International Journal of Pure and Applied Mathematics
Volume 20 No. 1 2005, 97-107

NEW TRUNCATED EXPANSION METHOD AND SOLITON-LIKE SOLUTION OF STOCHASTIC KdV-MKdV EQUATION

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Abstract: In this paper, the Wick-type stochastic KdV-MKdV equation is researched. New exact soliton-like solution is shown by using new truncated expansion method and Hermite transformation.

AMS Subject Classification: 47J35, 34G20,35K90
Key Words: stochastic KdV-MKdV equation, stochastic soliton solution, white noise, truncation expansion method, Hermite transformation

1. Introduction

It is well known that KdV equation is one of the most popular soliton equations. It describes shallow water, acoustic wave in anhormonic lattice, ion acoustic wave, magnetio-hydrodynamic wave in plasma, etc. However, the solitons are stable against mutual collisions and behave like particles. In [10], M. Wadati first answered the interesting question, "how does external noise affect the motion of solitons?" and studied the diffusion of soliton of the KdV equation under Gaussian noise, which satisfies a diffusion equation in transformed coordinates. M. Wadati also studied the behaviors of solitons under the Gaussian white noise of the stochastic KdV equations with and without damping [11].

Received: March 6, 2005

Recently, many researchers pay more attention to the study of the random waves, which are important subjects of stochastic partial differential equation. A number of soliton solutions of nonlinear stochastic partial differential equations have been obtained by many authors [2], [3], [5], [6], [8], [9], [12], [7], [4]. In [7], Holden et al used the white noise functional approach to study stochastic partial differential equations in Wick versions.

In this paper, we consider the Wick-type stochastic KdV-MKdV equation in following form:

$$U_t + K_1(t)\diamondsuit[U_{xxx} - a_1 U^{\diamond 2}\diamondsuit U_x + 2a_2(U_x^{\diamond 2} + U\diamondsuit U_{xx})] + a_3 K_2(t)\diamondsuit H(t)\diamondsuit U\diamondsuit U_x \\ + [K_2(t) + K_3(t)x]\diamondsuit U_x + K_3(t)\diamondsuit U = 0, \quad (1.1)$$

where $K_i(t)$ $(i = 1,2,3)$ are white noise functionals, $H(t) = \exp^{\diamond}[-\int^t K_3(s)ds]$, and $\diamondsuit$ is the Wick product on the Hida distribution space $(S(\mathbb{R}^d))^*$ which will be defined in the second section. Chen et al used homogenous balance principle to study equation (1.1) and gave some exact solutions, see [4]. In this paper we will use new truncated expansion method to study equation (1.1) and give soliton-like solution.

The rest of the paper is organized as follows. In next Section, we give some basic concepts on white noise in SPDEs. In Section 3, we apply the method to study the Wick-type stochastic KdV-MKdV equation and bring out some soliton-like solutions to the equation. The conclusion is presented in the last Section.

2. Some Basic Concepts on White Noise in SPDEs

In this section we will summarize the main matters for stochastic partial differential equations which use white noise functional approach. More detail, please see Holden et al, see [13].

Suppose that $(S(\mathbb{R}^d))$ and $(S(\mathbb{R}^d))^*$ are the Hida test function space and the Hida distribution space on $\mathbb{R}^d$, respectively. Let $h_n(x)$ be the d-order Hermite polynomials and put

$$\xi_n(x) = e^{-\frac{1}{2}x^2} h_n(\sqrt{2}x)/(\pi(n-1)!)^{1/2}, \quad n \geq 1,$$

then, the collection $\{\xi_n\}_{n\geq 1}$ constitutes an orthogonal basis for $L^2(\mathbb{R})$.

Let $\alpha = (\alpha_1, \cdots, \alpha_d)$ denote d-dimensional multi-indices with $\alpha_1, \cdots, \alpha_d \in \mathbb{N}$. The family of tensor products $\xi_\alpha = \xi_{(\alpha_1,\cdots,\alpha_d)} = \xi_{\alpha_1}\otimes\cdots\otimes\xi_{\alpha_d}(\alpha \in \mathbb{N}^d)$ forms an orthogonal basis for $L^2(\mathbb{R})$. Suppose that $\alpha^{(i)} = (\alpha_1^{(i)}, \cdots, \alpha_d^{(i)})$ is the i-th multi-index number in some fixed ordering of all d-dimensional multi-indices α. We can, and will, assume that this ordering has the property that

$$i < j \Rightarrow \alpha_1^{(i)} + \cdots + \alpha_d^{(i)} \leq \alpha_1^{(j)} + \cdots + \alpha_d^{(j)},$$

i.e., the $\{\alpha^{(j)}\}_{j=1}^{\infty}$ occurs in an increasing order. Now define

$$\eta_i = \xi_{\alpha^{(i)}} = \xi_{\alpha_1^{(i)}} \otimes \cdots \otimes \xi_{\alpha_d^{(i)}}, \quad i \geq 1.$$

We need to consider multi-indices of arbitrary length. For simplification of notation, we regard multi-indices as elements of the space $(\mathbb{N}_0^{\mathbb{N}})_c$ of all sequences $\alpha = (\alpha_1, \alpha_2, ...)$ with elements $\alpha_i \in \mathbb{N}_0$ and with compact support, i.e., with only finitely many $\alpha_i \neq 0$. We write $\jmath = (\mathbb{N}_0^{\mathbb{N}})_c$, for $\alpha \in \jmath$, define

$$H_\alpha(\omega) = \prod_{i=1}^{\infty} h_{\alpha_i}(\langle \omega, \eta_i \rangle), \quad \omega = (\omega_1, \cdots, \omega_m) \in (S(\mathbb{R}^d))^*.$$

For a fixed $n \in \mathbb{N}$ and $\forall k \in \mathbb{N}$, suppose the space $(\mathbb{S})_1^n$ consists of those $f(\omega) = \sum_\alpha c_\alpha H_\alpha(\omega) \in \bigoplus_{k=1}^n L^2(\mu)$ with $c_\alpha \in \mathbb{R}^n$ such that $||f||_{1,k}^2 = \sum_\alpha c_\alpha^2 (\alpha!)^2 (2\mathbb{N})^{k\alpha} < \infty$, where $c_\alpha^2 = |c_\alpha|^2 = \sum_{k=1}^n (c_\alpha^{(k)})^2$ if $c_\alpha = (c_\alpha^{(1)}, ..., c_\alpha^{(n)}) \in \mathbb{R}^n$, and μ is the white noise measure on $(S^*(\mathbb{R}), \mathbf{B}(S^*(\mathbb{R})),$ $\alpha! = \prod_{k=1}^{\infty} \alpha_k!$ and $(2\mathbb{N})^\alpha = \prod_j (2j)^{\alpha_j}$ for $\alpha \in \jmath$.

The space $(S)_{-1}^n$ consists of all formal expansions $F(\omega) = \sum_\alpha b_\alpha H_\alpha(\omega)$ with $b_\alpha \in \mathbb{R}^n$ such that $||F||_{-1,-q} = \sum_\alpha b_\alpha^2 (2\mathbb{N})^{-q\alpha} < \infty$ for some $q \in \mathbb{N}$. The family of seminorms $||f||_{1,k}, k \in \mathbb{N}$ gives rise to a topology on $(S)_1^n$, and we can regard $(S)_{-1}^n$ as the dual of $(S)_1^n$ by the action

$$\langle F, f \rangle = \sum_\alpha (b_\alpha, c_\alpha) \alpha!,$$

where (b_α, c_α) is the ordinary inner product in $\mathbb{R}^n$.

The Wick product $f \diamondsuit F$ of two elements $f = \sum_\alpha a_\alpha H_\alpha$, $F = \sum_\alpha b_\alpha H_\alpha \in (S)_{-1}^n$ with $a_\alpha, b_\alpha \in \mathbb{R}^n$, is defined by

$$f \diamondsuit F = \sum_{\alpha,\beta} (a_\alpha, b_\beta) H_{\alpha+\beta}.$$

We can prove that the spaces $(S(\mathbb{R}^d)), (S(\mathbb{R}^d))^*, (S)_1$, and $(S)_{-1}$ are closed under Wick products.

For $F = \sum_\alpha b_\alpha H_\alpha \in (S)_{-1}^N$, with $b_\alpha \in \mathbb{R}^N$, the Hermite transformation of F, denoted by $\mathcal{H}(F)$ or $\widetilde{F}$ is defined by

$$\mathcal{H}(F) = \widetilde{F}(z) = \sum_\alpha b_\alpha z^\alpha \in \mathbb{C}^N \text{ (when convergent)},$$

where $z = (z_1, z_2, ...) \in \mathbb{C}^\mathbb{N}$ (the set of all sequences of complex numbers) and $z^\alpha = z_1^{\alpha_1} z_2^{\alpha_2} \cdots\ z_n^{\alpha_n} \cdots$, if $\alpha \in \jmath$, where $z_j^0 = 1$.

For $F, G \in (S)_{-1}^N$, by this definition we have

$$\widetilde{F \diamond G}(z) = \widetilde{F}(z) \cdot \widetilde{G}(z)$$

for all z such that $\widetilde{F}(z)$ and $\widetilde{G}(z)$ exist. The product on the right-hand side of the above formula is the complex bilinear product between two elements of $\mathbb{C}^\mathbb{N}$ defined by $(z_1^1, ..., z_n^1) \cdot (z_1^2, ..., z_n^2) = \sum_{k=1}^n z_k^1 z_k^2$, where $z \in \mathbb{C}$.

Let $X = \sum_\alpha a_\alpha H_\alpha \in (S)_{-1}^N$, then the vector $c_0 = \tilde{X}(0) \in \mathbb{R}^N$ is called the generalized expectation of X which denoted by $E(X)$. Suppose that $g : U \to \mathbb{C}^M$ is an analytic function, where U is a neighborhood of $\xi_0 := E(X)$. Assume that the Taylor series of g around ξ_0 have coefficients in $\mathbb{R}^M$. Then the Wick version $g^\diamond(X) = \mathcal{H}^{-1}(g \circ \widetilde{X}) \in (S)_{-1}^M$. In other words, if g has the power series expansion $g(z) = \sum a_\alpha (z - \xi_0)^\alpha$ with $a_\alpha \in \mathbb{R}^M$, then $g^\diamond(X) = \sum a_\alpha (X - \xi_0)^{\diamond\alpha} \in (S)_{-1}^M$.

Suppose that modelling consideration leads us to consider an SPDE as follows:

$$A(t, x, \partial t, \nabla x, U, \omega) = 0, \tag{2.1}$$

where A is some given function, $U = U(t, x, \omega)$ is an unknown (generalized) stochastic process, and the operators $\partial t = \frac{\partial}{\partial t}, \nabla x = (\frac{\partial}{\partial x_1}, ..., \frac{\partial}{\partial x_d})$ when $x = (x_1, ..., x_d) \in \mathbb{R}^d$.

Firstly, we interpret all products as Wick products and all functions as their Wick versions. Wick version of equation (2.1) is written as follows:

$$A^\diamond(t, x, \partial t, \nabla x, U, \omega) = 0. \tag{2.2}$$

Secondly, we take the Hermite transformation of equation (2.1), which turns Wick products into ordinary products (between complex numbers), so the equation takes the form

$$\tilde{A}(t, x, \partial t, \nabla x, \tilde{U}, z_1, z_2, ...) = 0, \tag{2.3}$$

where $\tilde{U} = \mathcal{H}(U)$ is the Hermite transformation of U and $z_1, z_2, ...$ are complex numbers. Suppose that we can find a solution $u = u(t, x, z)$ of equation (2.3) for each $z \in \mathbb{K}_q(r)$, where $\mathbb{K}_q(r) = \{z \in \mathbb{C}^{\mathbb{N}}$ and $\sum_{\alpha \neq 0} |z^\alpha|^2 (2\mathbb{N})^{q\alpha} < r^2\}$ for some q, r. Then, under certain conditions, we can take the inverse Hermite transformation $U = \mathcal{H}^{-1} u \in (S)_{-1}$ and thereby obtain a solution U of the original Wick equation (2.2). We have the following theorem, which was proved by Holden et al [7].

Theorem 2.1. *Suppose u(t,x,z) is a solution (in the usual strong, pointwise sense) of equation (2.2) for (t, x) in some bounded open set $G \subset \mathbb{R} \times \mathbb{R}^d$, and for all $z \in K_q(r)$, for some q, r. Moreover, suppose that $u(t, x, z)$ and all its partial derivatives, which are involved in equation (2.2), are (uniformly) bounded for $(t, x, z) \in G \times K_q(r)$, continuous with respect to $(t, x) \in G$ for all $z \in K_q(r)$ and analytic with respect to $z \in K_q(r)$, for all $(t, x) \in G$. Then there exists $U(t, x) \in (S)_{-1}$ such that $u(t, x, z) = U(t, x))(z)$ for all $(t, x, z) \in G \times K_q(r)$ and $U(t, x)$ solves (in the strong sense in $(S)_{-1}$) equation (2.1) in $(S)_{-1}$.*

3. New Exact Soliton-Like Solution of Wick-Type Stochastic KdV-MKdV Equation

In this section, we will give exact solutions of equation (1.1). Taking the Hermite transformation of equation (1.1), we get the following equation,

$$\widetilde{U}_t + \widetilde{K}_1(t)[\widetilde{U}_{xxx} - a_1 \widetilde{U}^2 \widetilde{U}_x + 2a_2(\widetilde{U}_x^2 + \widetilde{U}\widetilde{U}_{xx})] + a_3 \widetilde{K}_2(t)\widetilde{H}(t)\widetilde{U}\widetilde{U}_x + [\widetilde{K}_2(t) + \widetilde{K}_3(t)x]\widetilde{U}_x + \widetilde{K}_3(t)\widetilde{U} = 0, \quad (3.1)$$

where $z = (z_1, z_2, \cdots) \in (\mathbb{C}^{\mathbb{N}})_c$ is a vector parameter.

Now we use new truncated expansion method (see [14]) to solve equation (3.1), for the sake of simplicity, let us denote $u(t, x, z) = \widetilde{U}(t, x, z)$, $K_i(t, z) = \widetilde{K}_i(t, z)(i = 1, 2, 3)$ and $H(t, z) = \widetilde{H}(t, z)$.

Let the equation (3.1) have the formal solutions

$$\begin{aligned} &u = u(t, x, z) = \sum_{i=0}^{n} v_i(t, z) F^i, \quad F = F(\xi) = \frac{1}{1 + e^\xi}, \\ &\xi = f(t, z)x + g(t, z), \end{aligned} \quad (3.2)$$

where $v_i(t, z)$ $(i = 1, \cdots, n)$, $f(t, z)$, $g(t, z)$ are functions to be determined later.

Substituting equation (3.2) into equation (3.1), it is easy to find that $n = 1$ by balancing the highest order nonlinear term and linear term in equation (3.1). Therefore, it is obtained that

$$u = v_0(t, z) + v_1(t, z)F. \quad (3.3)$$

From equation (3.3), we can obtain

$$\begin{cases} u_t = v_{0t} + v_{1t}F + v_1\xi_t F^2 - v_1\xi_t F, \\ u_x = v_1\xi_x F^2 - v_1\xi_x F, \\ u_{xx} = 2v_1\xi_x^2 F^3 - 3v_1\xi_x^2 F^2 + v_1\xi_t F^2, \\ u_{xxx} = 6v_1\xi_x^3 F^4 - 12v_1\xi_x^3 F^3 + 7v_1\xi_x^3 F^2 - v_1\xi_x^3 F. \end{cases} \tag{3.4}$$

Substituting equation (3.4) into equation (3.1) and setting the coefficient of F power times to zero, give rise the differential equations:

$$F^4 : 6K_1v_1\xi_x^3 - a_1K_1v_1^3\xi_x - 2a_2K_1v_1^2\xi_x^2 = 0, \tag{3.5}$$

$$\begin{aligned} F^3 : 12K_1v_1\xi_x^3 - a_1K_1v_1^3\xi_x - 2a_2K_1v_1^2\xi_x^2 + 4a_2K_1v_0v_1\xi_x^2 \\ - a_3HK_1v_1^2\xi_x + 2a_1K_1v_0v_1^2\xi_x^2 = 0, \end{aligned} \tag{3.6}$$

$$\begin{aligned} F^2 : v_1\xi_t + K_3xv_1\xi_x + K_2v_1\xi_x + 7K_1v_1\xi_x^3 - a_1K_1v_0^2v_1\xi_x + 2a_1K_1v_0v_1^2\xi_x \\ + 6a_2K_1v_0v_1\xi_x^2 + a_3HK_1v_0v_1\xi_x^2 - a_3HK_1v_1^2\xi_x = 0, \end{aligned} \tag{3.7}$$

$$\begin{aligned} F : v_{1t} - v_1\xi_t + K_3v_1 - K_3xv_1\xi_x - K_2v_1\xi_x - a_3K_1v_0v_1\xi_x \\ + 2a_2K_1v_0v_1\xi_x^2 + a_1K_1v_0^2v_1\xi_x - K_1v_1\xi_x^3 = 0, \end{aligned} \tag{3.8}$$

$$F^0 : v_{0t} + 2K_3v_0 = 0. \tag{3.9}$$

From equation (3.5) and equation (3.6), we obtain

$$v_1 = \frac{6\xi_x^2 + 4a_2v_0\xi_x}{a_3H - 2a_1v_0}. \tag{3.10}$$

From equation (3.7), equation (3.8) and equation (3.10), we get

$$v_1(t,z) = c_1 \exp(-\int^t K_3(s,z)ds), \tag{3.11}$$

where c_1 is a constant to be determined later. Solving equation (3.9) yields

$$v_0(t,z) = c_0 \exp(-\int^t K_3(s,z)ds). \tag{3.12}$$

From equation (3.7), we also obtain

$$\xi_t = -K_3 x\xi_x - K_2\xi_x - 7K_1\xi_x^3 + a_1K_1v_0^2\xi_x - 2a_1K_1v_0v_1\xi_x \\ - 6a_2K_1v_0\xi_x^2 - a_3K_1Hv_0\xi_x + a_3HK_1v_1\xi_x. \quad (3.13)$$

Since $\xi_x = f(t,z), \xi_t = f_t(t,z)x + g_t(t,z)$, we have

$$f_t(t,z) = K_3(t,z)f(t,z), \quad (3.14)$$

and

$$g_t(t,z) = -K_2\xi_x - 7K_1\xi_x^3 + a_1K_1v_0^2\xi_x - 2a_1K_1v_0v_1\xi_x \\ - 6a_2K_1v_0\xi_x^2 - a_3HK_1v_0\xi_x + a_3HK_1v_1\xi_x. \quad (3.15)$$

Solving equation (3.14) yields

$$f(t,z) = \lambda\exp(-\int^t K_3(s,z)ds), \quad (3.16)$$

where c_0 and λ are arbitrary integrable constants. Substituting equation (3.12), equation (3.16) and $H(t,z)$ into equation (3.10), and comparing it with equation (3.11), we have

$$c_1 = (6\lambda^2 + 4a_2c_0\lambda)/(a_3 - 2a_1c_0). \quad (3.17)$$

From equation (3.15) and equation (3.16), we obtain

$$g(t,z) = \int^t \Big\{ -\lambda K_2(s,z)\exp(-\int K_3(\tau,z)d\tau) - \lambda(\lambda^2 + 2\lambda a_2c_0 \\ - a_1c_0^2 + a_3c_0)K_1(s,z)\exp(-\int^s 3K_3(\tau,z)d\tau)\Big\} ds + c_3, \quad (3.18)$$

where c_3 is an arbitrary constant.

Substituting equation (3.13) and equation (3.18) into equation (3.2), we obtain

$$\xi = \lambda\exp(-\int^t K_2(s,z)ds)x + \int^t \Big\{ -\lambda K_3(s,z)\exp(-\int K_3(\tau,z)d\tau) \\ - \lambda(\lambda^2 + 2\lambda a_2c_0 - a_1c_0^2 + a_3c_0)K_1(s,z)\exp(-\int^s 3K_3(\tau,z)d\tau)\Big\} ds \\ + c_3. \quad (3.19)$$

It notes that

$$\text{th}\,(\frac{1}{2}\xi) = 1 - 2F, \quad \text{sech}^2(\frac{1}{2}\xi) = 4F - 4F^2, \quad \frac{1}{1+\text{ch}(\xi)} = 2F - 2F^2. \quad (3.20)$$

Therefore, the solution of truncation expansion can be expressed by the polynomial of $\text{th}(0.5\xi)$, $\text{sech}^2(0.5\xi)$, $1/[1+\text{ch}(\xi)]$.

From equation (3.3) and equation (3.19), we can obtain the explicit exact solution to equation (3.1) is presented as follows:

$$u(t,x,z) = \exp(-\int^t K_3(s,z)ds)[\frac{2c_0+c_1}{2} - \frac{c_1}{2}th(\frac{1}{2}\xi)], \quad (3.21)$$

where ξ is given by equation (3.19).

In order to get exact solutions of equation (1.1), we give the following condition: (a) Suppose that for (t,x) in a bounded open set $\mathbb{G} \subset \mathbb{R}_+ \times \mathbb{R}$, and for all $z \in \mathbb{K}_q(r)$ for some $q > 0$ and $r > 0$ such that $K_i(t,z)(i = 1,2,3), H(t,z)$ satisfy that $u(t,x,z)$ and all its partial derivatives, which are involved in equation (3.1), are uniformly bounded for $(t,x,z) \in \mathbb{G} \times \mathbb{K}_q(r)$, continuous with respect to $(t,x) \in \mathbb{G}$ for all $z \in \mathbb{K}_q(r)$ and analytic with respect to $z \in \mathbb{K}_q(r)$ for all $(t,x) \in G$. From condition (a), Theorem 2.1 implies that there exists $U(t,x) \in (S)_{-1}$ such that $u(t,x,z) = (\mathcal{H}U(t,x))(z)$ for all $(t,x,z) \in \mathbb{G} \times \mathbb{K}_q(r)$, where $U(t,x)$ is the inverse Hermite transformation of $u(t,x,z)$. Consequently, $U(t,x)$ solves equation (1.1). Hence, by equation (3.19) and equation (3.21) we have that a stochastic solution of equation (1.1) is

$$U(t,x) = \exp^{\diamond}(-\int^t K_3(s)ds)[\frac{2c_0+c_1}{2} - \frac{c_1}{2}\text{th}^{\diamond}(\frac{1}{2}\overline{\xi})], \quad (3.22)$$

where

$$\begin{aligned}\overline{\xi} = \lambda \exp^{\diamond}(-\int^t K_3(s)ds)x + \int^t \{-\lambda K_2(s)\diamondsuit \exp^{\diamond}(-\int K_3(\tau)d\tau) \\ - \lambda(\lambda^2 + 2a_2c_0\lambda - a_1c_0^2 + a_3c_0)K_1(s)\diamondsuit \exp^{\diamond}(-\int^s 3K_3(\tau,z)d\tau)\}ds \\ + c_3. \quad (3.23)\end{aligned}$$

Let $k(t)$ be integrable functions on $\mathbb{R}_-$ and put

$$K_1(t) = b_1W(t), \quad K_2(t) = b_2W(t), \quad K_3(t) = k(t) + b_3W(t), \quad (3.24)$$

where $W(t)$ is Gaussian white noise, i.e., $W(t)=\dot{B}(t)$, $B(t)$ is Brown motion. Considering the Hermite transformation of equation (3.24), we have

$$\begin{aligned}&K_1(t,z)(t)=b_1\widetilde{W}(t,z),\quad K_2(t,z)=b_2\widetilde{W}(t,z),\\&K_3(t,z)=k(t)+b_3\widetilde{W}(t,z),\end{aligned}\tag{3.25}$$

where $\widetilde{W}(t,z)=\sum_{k=1}^{\infty}\int_0^t\eta_k(s)dsz_k$. Since $\exp^{\diamond}\{B(t)\}=\exp\{B(t)-\frac{1}{2}t^2\}$ (see Lemma 2.6.16 in [7]), we get the stochastic soliton-like solution of equation (1.1) is

$$\begin{aligned}U(t,x)=\exp\left(-\int^t k(s)ds-b_3B(t)+\frac{b_3}{2}t^2\right)\\ \times[\frac{2c_0+c_1}{2}-\frac{c_1}{2}\mathrm{th}(\frac{1}{2}\theta)],\end{aligned}\tag{3.26}$$

where

$$\begin{aligned}\theta=&\lambda\exp(-\int^t k(s)ds-b_3B(t)+\frac{b_3}{2}t^2)x+\int^t\{-[\lambda b_2+\lambda b_1(\lambda^2+2a_2c_0\lambda\\&-a_1c_0^2+a_3c_0)]W(s)\diamondsuit\exp(-\int^s k(\tau)d\tau-b_3B(s)+\frac{b_3}{2}s^2)\}ds\\=&\lambda\exp(-\int^t k(s)ds-b_3B(t)+\frac{b_3}{2}t^2)x+\int^t\{-[\lambda b_2+\lambda b_1(\lambda^2+2a_2c_0\lambda\\&-a_1c_0^2+a_3c_0)]\exp(-\int^s k(\tau)d\tau-b_3B(s)+\frac{b_3}{2}s^2)\}\delta B(s)+c_3.\end{aligned}\tag{3.27}$$

In equation (3.27), we have already used the following relation

$$\int_{\mathbb{R}}\Psi(t)\delta B(t)=\int_{\mathbb{R}}\Psi(t)\diamond W(t)dt,\quad\Psi(t)\in L^2(\mathbb{R}),$$

where the stochastic integral $\int(\cdot)\delta B(t)$ is the Skorohod integral.

Remark 1. When Wick product $\diamondsuit$ is an ordinary product $\cdot$ in the equation (1.1), we obtain the following KdV-MKdV equation with variable coefficients, which is written as:

$$\begin{aligned}u_t+k_1(t)[u_{xxx}-a_1u^2u_x+2a_2(u_x^2+uu_{xx})]\\+a_3k_2(t)h(t)uu_x+[k_2(t)+k_3(t)x]u_x+k_3(t)u=0,\end{aligned}\tag{3.28}$$

where $k_i(t)(i=1,2,3)$ are integrable functions on $\mathbb{R}_+$, $a_i(i=1,2,3)$ are arbitrary constants and $h(t)=\exp[-\int^t k_3(s)ds]$. And Yan et al. [13] and Zhang et al [14] studied and gave exact solutions for equation (1.2) by using different methods, respectively.

Remark 2. Equation (1.1) can be regarded as the perturbation of equation (3.28).

4. Conclusion

We have discussed some stochastic exact soliton-like solutions of the Wick-type stochastic KdV-MKdV equation by using new truncated expansion method and Hermite transformation. Noting that there exists a unitary mapping between the Wiener white noise space and the Poisson white noise space, we can obtain the solution of the Poisson SPDE simply by applying this mapping to the solution of the corresponding Gaussian SPDE. A nice and concise account of this connection was given by Benth and Gjerde [1]. We can see this in Section 4.9 of [7] as well. Hence, we can attain stochastic soliton solutions as we do in this section if the coefficient $K_i(t)$ $(i = 1, 2)$, and $H(t)$ are Poisson white noise functions in equation (1.1).

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