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ON CONTINUOUS DEFORMATION OF RICHARDS FAMILY

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Abstract: In this paper we study on Richards family which is important curves in logistic research, for obtaining continuous deformation by equivalent relation.

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1. Introduction

An approximation problem to logistic models for fitting best model is very diffucult. Many authors asked: How we get fitting logistic model? The answers are generally depressing. In many cases the available computing algorithms for estimation had unsatisfactory convergence properties sometimes not covering at all. However, the general procedure of theoretical approximation and the

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relation to the growth curve and growth rate are very important role for general description. There is a new appreciation of the role of curvature in sigmodial curves and its effect on inferential procedures. Curvature cames in a two-piece suite: intrinsic curvature, which relates to the geometry of the logistic model, and parameter-effects curvature, which depends on the parametrization of the model. For many types of growth data, the growth rate does not steadily decline, but rather increases to a maximum before steadily declining to zero. Such a model adds another recognizable feature to the curve, the position of the point of inflection being the time when the growth rate is greatest in.

For many models described in the literature achieve this sigmoidal behavior by modeling the current growth rate as the product of functions of the current size and remaining growth, namely

$$\frac{df}{dx} = g\,(f)\,\{h\,(\alpha)\; - \; h\,(f)\}\ , \tag{1}$$

where g and h are increasing functions with $g\,(0) = h\,(0) = 0$. This is a general concept which is described by Seber and Wild [1]. The simplest form of (1) is with $g\,(f) = h\,(f) = f$, so that we obtained logistic model by $\frac{df}{dx} = \frac{k}{\alpha} f\,(\alpha - f)$.

An obvious way of describing a sigmoidal shape is to use the distribution function $F\,(x\,;\theta\,)$ of an absolutely continuous random variable with a unimodial distribution. Then $f\,(x) = \alpha\,F\,(\,kx\,;\,\theta\,)$ gives us a sigmoidal growth curve for emprical use. The parametric family $F\,(x\,;\theta\,)$ can be chosen to give as much flexibility as desired. The Richards family defined by

$$F\,(\,x\,;\,m\,) = \left\{1 + (m-1)\;e^{-x}\right\}^{1/(1-m)}. \tag{2}$$

Imposing (1) we can write Richards family as

$$\frac{df}{dx} = \frac{k}{(m-1)\,\alpha^{m-1}} f\left(\alpha^{m-1}\; - \;f^{m-1}\right)\ , \quad m \neq 1\,, \tag{3}$$

where k, α and m are real parameters (see Seber and Wild [1]). The Richards curves may be defined continuous deformation by choosing a real parameter m.

2. Continuous Deformation of Richards Family

Let $S\, =\, \{(\,x\,;\,y\,)\,:\,x \in IR,\ 0 \le y \le \alpha\,\}$ be a closed subset of IR^2, (2) with $m \neq 1$ and $I = [\,0\,,\,1\,]$. Suppose T is a continuous deformation defined by

$$\begin{aligned} &T\;(x,t) : IR * I \to S\,, \\ &(x,t) \to T\,(x,t) = f_t\,(x)\ , \end{aligned} \tag{4}$$

where $f_0(x) = F(x;\tau = 0)$ and $f_1(x) = F(x;\tau = 1)$ with $\tau = 1/(1+e^{-m})$. There is an important result for Richards family which is obtained the optimal Richards curve when the starting curve $f_0(x)$ tends to final curve $f_1(x)$. Then f_0 equivalence to f_1 denote by $f_0 \sim f_1$ if there is some T, see Spainer [2]. The collection $\{f_t(x)\}_t$ is a defined continuous mapping of Richards family. Certainly the optimal Richards curve belong to the collection $\{f_t(x)\}_t$ and problem of fitting best logistic model tends to optimization problem of Richards family on $\{f_t(x)\}_t$.

Result 1. Maps of Richards family are an equivalence relation in the $\{f_t(x)\}_t$.

Result 2. Let $t_1 > t_2 > \cdots > t_k > \cdots$ be a sequence of real numbers such that $0 < t_i < 1$ for all and $E_i^k = \{x : f_{t_i}(x) < k, k \in IR\}$. Then $E_1^k \subset E_2^k \subset \cdots$. Note that every E_i^k be a measurable set and every A is a subset such that $A = \bigcup_i u_i$ with $u_i = (E_i - E_{i-1})\,A$.

Result 3. The collection $\{f_t(x)\}_t$ is a convex.

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