future periods, since the quality of expected data (i.e. fluctuating demands, etc.) tends to deteriorate as they are taken further into the future. Consequently, the plan must be constantly revised on a rolling horizon basis, and only the first period (or first several periods) of the plan is to be effectively implemented.

As formulated, through whichever the deterministic model, a frozen, long-term aggregate plan can be provided. As discussed before, the solution tends to deteriorate over the time periods. Consequently, adapting the model to incorporate updated information in the near periods is required. This means it works more effectively on a rolling horizon basis.

Another way to use the near-optimal solution of the model is to create scenarios of the aggregate planning. By means of these scenarios, managers can get insights about how to make an employment plan to meet the demand on time. Then, varying the values of some parameters, it is possible to build different scenarios.

7.2. Simulation Results

We have studied the effectiveness of the feedback control scheme for the problem SMP by numerical experiments. The mean optimal solution can be used as target trajectories during the simulation process. A set of examples is solved through the approach we presented. The objective functions used in the examples are
\[ h_{it}(X_{it}) = \alpha_i X_{it} + b_i \delta_{it}, \]
\[ p_{it}(u_{it}) = \beta^+_i \max\{u_{it}, 0\} + \beta^-_i \max\{-u_{it}, 0\} \]
and
\[ c_{it}(X_{it}) = \gamma_i \max\{D_{it} - X_{it}, 0\}. \]
The cost coefficients are \( \alpha_1 = 800, \alpha_2 = 1000, \beta^+_1 = 1000, \beta^-_1 = 1500, \beta^+_2 = 1600, b_1 = 2000, b_2 = 4000, \gamma_1 = 4000 \) and \( \gamma_2 = 6000 \). The variances of the two demands are monotonically increasing along with the planning horizon and \( \eta_t = 0.6 \). A computational result obtained is illustrated in Figure 1, where the demand tra-
Figure 1: An example for the mean value model approach

...jectors are shown. States in the figure are the number of Full-time employees. The simulation trajectory is compared with mean optimal state trajectory.

The results from a set of experiments with different satisfactory chance $\eta_t$ show that the feedback control scheme has a better effect when the parameter $\eta_t$ becomes larger along with the entire planning horizon $T$.

Then a same example is computed through the mean value model approach and sampling model approach, respectively. In the sampling model approach, we let $L = 5$ and the coefficients $w_l$ are generated through a beta distribution $B(2, 2)$. The result is illustrated in Figure 2, which shows that the state trajectory obtained from the sampling approach are more closer to the mean optimal state trajectory than that obtained from the mean value model approach with the satisfactory chance $\eta_t > 0.5$.

8. Conclusions

The paper has described an aggregated manpower planning problem with stochastic demands. Due to the complex nature of the problem, mean value model approach and sampling model approach are presented as a deterministic alternative to obtain a sub-optimal solution to the stochastic problem respectively. The mixed model, not only is closest to the realistic situation, but also can provide near-optimal solution that is closer to the real optimal one. For a kind
of problem with a special objective function, the optimization method proposed for the deterministic problem can be applied to solve the approximated deterministic models. Many potential improvements and enrichments from various directions can be made to enhance the models' applicability. These models not only can be applied to solve the problem with any complicated objective function and with any probability distribution of demands, but also can be applied to solve the problems with more than two types of employee.

9. Acknowledgements

This project was supported by the National Natural Science Foundation of China (NSFC) under contract 60074018.

References


