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# STABLE COHERENT SYSTEMS ON INTEGRAL PROJECTIVE VARIETIES: AN ASYMPTOTIC EXISTENCE THEOREM

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**Abstract:** Fix integers $k > n \geq 2$, an integral projective variety $X$, a rank $n$ vector bundle $E$ on $X$ and an ample line bundle $H$ on $X$. Here we prove the existence of an integer $t_0$ (depending only from $k, n, X, H, E$) such that for all integers $t \geq t_0$ a general $k$-dimensional linear subspace $V$ of $E(tH)$ spans $E(tH)$, the coherent system $(E, V)$ is $\alpha$-$H$-stable for every $\alpha \gg 0$ and the natural map $\bigwedge^n(V) \to H^0(X, \det(E)(ntH))$ is injective.

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**Key Words:** coherent system, stable vector bundle, stable coherent system

## 1. Stable Coherent Systems

See [2] for the general theory of coherent systems. Here we use the results and methods of [1] (concerning the one-dimensional case) to obtain same results for higher dimensional integral projective varieties, i.e. we will prove the following results.

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**Theorem 1.** *Fix integers $k > n \geq 2$, an integral projective variety $X$, a rank $n$ vector bundle $E$ on $X$ and an ample line bundle $H$ on $X$. There is an integer $t_0$ (depending only from $k, n, X, H, E$) such that for all integers $t \geq t_0$ a general $k$-dimensional linear subspace $V$ of $H^0(X, E(tH))$ spans $E(tH)$ and the coherent system $(E(tH), V)$ is $\alpha$-$H$-stable for every $\alpha \gg 0$.*

**Theorem 2.** *Fix integers $k > n \geq 2$, an integral projective variety $X$, a rank $n$ vector bundle $E$ on $X$ and an ample line bundle $H$ on $X$. There is an integer $t_0$ (depending only from $k, n, X, H, E$) such that for all integers $t \geq t_0$ a general $k$-dimensional linear subspace $V$ of $E(tH)$ spans $E(tH)$ and the natural map $u_{(E,V)} : \bigwedge^n(V) \to H^0(X, \det(E)(ntH))$ is injective.*

As in [1] we could easily show that Theorem 2 is an direct consequence of Theorem 1, but we prefer to show that both results are easy consequences of the case $\dim(X) = 1$ proved in [1].

*Proofs of Theorem 1 and Theorem 2.* Since the case $\dim(X) = 1$ is just [1], Theorem 1, we may assume $\dim(X) \geq 2$. Fix an integral curve $C \subset X$. Since $H$ is ample, there is an integer $t_1 \geq 0$ (depending only from $E$ and $H$) such that $E(tH)$ is spanned and $h^i(X, E(tH)) = 0$ for all $i > 0$ and all integers $t \geq t_1$. Since a multiple of $H$ is very ample, it is easy to check the existence of an integer $t_2 \geq t_1$ such that $h^1(X, \mathcal{I}_C \otimes E(tH)) = 0$ for all integers $t \geq t_2$. Since $C$ is fixed, the integer $t_2$ depends only from $E$ and $H$. Hence for all integers $t \geq t_2$ the restriction map $\rho_{E(tH),C} : H^0(X, E(tH)) \to H^0(C, E(tH)|C)$ is surjective. This implies that $\dim(\rho_{E(tH),C}(V) = \min\{k, h^0(C, E(tH)|C)\}$ for a general $k$-dimensional linear subspace $V \subseteq H^0(X, E(tH))$. By [1], Theorem 2, there is an integer $t_0 \geq t_2$ depending only from $E|C$ and $H|C$, such that Theorem 2 is true for $C$ with this integer $t_0$ and $h^0(C, E(tH)|C) \geq k$ for all integers $t \geq t_0$. Since $C$ is fixed, this integer $t_0$ depends only from $E$ and $H$. Since $\dim(\rho_{E(tH),C}(V) = \min\{k, h^0(C, E(tH)|C)\}$ for a general $k$-dimensional linear subspace $V \subseteq H^0(X, E(tH))$, we immediately get Theorem 2 for $X$. To obtain Theorem 1 use a covering family $\mathcal{F}$ of integral curve of $X$ instead of just one curve and then apply [1], Theorem 1, to $E|C$ and $H|C$ for all $C \in \mathcal{F}$. By semicontinuity (or the proofs in [1]) we may find an integers $t_0$ which works for all $C \in \mathcal{F}$.

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## References

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