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# MATRIX RATIO TEST FOR FINDING BASIC FEASIBLE SOLUTION WITHOUT ARTIFICIAL VARIABLE(S) IN LP PROBLEMS

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**Abstract:** Note, that each artificial variable creates one extra dimension for LP problem. In this paper, we show how can find Basic Feasible Solution *without adding artificial variable(s).*

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## 1. Introduction

Consider the following LP problem:

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$$\begin{aligned} \text{Min} \quad & cx \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

that $A$ is the $m \times n$ Matrix and Rank$(A) = m$.

Suppose that we can partition the constraint matrix $A$ into $A = [B, N]$. Where $B$ is a Basic matrix, not necessarily feasible.

We known three methods to initiate the simplex method algorithms, by the use of artificial variable(s):

i) Big_M method [3, Section 2.7].

ii) Two phase Method [3, Section 2.4].

iii) Single variable technique [1, Section 4.4], [2, Section 4.5] and [2, Exercise 4.33].

We want to show in many problems, *with our technique,* we can find *basic feasible solution,* without using artificial variable(s).

## 2. Maximum Ratio Test (MRT)

The goal is to find *basic feasible solution.* Without adding artificial variable(s). Consider following tableau:

| | $x_k$ | RHS |
|---|---|---|
| | $y_{1k}$ | $\bar{b}_1$ |
| . | . | . |
| . | . | . |
| $x_{B_p}$ | $y_{pk}$ | $\bar{b}_p$ |
| . | . | . |
| . | . | . |
| . | $y_{rk}$ | $\bar{b}_r$ |
| . | $y_{rk}$ | $\bar{b}_r$ |
| . | $y_{(r+1)k}$ | $\bar{b}_{r+1}$ |
| . | . | . |
| . | . | . |
| . | $y_{mk}$ | $\bar{b}_m$ |

Suppose $\bar{b} \ngeq 0$:

If $\bar{b}_i < 0$ then $i \in J_1 = \{1, \dots, r\}$.

If $\bar{b}_i \geq 0$ then $i \in J_2 = \{r+1, \dots, m\}$.

In tableau, if there was $y_k$ subject to $y_{ik} < 0$ for $i \in J_1$ and $y_{ik} \leq 0$ for $i \in J_2$. We can use MRT to find a basic feasible solution:

$$\frac{\bar{b}_p}{y_{pk}} = \text{ Maximum } \{\frac{\bar{b}_i}{y_{ik}}, i \in J_1\}.$$

## 3. Validation of MRT

In previous tableau, if $x_k$ enters the basic and $x_{B_p}$, leaves the basic, then pivoting can be done on $y_{pk}$:

| | $x_k$ | RHS |
|---|---|---|
| . | 0 | . |
| . | . | . |
| $x_k$ | 1 | $\frac{\bar{b}_p}{y_{pk}}$ |
| . | . | . |
| . | . | . |
| $x_{B_i}$ | . | $\bar{b}_i - \frac{\bar{b}_p}{y_{pk}} y_{ik}$ |
| . | . | . |
| . | 0 | . |

We show that after pivoting, right hand side becomes positive:

1) $p \in J_1 \Longrightarrow \frac{\bar{b}_p}{y_{pk}} > 0.$

2) $p \in J_1$ & $i \in J_1 \Longrightarrow \bar{b}_i - \frac{\bar{b}_p}{y_{pk}} y_{ik} = y_{ik}(\frac{\bar{b}_i}{y_{ik}} - \frac{\bar{b}_p}{y_{pk}}) \geq 0.$

3) $p \in J_1$ & $i \in J_2 \Longrightarrow \bar{b}_i - \frac{\bar{b}_p}{y_{pk}} y_{ik} = y_{ik}(\frac{\bar{b}_i}{y_{ik}} - \frac{\bar{b}_p}{y_{pk}}) \geq 0.$

**Corollary.** *Each basic solution that has got this $y_k$, its problem is feasible.*

## 4. Numerical Example

**Example.** Solve the following problem:

$$\begin{aligned} \text{Maximum} \quad & 2x_2 - x_2 + x_3, \\ s.t. \quad & x_1 + x_2 - x_3 \geq 4, \\ & x_1 - 4x_2 + x_3 \leq 2, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

$$\begin{aligned} \text{Maximum} \quad & 2x_1 - x_2 + x_3, \\ s.t. \quad & -x_1 - x_2 + x_3 \leq -4, \\ & x_1 - 4x_2 + x_3 \leq 2, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | RHS |
|---|---|---|---|---|---|---|
| $z$ | -2 | 1 | -1 | 0 | 0 | 0 |
| $x_4$ | -1 | -1 | 1 | 1 | 0 | -4 |
| $x_5$ | 1 | -4 | 1 | 0 | 1 | 2 |

We enter $x_2$ in to the basic and with MRT $x_4$ leave the basic.

| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | RHS |
|---|---|---|---|---|---|---|
| $z$ | -3 | 0 | 0 | 1 | 0 | -4 |
| $x_2$ | 1 | 1 | -1 | -1 | 0 | 4 |
| $x_5$ | 5 | 0 | -3 | -4 | 1 | 18 |

RHS becames positive, and we can solve the problem with simplex method.

## 5. Conclusion

In many basic solutions, we do not have such a $y_k$, that defined in Section 2, and our technique can be used just for basic solutions that have such a $y_k$.

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