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# A COORDINATED POLICY MODEL IN DISTRIBUTION SYSTEM WITH STOCHASTIC DEMAND

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**Abstract:** This paper addresses a coordinated inventory and ordering policies in two-level distribution system with stochastic demand composing of a single supplier and multiple retailers, in which the supplier (distribution center) takes base-stock policy and deals in multiple products. A mathematical model to maximize expected system profit is developed. Some properties of the model and existence of solutions of the model are analized. Then corresponding solution methods are proposed. Last, three cases on demand are studied and some future researches areas are identified. This work can provide insight for purchasing and inventory decisions of distribution system and supply chain management.

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**Key Words:** supply chain management, distribution system, stochastic demand, coordinated inventory policies, mathematical model

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## 1. Introduction

In downstream supply chain, distribution decisions is an important factor affecting performance of supply chain management. A number of researchers have developed models to study distribution system. Earlyer researches with respect to distribution decisions focus on EOQ under deterministic demand or constant demand rate. For example see: Goyal (1976), [3], Monahan (1984), [10], Lal and Richard (1984), [6], Lee and Rosenblatt (1986), [7]. They assumed that the supplier had retailers' complete information and had no backlog and stockout to the retailer or the buyer. The supplier induced the retailer or the buyer to increase order quantity at a time to reduce order frequency so that transaction costs such as production set-up cost, order processing cost and so forth can be lowered by various price discount. Kohli and Park (1989), see [4], Kohli and Park (1994), see [5] attempted to lower costs and increase profits by cooperative game theory and coordinated decisions respectively, but they did still consider only the situation that total demand is deterministic within some period. Fangruo Chen (2000), see [1] considered a distribution system with a central warehouse and multiple retailers, in which customer demand arrived at each retailers continuously at constant rate, a single product was dealt in and stockouts at the retailers were backlogged. Chia-Shin Chung et al (2001), see [2] addressed the inventory placement problem in a serial supply chain facing a stochastic demand for a single period. All stocking decisions were made before the demand occurs, all costs were proportional. There were no fixed costs and no backlog, all unsatisfied demands were lost, and proposed an algorithm to determine inventories in different stages to maximize the expected profit. Lee and Kim (2002), see [8] tried to find a production and distribution policy to minimize total cost by combining analytic method with simulation method. Thomas and Hackman (2003), see [11] analyzed a supply chain environment in which a distributor facing price-sensitive demand obtained fixed order frequency and deterministic order quantity by favorable prices. Moreover, there are much work concerning distribution or distribution system, but most of them address such situation in which only a single product is dealt in under deterministic demand or constant demand rate. Varieties of demands, diversification of product, disposal of excess demand and otherness of life cycle of product are not considered adequately.

This paper addresses a coordinated inventory and ordering policies in two-level distribution system with stochastic demand composing of a single supplier and multiple retailers, in which the supplier (distribution center) takes base-stock policy and deals in multiple products with short life cycle, replenishes periodically at each stage within a period, and gains deterministic base stock at the beginning of each stage within a period. A retailer deals in a sort of product only. Retailer does not hold inventory or his inventory can be neglected. Retailer decides a deterministic quantity ordering from the distribution center at each stage within a period. The part of demand over the deterministic quantity may be met by placing a second order from the distribution center timely in this stage within this period; some backorder cost must be paid for the backlog due to a second order. If demand exceeds the base stock in this stage at distribution center, all unsatisfied demands are lost, but both distribution center and retailer must pay some punishment cost. Owing to short life cycle, surplus products both at distribution center and at retailer must be disposed by lower price or even spending extra costs so that new products are dealt in next stage. Demand only occurring at retailer follows some stochastic distribution.

## 2. Model

To determine the supplier's base stock and the retailer's deterministic order quantity to maximize the expected total profit within a period, corresponding mathematical model must be developed. For convenience, primary notations used in this paper are summarized as follows:

$i = 1, 2, \cdots, N$: index the product variety and retailers;

$t = 1, 2, \cdots, T$: index stages in a period;

$\beta_i$: the increased proportion of price of product $i$ placed a second order within the same stage;

$bs_{it}, Q_{it}$: the supplier's base stock and the retailer's deterministic order quantity of product $i$ at stage $t$ respectively;

$s_i^s, s_i^{b1}, s_i^{b2}$: the supplier's purchasing set-up cost and the retailer's first and second order set-up cost of product $i$ within the same stage respectively;

$c_i, w_i, p_i$: the supplier's purchasing unit price and wholesale unit price and the retailer's sale unit price of product $i$ respectively;

$h_i, \theta_i^s, \theta_i^b$: inventory holding cost of product $i$at distribution center, unit punishment cost of product $i$ paid by the distribution center and the retailer respectively;

$b_i$: unit backlogging cost of product paid to customers by the retailer;

$d_{it}^s, d_{it}^b$: unit value of product $i$ at end of stage $t$ at distribution center and retailer respectively, which may be negative;

$x_{it}, f_{it}(x_{it}), F_{it}(x_{it})$: stochastic demand quantity, distribution density and distribution function of product $i$ at stage $t$ only occurring at retailer;

$\pi_{it}^s, \pi_{it}^b, \pi_{it}$: profits of distribution center, retailer and system from product $i$ at stage $t$;

Both profits of distribution center and retailer are the difference between sale revenue and costs. According to demand within a stage, one can get three condition profit functions of distribution center and retailer respectively as follows:

$$\begin{aligned}\pi_{it}^s(x_{it}\,|x_{it} \le Q_{it})\\ &= (w_i - c_i)Q_{it} + (d_{it}^s - c_i)(bs_{it} - Q_{it}) - s_i^s - h_i(bs_{it} - Q_{it})\,, \end{aligned} \tag{1}$$

$$\begin{aligned}\pi_{it}^s(x_{it}\,|Q_{it} < x_{it} \le bs_{it})\\ &= (w_i - c_i)Q_{it} + ((1+\beta_i)w_i - c_i)(x_{it} - Q_{it}) + (d_{it}^s - c_i)(bs_{it} - x_{it})\\ &\quad - s_i^s - h_i(bs_{it} - Q_{it})\,, \end{aligned} \tag{2}$$

$$\begin{aligned}\pi_{it}^s(x_{it}\,|x_{it} > bs_{it}) &= (w_i - c_i)Q_{it} + ((1+\beta_i)w_i - c_i)(bs_{it} - Q_{it}) - s_i^s\\ &\quad - h_i(bs_{it} - Q_{it}) - \theta_i^s(x_{it} - bs_{it})\,, \end{aligned} \tag{3}$$

$$\pi_{it}^b(x_{it}\,|x_{it} \le Q_{it}) = (p_i - w_i)x_{it} + (d_{it}^b - w_i)(Q_{it} - x_{it}) - s_i^{b1}\,, \tag{4}$$

$$\begin{aligned}\pi_{it}^b(x_{it}\,|Q_{it} < x_{it} \le bs_{it}) &= (p_i - w_i)Q_{it} + (p_i - (1+\beta_i)w_i)(x_{it} - Q_{it})\\ &\quad - s_i^{b1} - s_i^{b2} - b_i(x_{it} - Q_{it})\,, \end{aligned} \tag{5}$$

$$\begin{aligned}\pi_{it}^b(x_{it}\,|x_{it} > bs_{it}) &= (p_i - w_i)Q_{it} + (p_i - (1+\beta_i)w_i)(bs_{it} - Q_{it}) - s_i^{b1} - s_i^{b2}\\ &\quad - b_i(bs_{it} - Q_{it}) - \theta_i^b(x_{it} - bs_{it})\,. \end{aligned} \tag{6}$$

Thus

$$\begin{aligned}\pi_{it} &= \int_0^{Q_{it}} [\pi_{it}^b(x_{it}\,|x_{it} \le Q_{it}) + \pi_{it}^s(x_{it}\,|x_{it} \le Q_{it})]f_{it}(x_{it})dx_{it}\\ &+ \int_{Q_{it}}^{bs_{it}} [\pi_{it}^b(x_{it}\,|Q_{it} < x_{it} \le bs_{it}) + \pi_{it}^s(x_{it}\,|Q_{it} < x_{it} \le bs_{it})]f_{it}(x_{it})dx_{it}\\ &\quad + \int_{bs_{it}}^{\infty} [\pi_{it}^b(x_{it}\,|x_{it} > bs_{it}) + \pi_{it}^s(x_{it}\,|x_{it} > bs_{it})]f_{it}(x_{it})dx_{it}\end{aligned}$$

$$
\begin{aligned}
&= (p_i - w_i)Q_{it} + [(p_i - (1+\beta_i)w_i - b_i)(bs_{it} - Q_{it}) - s_i^{b2}][1 - F_{it}(Q_{it})] \\
&\quad - s_i^{b1} - h_i(bs_{it} - Q_{it}) - (p_i - d_{it}^b)\int_0^{Q_{it}} (Q_{it} - x_{it})f_{it}(x_{it})dx_{it} \\
&\quad - (p_i - (1+\beta_i)w_i - b_i)\int_{Q_{it}}^{bs_{it}} (bs_{it} - x_{it})f_{it}(x_{it})dx_{it} - (\theta_i^b + \theta_i^s) \\
&\quad \times \int_{bs_{it}}^{\infty} (x_{it} - bs_{it})f_{it}(x_{it})dx_{it} + (w_i - c_i)Q_{it} + ((1+\beta)w_i - c_i) \\
&\quad \times (bs_{it} - Q_{it})[1 - F_{it}(Q_{it})] - ((1+\beta_i)w_i - c_i)\int_{Q_{it}}^{bs_{it}} (bs_{it} - x_{it})f_{it}(x_{it})dx_{it} \\
&\quad + (d_{it}^s - c_i)(bs_{it} - Q_{it})F_{it}(bs_{it}) - s_i^s - (d_{it}^s - c_i)\int_{Q_{it}}^{bs_{it}} (x_{it} - Q_{it})f_{it}(x_{it})dx_{it} \\
&= (p_i - c_i)Q_{it} - s_i^s - s_i^{b1} + [(p_i - c_i - b_i)(bs_{it} - Q_{it}) - s_i^{b2}][1 - F_{it}(Q_{it})] \\
&\quad - h_i(bs_{it} - Q_{it}) - (p_i - d_{it}^b)\int_0^{Q_{it}} (Q_{it} - x_{it})f_{it}(x_{it})dx_{it} \\
&\quad - (p_i - c_i - b_i)\int_{Q_{it}}^{bs_{it}} (bs_{it} - x_{it})f_{it}(x_{it})dx_{it} - (\theta_i^b + (d_{it}^s - c_i) \\
&\quad \times (bs_{it} - Q_{it})F_{it}(bs_{it}) + \theta_i^s)\int_{bs_{it}}^{\infty} (x_{it} - bs_{it})f_{it}(x_{it})dx_{it} \\
&\quad - (d_{it}^s - c_i)\int_{Q_{it}}^{bs_{it}} (x_{it} - Q_{it})f_{it}(x_{it})dx_{it}\,. \qquad (7)
\end{aligned}
$$

Let

$$
\pi_t = \sum_{i=1}^{N} \pi_{it}, \quad \pi_i = \sum_{t=1}^{T} \pi_{it}, \quad \pi = \sum_{t=1}^{T}\sum_{i=1}^{N} \pi_{it} = \sum_{i=1}^{N}\sum_{t=1}^{T} \pi_{it} = \pi(BS, Q),
$$
$$
U_N = \{1, 2, \cdots, N\}, \quad U_T = \{1, 2, \cdots, T\}\,,
$$

where $BS = (bs_{it})_{N\times T}$; $Q = (Q_{it})_{N\times T}$

Hence, a mathematical model to maximize expected system profit can be developed as follows:

$$
(P)\ \text{Max}\ \pi = \pi(BS, Q)\,, \qquad (8)
$$

$$
s.t. \sum_{i=1}^{N} (c_i bs_{it} + s_i^s) \le A(t)\,, \qquad (9)
$$

$$
\sum_{i=1}^{N} \alpha_i bs_{it} \le B(t)\,, \qquad (10)
$$

$$0 < Q_{it} \leq bs_{it}\,, \tag{11}$$

$$bs_{it} \in U_{it}^{BS} \subset R^{+}\,, \tag{12}$$

$$Q_{it} \in U_{it}^{Q} \subset R^{+}\,, \tag{13}$$

$$i \in U_N, t \in U_T\,, \tag{14}$$

where, (9) is a capital constraint of distribution center, $A(t)$is capital quantity that can be spent on purchasing at distribution center in stage $t$within a period; (10) is inventory placement constraint of distribution center, $\alpha_i (i = 1, 2, \cdots, N)$ is a constant for converting volume, $B(t)$ is inventory bulk that can be scheduled by distribution center in stage $t$within a period; (11) is a relation constraint between $Q_{it}$ and $bs_{it}$, $U_{it}^{BS}$ and $U_{it}^{Q}$ are bounded and closed, $bs_{it}$ take its numerical value in $U_{it}^{BS}$ and $Q_{it}$ take its numerical value in $U_{it}^{Q}$.

## 3. Solutions

Obviously, obtaining solution of problem (P) is quite hard. Hence, one considers the following relaxed problem (RP) first:

$$(RP)\ \text{Max}\ \pi = \pi(BS, Q)\,, \tag{15}$$

$$\text{s.t.}\quad bs_{it},\ Q_{it} \in R\,. \tag{16}$$

At the same time, assume:

(A1) $d_{it}^{b} = d_{it}^{s} \leq c_i < w_i < p_i$, $p_i - c_i - h_i - b_i > 0$, $\forall i \in U_N$, $t \in U_T$.

(A2) $f_{it}(\cdot)$ continuously differentiable, $F_{it}(\cdot)$is an increasing concave function, and $F_{it}(0) \approx 0$,$\forall i \in U_N$,$t \in U_T$.

Distinctly, (A1) accords with almost all reality and many stochastic distributions such as uniform distribution, exponential distribution, normal distribution and so on, can satisfy (A2) under some conditions. For example, distribution function of normal distribution $N(\mu, \sigma^2)$ is a increasing concave function on $[\mu, +\infty)$, and if $\mu$ is very great, while $\sigma$ is very small (e.g. $\mu \geq 3\sigma$), then $F_{it}(0) \approx 0$. Thus, one can derive the following theorem.

**Theorem 1.** *When (A1), (A2) are hold,*

$$(BS^*, Q^*) = ((bs_{it}^*)_{N\times T}, (Q_{it}^*)_{N\times T})$$

*is solution of (RP) if and only if*

$$s_i^{b2} f_{it}(Q_{it}^*) - b_i F_{it}(Q_{it}^*) + h_i = 0, \quad \forall i \in U_N, \quad t \in U_T \tag{17}$$

*and*

$$p_i + \theta_i^b + \theta_i^s - c_i - b_i - h_i - (p_i + \theta_i^b + \theta_i^s - b_i - d_{it}^s) F_{it}(bs_{it}^*) = 0, \quad \forall i \in U_N,\ t \in U_T . \tag{18}$$

*Proof.* From (7), (8) and (A1), one has

$$\partial\pi/\partial Q_{it} = s_i^{b2} f_{it}(Q_{it}) - b_i F_{it}(Q_{it}) + h_i \,, \tag{19}$$

$$\partial\pi/\partial bs_{it} = (p_i + \theta_i^b + \theta_i^s - c_i - b_i - h_i) - (p_i + \theta_i^b + \theta_i^s - b_i - d_{it}^s) F_{it}(bs_{it}) \,. \tag{20}$$

Thus

$$\partial^2\pi/\partial bs_{it}\partial Q_{j\tau} = \partial^2\pi/\partial Q_{it}\partial bs_{lm} = 0 \,, \tag{21}$$

where $i, j, l \in \{1, 2, \cdots, N\}$, $t, \tau, m \in \{1, 2, \cdots, T\}$ and $(i-j)^2 + (t-\tau)^2 \neq 0$, $(i-l)^2 + (t-m)^2 \neq 0$,

$$\partial^2\pi/\partial Q_{it}^2 = s_i^{b2} f'_{it}(Q_{it}) - b_i f_{it}(Q_{it}) < 0 \,, \tag{22}$$

$$\partial^2\pi/\partial bs_{it}^2 = -(p_i + \theta_i^b + \theta_i^s - b_i - d_{it}^s) f_{it}(bs_{it}) < 0 \,. \tag{23}$$

Hence, the objective function of problem (RP) is a concave function on feasible domain (a convex set), this completes the proof. □

Furthermore, one can derive following theorem.

**Theorem 2.** *When (A1) and (A2) hold, solution of original problem (P) is one and only.*

*Proof.* In fact, if the feasible domain of problem (P) is extended to a continuous bounded and closed space, then the objective function is obviously continuous on this space. Add concave of the objective function to this, one can gains that solution of original problem (P) is one and only one.

In practice, both $U_{it}^{BS}$ and $U_{it}^{Q}$ are often bounded discrete space, if potencies of them, $N$ and $T$ are small, one can solves problem (P) easily. Another situation can be dealt by using Theorem 1. If $(BS^*, Q^*) = ((bs_{it}^*)_{N\times T}, (Q_{it}^*)_{N\times T})$ determined by (17) and (18) of Theorem 1 is not in the feasible domain of problem (P), then problem (P) can be solved by precision searching algorithm or heuristic method. □

## 4. Cases Study

For the situation that $(BS^*, Q^*) = ((bs^*_{it})_{N\times T}, (Q^*_{it})_{N\times T})$ determined by (17) and (18) of Theorem 1 is in the feasible domain of problem (P), one analyzes three cases including uniform distribution, exponential distribution, normal distribution, and then presents detailed solution procedures.

### 4.1. Uniform Distribution

Let

$$f_{it}(x_{it}) = \begin{cases} 1/(b-a), a \le x_{it} \le b\,, \\ 0,\ x_{it} < a \ or \ x_{it} > b\,, \end{cases}$$

then

$$F_{it}(x_{it}) = \begin{cases} 0, & x_{it} \le a\,, \\ (x_{it}-a)/(b-a), & a < x_{it} \le b\,, \\ 1, & x_{it} > b\,. \end{cases}$$

From (17) one has

$$s_i^{b2}/(b-a) - b_i(Q^*_{it}-a)/(b-a) + h_i = 0\,. \tag{24}$$

Thus

$$Q^*_{it} = a + (h_i(b-a) + s_i^{b2})/b_i, \forall i \in U_N, t \in U_T\,. \tag{25}$$

From (18) one has

$$p_i + \theta_i^b + \theta_i^s - c_i - b_i - h_i - (p_i + \theta_i^b + \theta_i^s - b_i - d^s_{it})(bs^*_{it} - a)/(b-a) = 0\,. \tag{26}$$

Hence

$$bs^*_{it} = a + (p_i + \theta_i^b + \theta_i^s - c_i - b_i - h_i)(b-a)/(p_i + \theta_i^b + \theta_i^s - b_i - d^s_{it}), \\ \forall i \in U_N, \quad t \in U_T\,. \tag{27}$$

### 4.2. Exponential Distribution

Let

$$f_{it}(x_{it}) = \begin{cases} \lambda_{it} e^{-\lambda_{it} x_{it}}, & x_{it} > 0\,, \\ 0, & x_{it} \le 0\,, \end{cases}$$

then

$$F_{it}(x_{it}) = \begin{cases} 1 - e^{-\lambda_{it} x_{it}}, & x_{it} > 0\,, \\ 0, & x_{it} \le 0\,. \end{cases}$$

From (17) one has

$$s_i^{b2}\lambda_{it}e^{-\lambda_{it}Q_{it}^*} - b_i(1 - e^{-\lambda_{it}Q_{it}^*}) + h_i = 0\,. \tag{28}$$

Thus

$$Q_{it}^* = (1/\lambda_{it})\ln((\lambda_{it}s_i^{b2} + b_i)/(h_i + b_i)), \quad \forall i \in U_N,\ t \in U_T\,. \tag{29}$$

From (18) one has

$$p_i + \theta_i^b + \theta_i^s - c_i - b_i - h_i - (p_i + \theta_i^b + \theta_i^s - b_i - d_{it}^s)(1 - e^{-\lambda_{it}Q_{it}^*}) = 0\,. \tag{30}$$

Hence

$$bs_{it}^* = (1/\lambda_{it})\ln((p_i + \theta_i^s + \theta_i^b - b_i - d_{it}^s)/(c_i + h_i - d_{it}^s)), \quad \forall i \in U_N, \quad t \in U_T\,. \tag{31}$$

### 4.3. Normal Distribution

Let

$$f_{it}(x_{it}) = \begin{cases} (1/(\sigma\sqrt{2\pi}))e^{-(x_{it}-\mu)^2/(2\sigma^2)}, & x_{it} > 0\,, \\ 0, & x_{it} \le 0\,, \end{cases}$$

then

$$F_{it}(x_{it}) = \begin{cases} (1/(\sigma\sqrt{2\pi}))\int_0^{x_{it}} e^{-(\tau_{it}-\mu)^2/(2\sigma^2)}d\tau_{it}, & x_{it} > 0\,, \\ 0, & x_{it} \le 0\,. \end{cases}$$

From (17) one has

$$s_i^{b2}e^{-(Q_{it}^*-\mu)^2/(2\sigma^2)} - b_i\int_0^{Q_{it}^*} e^{-(x-\mu)^2/(2\sigma^2)}dx + h_i\sigma\sqrt{2\pi} = 0\,, \quad \forall i \in U_N,\ t \in U_T\,. \tag{32}$$

Through this formula and by dint of function value table of standard normal distribution, one can obtain $Q_{it}^*$ using one-dimensional searching algorithm.

From (18) one has

$$p_i + \theta_i^b + \theta_i^s - c_i - b_i - h_i - (p_i + \theta_i^b + \theta_i^s - b_i - d_{it}^s) \times (1/(\sigma\sqrt{2\pi}))\int_0^{bs_{it}^*} e^{-(x-\mu)^2/(2\sigma^2)}dx = 0\,. \tag{33}$$

Hence

$$
\begin{aligned}
& bs_{it}^* \\
& \quad = \mu + \sigma\Phi^{-1}((p_i + \theta_i^s + \theta_i^b - c_i - b_i - h_i)/(p_i + \theta_i^s + \theta_i^b - b_i - d_{it}^s)), \\
& \qquad \forall i \in U_N,\ t \in U_T\,. \quad (34)
\end{aligned}
$$

## 5. Conclusions and Future Researches

This paper addresses a coordinated inventory and ordering policies in two-level distribution system with stochastic demand composing of a single supplier and multiple retailers, in which the supplier (distribution center) takes base-stock policy and deals in multiple products with short life cycle, replenish periodically at each stage within every period, and gains deterministic base stock at the beginning of each stage within a period. A retailer deals in a sort of product only. Retailer does not hold inventory or his inventory can be neglected. Retailer combines fixed order and instant order from distribution center in each stage within a period. Some backorder cost must be paid for the backlog due to a second order. If demand exceed the base stock in this stage at distribution center, all unsatisfied demands are lost, but both distribution center and retailer must pay some punishment cost. Owing to short life cycle, surplus products both at distribution center and at retailer must be disposed by lower price or even spending extra cost so that new products are dealt in the stage. This paper developed a mathematical model to maximize expected system profit and analyzed existence and uniqueness of solutions of the model, and then proposed corresponding solution methods. Last, three cases on demand are studied. The situation considered by this paper has universality. This work can provide insight for purchasing and inventory decisions of distribution system and supply chain management. In next work, we will consider uncertain lead time, combine production with transportation and so on.

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