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NEW SOLITON-LIKE SOLUTIONS FOR GENERALIZED STOCHASTIC KdV EQUATION

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Abstract: In this paper, by using Hermite transformation, generalized Wick-type stochastic KdV equation is researched. Some soliton-like solutions are obtained via extended tanh method and Hermite transformation.

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1. Introduction

In recent years, many authors have researched a number of soliton solutions of nonlinear stochastic partial differential equations. Especially, stochastic KdV equations have been studied by Wadati [12], [13], de Bouard and Debussche [2], [3], Debussche and Printems [4], [5], Konotop and Vźquez [8], Printems [9], Xie [10], Gao [6], and so on. In [7], Holden et al. gave the white noise functional approach to study stochastic partial differential equations in Wick versions.

In this paper, we consider the generalized Wick-type stochastic KdV equation as follows:

$$(U_t + 6U \diamondsuit U_x + U_{xxx})_x + H_1(t) \diamondsuit U_x + H_2(t) \diamondsuit U_{yy} = 0, \tag{1.1}$$

where $H_i(t)$ $(i = 1, 2)$ are the white noise functionals, $\diamondsuit$ is the Wick product on the Hida distribution space $((S(\mathbb{R}^d)))^*$, which will be defined in the second section. Equation (1.1) is regarded as the perturbation of the the generalized

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KdV equation with variable coefficients, which is written as

$$(u_t + 6uu_x + u_{xxx})_x + e(t)u_x + n(t)u_{yy} = 0, \tag{1.2}$$

where $e(t)$ and $n(t)$ are functions of variable t only. E. Yomba [11] studied and gave the explicit solutions of equation (1.2) by using extended tanh method. In this paper, we will discuss some exact soliton solutions of equation (1.1) via extended tanh method and white noise analysis method.

The rest of the paper is organized as follows. In Section 2, we give some preliminaries. In Section 3, by using extended tanh method and white noise analysis method, some new exact solutions for equation (1.1) are obtained. Finally, we draw the conclusion.

2. Preliminaries

Suppose that $(S(\mathbb{R}^d))$ and $(S(\mathbb{R}^d))^*$ are the Hida test function space and the Hida distribution space on $\mathbb{R}^d$, respectively. Let $h_n(x)$ be the n-order Hermite polynomials and put $\xi_n(x) = e^{-\frac{1}{2}x^2}h_n(\sqrt{2}x)/(\pi(n-1)!)^{1/2}, n \geq 1$, then, the collection $\{\xi_n\}_{n\geq 1}$ constitutes an orthogonal basis for $L^2(\mathbb{R})$.

Let $\alpha = (\alpha_1, \cdots, \alpha_d)$ denote d-dimensional multi-indices with $\alpha_1, \cdots, \alpha_d \in \mathbb{N}$. The family of tensor products $\xi_\alpha = \xi_{(\alpha_1,\cdots,\alpha_d)} = \xi_{\alpha_1}\otimes\cdots\otimes\xi_{\alpha_d}(\alpha \in \mathbb{N}^d)$ forms an orthogonal basis for $L^2(\mathbb{R})$. Suppose that $\alpha^{(i)} = (\alpha_1^{(i)}, \cdots, \alpha_d^{(i)})$ is the i-th multi-index number in some fixed ordering of all d-dimensional multi-indices α. We can, and will, assume that this ordering has the property that

$$i < j \Rightarrow \alpha_1^{(i)} + \cdots + \alpha_d^{(i)} \leq \alpha_1^{(j)} + \cdots + \alpha_d^{(j)},$$

i.e., the $\{\alpha^{(j)}\}_{j=1}^\infty$ occurs in an increasing order. Now define

$$\eta_i = \xi_{\alpha^{(i)}} = \xi_{\alpha_1^{(i)}} \otimes \cdots \otimes \xi_{\alpha_d^{(i)}}, i \geq 1.$$

We need to consider multi-indices of arbitrary length. For simplification of notation, we regard multi-indices as elements of the space $(\mathbb{N}_0^{\mathbb{N}})_c$ of all sequences $\alpha = (\alpha_1, \alpha_2, ...)$ with elements $\alpha_i \in \mathbb{N}_0$ and with compact support, i.e., with only finitely many $\alpha_i \neq 0$. We write $\jmath = (\mathbb{N}_0^{\mathbb{N}})_c$, for $\alpha \in \jmath$, define

$$H_\alpha(\omega) = \prod_{i=1}^\infty h_{\alpha_i}(\langle\omega, \eta_i\rangle), \quad \omega = (\omega_1, \cdots, \omega_m) \in (S(\mathbb{R}^d))^*.$$

For a fixed $n \in \mathbb{N}$ and $\forall k \in \mathbb{N}$, suppose the space $(\mathbb{S})_1^n$ consists of those $f(\omega) = \sum_\alpha c_\alpha H_\alpha(\omega) \in \bigoplus_{k=1}^n L^2(\mu)$ with $c_\alpha \in \mathbb{R}^n$ such that $||f||_{1,k}^2 = \sum_\alpha c_\alpha^2(\alpha!)^2(2\mathbb{N})^{k\alpha} <$

∞, where $c_\alpha^2 = |c_\alpha|^2 = \sum_{k=1}^n (c_\alpha^{(k)})^2$ if $c_\alpha = (c_\alpha^{(1)}, ..., c_\alpha^{(n)}) \in \mathbb{R}^n$, and μ is the white noise measure on $(S^*(\mathbb{R}), \mathbf{B}(S^*(\mathbb{R}))$, $\alpha! = \prod_{k=1}^\infty \alpha_k!$ and $(2\mathbb{N})^\alpha = \prod_j (2j)^{\alpha_j}$ for $\alpha \in \jmath$.

The space $(S)_{-1}^n$ consists of all formal expansions $F(\omega) = \sum_\alpha b_\alpha H_\alpha(\omega)$ with $b_\alpha \in \mathbb{R}^n$ such that $||F||_{-1,-q} = \sum_\alpha b_\alpha^2 (2\mathbb{N})^{-q\alpha} < \infty$ for some $q \in \mathbb{N}$. The family of seminorms $||f||_{1,k}, k \in \mathbb{N}$ gives rise to a topology on $(S)_1^n$, and we can regard $(S)_{-1}^n$ as the dual of $(S)_1^n$ by the action

$$\langle F, f \rangle = \sum_\alpha (b_\alpha, c_\alpha) \alpha!\,,$$

where (b_α, c_α) is the ordinary inner product in $\mathbb{R}^n$.

The Wick product $f \Diamond F$ of two elements $f = \sum_\alpha a_\alpha H_\alpha$, $F = \sum_\alpha b_\alpha H_\alpha \in (S)_{-1}^n$ with $a_\alpha, b_\alpha \in \mathbb{R}^n$, is defined by

$$f \Diamond F = \sum_{\alpha,\beta} (a_\alpha, b_\beta) H_{\alpha+\beta}.$$

We can prove that the spaces $(S(\mathbb{R}^d)), (S(\mathbb{R}^d))^*, (S)_1$, and $(S)_{-1}$ are closed under Wick products.

For $F = \sum_\alpha b_\alpha H_\alpha \in (S)_{-1}^N$, with $b_\alpha \in \mathbb{R}^N$, the Hermite transformation of F, denoted by $\mathcal{H}(F)$ or $\widetilde{F}$ is defined by

$$\mathcal{H}(F) = \widetilde{F}(z) = \sum_\alpha b_\alpha z^\alpha \in \mathbb{C}^N \text{ (when convergent)},$$

where $z = (z_1, z_2, ...) \in \mathbb{C}^\mathbb{N}$ (the set of all sequences of complex numbers) and $z^\alpha = z_1^{\alpha_1} z_2^{\alpha_2} \cdots\ z_n^{\alpha_n} \cdots$, if $\alpha \in \jmath$, where $z_j^0 = 1$.

For $F, G \in (S)_{-1}^N$, by this definition we have

$$\widetilde{F \diamond G}(z) = \widetilde{F}(z) \cdot \widetilde{G}(z)$$

for all z such that $\widetilde{F}(z)$ and $\widetilde{G}(z)$ exist. The product on the right-hand side of the above formula is the complex bilinear product between two elements of $\mathbb{C}^\mathbb{N}$ defined by $(z_1^1, ..., z_n^1) \cdot (z_1^2, ..., z_n^2) = \sum_{k=1}^n z_k^1 z_k^2$, where $z \in \mathbb{C}$.

Let $X = \sum_\alpha a_\alpha H_\alpha \in (S)_{-1}^N$, then the vector $c_0 = \tilde{X}(0) \in \mathbb{R}^N$ is called the generalized expectation of X which denoted by $E(X)$. Suppose that $g : U \to \mathbb{C}^M$ is an analytic function, where U is a neighborhood of $\xi_0 := E(X)$. Assume that the Taylor series of g around ξ_0 have coefficients in $\mathbb{R}^M$. Then the Wick version $g^\diamond(X) = \mathcal{H}^{-1}(g \circ \widetilde{X}) \in (S)_{-1}^M$. In other words, if g has the power series expansion $g(z) = \sum a_\alpha (z-\xi_0)^\alpha$ with $a_\alpha \in \mathbb{R}^M$, then $g^\diamond(X) = \sum a_\alpha (X-\xi_0)^{\diamond\alpha} \in (S)_{-1}^M$.

Suppose that modelling consideration leads us to consider an SPDE as follows:

$$A(t, x, \partial t, \nabla x, U, \omega) = 0, \tag{2.1}$$

where A is some given function, $U = U(t, x, \omega)$ is an unknown (generalized) stochastic process, and the operators $\partial t = \frac{\partial}{\partial t}, \nabla x = (\frac{\partial}{\partial x_1}, ..., \frac{\partial}{\partial x_d})$ when $x = (x_1, ..., x_d) \in \mathbb{R}^d$.

Firstly, we interpret all products as Wick products and all functions as their Wick versions. Wick version of equation (2.1) is written as follows:

$$A^\diamond(t, x, \partial t, \nabla x, U, \omega) = 0. \tag{2.2}$$

Secondly, we take the Hermite transformation of equation (2.1), which turns Wick products into ordinary products (between complex numbers), so the equation takes the form

$$\tilde{A}(t, x, \partial t, \nabla x, \tilde{U}, z_1, z_2, ...) = 0, \tag{2.3}$$

where $\tilde{U} = \mathcal{H}(U)$ is the Hermite transformation of U and $z_1, z_2, ...$ are complex numbers. Suppose that we can find a solution $u = u(t, x, z)$ of equation (2.3) for each $z \in \mathbb{K}_q(r)$, where $\mathbb{K}_q(r) = \{z \in \mathbb{C}^\mathbb{N}$ and $\sum_{\alpha \neq 0} |z^\alpha|^2 (2\mathbb{N})^{q\alpha} < r^2\}$ for some q, r. Then, under certain conditions, we can take the inverse Hermite transformation $U = \mathcal{H}^{-1} u \in (S)_{-1}$ and thereby obtain a solution U of the original Wick equation (2.2). We have the following theorem, which was proved by Holden et al [7].

Theorem 2.1. *Suppose u(t,x,z) is a solution (in the usual strong, pointwise sense) of equation (2.2) for (t, x) in some bounded open set $G \subset \mathbb{R} \times \mathbb{R}^d$, and for all $z \in K_q(r)$, for some q, r. Moreover, suppose that $u(t, x, z)$ and all its partial derivatives, which are involved in equation (2.2), are (uniformly) bounded for $(t, x, z) \in G \times K_q(r)$, continuous with respect to $(t, x) \in G$ for all $z \in K_q(r)$ and analytic with respect to $z \in K_q(r)$, for all $(t, x) \in G$. Then there exists $U(t, x) \in (S)_{-1}$ such that $u(t, x, z) = U(t, x))(z)$ for all $(t, x, z) \in G \times K_q(r)$ and $U(t, x)$ solves (in the strong sense in $(S)_{-1}$) equation (2.1) in $(S)_{-1}$.*

3. Some New Exact Solutions of Equation (1.1)

Considering the Hermite transformation of equation (1.1), we obtain

$$(\widetilde{U}_t + 6\widetilde{U}\widetilde{U}_x + \widetilde{U}_{xxx})_x + \widetilde{H}_1(t, z)\widetilde{U}_x + \widetilde{H}_2(t, z)\widetilde{U}_{yy} = 0, \tag{3.1}$$

where $z \in (\mathbb{C}^{\mathbb{N}})_c$ is a vector parameter.

Now we use the extended tanh method [11] to solve equation (3.1). For simplicity, we assume that $u = \widetilde{U}(x,y,t,z), H_i = \widetilde{H}_i(t,z)$ $(i = 1,2)$, $H_1 = CH_2(t,z)$, where $C(C \neq 0)$ is a constant.

For solving equation (3.1), we introduce the following auxiliary ordinary equation:

$$(\frac{d\omega}{d\xi})^2 = a\omega^2(\xi) + b\omega^3(\xi) + c\omega^4(\xi), \tag{3.2}$$

where a, b and c are constants. We shall seek the exact solutions of equation (3.1) by using the following solutions of equation (3.2)

$$\omega(\xi) = \begin{cases} \dfrac{-ab\sec^2(\frac{\sqrt{a}}{2}\xi)}{b^2 - ac(1-\tanh(\frac{\sqrt{a}}{2}\xi))^2}, & a > 0, \\ \dfrac{2a\sec(\sqrt{a}\xi)}{\sqrt{b^2-4ac} - b\sec(\sqrt{a}\xi)}, & b^2 - 4ac > 0,\ a > 0. \end{cases} \tag{3.3}$$

By balancing the highest order derivative term u_{xxx} with the highest powder nonlinear term uu_x yields $n = 2$.

Therefore we may choose the solution of equation (3.1) in the form

$$\begin{aligned} &u = f(y,t,z) + h(y,t,z)\omega(\xi) + g(y,t,z)\omega^2(\xi), \\ &\xi = p(y,t,z)x + q(y,t,z), \end{aligned} \tag{3.4}$$

where f, h, g, p and q are functions to be determined later. Substituting equation (3.4) and equation (3.2) into equation (3.1) and collecting coefficients of power of $x^i\phi^m$, and

$$x^k\phi^l\sqrt{a\phi^2(\zeta) + b\phi^3(\zeta) + c\phi^4(\zeta)}\ (i = 0,1,2;\ m = 0,...,6;\ k = 0,1;\ l = 0,1)$$

with the aid of *Mathematica* (note:f, g, h, p, q are independent of x), then setting each coefficient to zero, we can deduce the following set of over-determined partial differential equations with respect to unknown derivative functions (f, h, g, p, q) (note: in the rest of this paper, p_y denotes $\frac{\partial p(y,t)}{\partial y}$, and so on).

$$\begin{aligned} &2ahp_y^2 = 0, && (8ag + 3bh)p_y^2 = 0, \\ &2(5bg + 2ch)p_y^2 = 0, && 12cgp_y^2 = 0, \\ &2ah(pp_t + 2H_2(t,z)p_yq_y) = 0, \\ &(8ag + 3bh)(pp_t + 2H_2(t,z)p_yq_y) = 0, \\ &2(5bg + 2ch)(pp_t + 2H_2(t,z)p_yq_y) = 0, \end{aligned} \tag{3.5}$$

$$12cg(pp_t + 2H_2(t,z)p_yq_y) = 0, \qquad 2(2g_yp_y + gp_{yy}) = 0,$$
$$2h_yp_y + hp_{yy} = 0, \qquad 2H_2(t,z)f_{yy} = 0,$$

$$\begin{aligned}
&H_1(t,z)hp + ph_t + hp_t + 2H_2(t,z)h_yq_y + hH_2(t,z)q_{yy} = 0,\\
&2H_1(t,z)gp + pg_t + gp_t + 2H_2(t,z)g_yq_y + gH_2(t,z)q_{yy} = 0,\\
&2(6afhp^2 + a^2hp^4 + ahpq_t + ahH_2(t,z)q_y^2 + H_2(t,z)h_{yy}) = 0,\\
&48afgp^2 + 18bfhp^2 + 24ah^2p^2 + 32a^2gp^4 + 15abhp^4 + 8agpq_t + 3bhpq_t\\
&\qquad + 8agH_2(t,z)q_y^2 + 2H_2(t,z)g_{yy} = 0,\\
&60bfgp^2 + 24cfhp^2 + 108aghp^2 + 30bh^2p^2 + 130abgp^4 + 15b^2hp^4 + 40achp^4\\
&\qquad + 10bgpq_t + 4chpq_t + 10bgH_2(t,z)q_y^2 + 4chH_2(t,z)q_y^2 = 0,\\
&3(24cfgp^2 + 32ag^2p^2 + 42bghp^2 + 12ch^2p^2 + 35b^2gp^4 + 80acgp^4 + 20bchp^4\\
&\qquad + 4cgpq_t + 4cgH_2(t,z)q_y^2) = 0,\\
&12p^2(9bg^2 + 12cgh + 28bcgp^2 + 4c^2hp^2) = 0,\\
&120cgp^2(g + 2cp^2) = 0\,.
\end{aligned} \tag{3.6}$$

From equation (3.5), we find

$$p_y = 0, \quad p_t = 0, \quad f_{yy} = 0\,. \tag{3.7}$$

Equation (3.7) indicates that p ($p = k \neq 0$) is a constant. Solving equation (3.6) with p, we obtain two cases

Case 1.

$$\begin{aligned}
&g = -2ck^2,\ h = -bk^2, b^2 = 4ac,\ f = 0,\\
&q = -\frac{kC}{2}y^2 + K_1\exp(2\int_0^t CH_2(s,z)ds)y - ak^3t\\
&\quad - \frac{K_1^2}{k}\int_0^t [H_2(s,z)\exp(4\int_0^s CH_2(\tau,z)d\tau)]ds + K_2,
\end{aligned} \tag{3.8}$$

where a, c, K_1 and K_2 are arbitrary constants. Therefore we obtain the soliton-like solutions of equation (3.1) as follows:

$$u_1 = -\frac{4ak\sec^2(\frac{\sqrt{a}}{2})\xi_1}{4 - [1 - \tanh(\frac{\sqrt{a}}{2})\xi_1]^2} - \frac{1}{2a}\{\frac{4ak\sec^2(\frac{\sqrt{a}}{2})\xi_1}{4 - [1 - \tanh(\frac{\sqrt{a}}{2})\xi_1]^2}\}^2, \qquad a > 0,\ c > 0, \tag{3.9}$$

where

$$\begin{aligned}
\xi_1 = kx &- \tfrac{kC}{2}y^2 + K_1\exp(2\int_0^t CH_2(s,z)ds)y - ak^3t\\
&- \frac{K_1^2}{k}\int_0^t [H_2(s,z)\exp(4C\int_0^s H_2(\tau,z)d\tau)]ds + K_2.
\end{aligned} \tag{3.10}$$

Case 2.

$$\begin{aligned}
&c=0, \quad g=0, \quad h=-\frac{bk^2}{2}, \quad q=-\frac{kC}{2}y^2+A_1(t,z)y+A_2(t,z),\\
&A_2(t,z)=-ak^3t-6k\int_0^t A_4(s,z)ds-\frac{1}{k}\int_0^t H_2(s,z)A_1^2(s)ds+A_5,\\
&A_3(t,z)=\frac{1}{6k}[2CH_2(t,z)A_1(t,z)-A_{1t}(t,z)],\\
&f=A_3(t,z)y+A_4(t,z),
\end{aligned} \tag{3.11}$$

where a, b, A_5 are arbitrary constants, and A_1, A_4 are arbitrary functions with respect to (t,z). Hence we obtain another soliton-like solutions of equation (3.1) as follows:

$$u_2(x,y,t,z)=A_3(t,z)y+A_4(t,z)+\frac{ak^2}{2}\sec^2(\frac{\sqrt{a}}{2}\xi_2), \quad a>0, \tag{3.12}$$

$$u_3(x,y,t,z)=A_3(t,z)y+A_4(t,z)+ak^2\frac{\sec(\sqrt{a}\xi_2)}{\sec(\sqrt{a}\xi_2)-1}, \quad a>0, \ b>0, \tag{3.13}$$

$$u_4(x,y,t,z)=A_3(t,z)y+A_4(t,z)+ak^2\frac{\sec(\sqrt{a}\xi_2)}{\sec(\sqrt{a}\xi_2)+1}, \quad a>0, \ b<0, \tag{3.14}$$

where

$$\begin{aligned}
\xi_2=kx-\frac{kC}{2}y^2+A_1(t,z)y-ak^3t-6k\int_0^t A_4(s,z)ds\\
-\frac{1}{k}\int_0^t H_2(s,z)A_1^2(s,z)ds+A_5.
\end{aligned} \tag{3.15}$$

Let $H_2(t)=h(t)+a_2W(t)$, then $H_1(t)=C(h(t)+a_2W(t))$. If $A_1(t)=b_1W(t)$, $A_4(t)=\gamma(t)+b_4W(t)$, from equation (3.11), we have

$$\begin{aligned}
A_2(t,z)&=-ak^3t-6k\int_0^t(\gamma(s)+b_4W(s))ds\\
&\quad-\frac{1}{k}\int_0^t H_2(s,z)b_1^2W^2(s)ds+A_5,\\
A_3(t,z)&=\frac{1}{6k}[2b_1C(h(t)+a_2W(t))W(t)-b_1W'(t)],
\end{aligned} \tag{3.16}$$

where $W(t)$ is a Gaussian white noise. Suppose $B(t)$ be a Brownain motion, we have $W(t)=\dot{B}(t)$. Considering the Hermite transformation of $H_1(t)$, $H_2(t)$,

$A_1(t)$ $A_4(t)$ and equation (3.16), we obtain $\widetilde{H}_2(t,z)=h(t)+a_2\widetilde{W}(t,z)$, $\widetilde{H}_1(t,z)=C(h(t)+a_2\widetilde{W}(t,z))$, $\widetilde{A}_1(t,z)=b_1\widetilde{W}(t,z)$, $\widetilde{A}_4(t,z)=\gamma(t)+b_4\widetilde{W}(t,z)$, and

$$\begin{aligned}\widetilde{A}_2(t,z) &= -ak^3t-6k\int_0^t(\gamma(s)+b_4\widetilde{W}(s,z))ds\\ &\quad -\frac{1}{k}\int_0^t\widetilde{A}_2(s,z)b_1^2\widetilde{W}^2(s,z)ds+A_5,\\ \widetilde{A}_3(t,z) &= \frac{1}{6k}[2b_1C(h(t)+a_2\widetilde{W}(t,z))\widetilde{W}(t,z)-b_1\widetilde{W}'(t,z)],\end{aligned}\tag{3.17}$$

where a_2,b_1,b_4 are arbitrary constants, and $\widetilde{W}(t,z)=\sum_{k=1}^{\infty}\int_0^t\eta_k(s)dsz_k$.

In order to get explicit solutions of equation (1.1), we assume that the following condition:

(F) Suppose that for (t,x,y) in a bounded open set $\mathbb{G}\subset\mathbb{R}\times\mathbb{R}_-^2$, for all $z\in\mathbb{K}_q(r)$ for some $q>0$ and $r>0$ such that $H_i(t,z)(i=1,2)$ satisfy that $u(t,x,y,z)$ and all its partial derivatives, which are involved in equation (3.1), are uniformly bounded for $(t,x,y,z)\in\mathbb{G}\times\mathbb{K}_q(r)$, continuous with respect to $(t,x,y)\in\mathbb{G}$ for all $z\in\mathbb{K}_q(r)$ and analytic with respect to $z\in\mathbb{K}_q(r)$ for all $(t,x,y)\in G$.

From **(F)**, Theorem 2.1 implies that there exists $U(t,x,y)\in(S)_{-1}$ such that $u(t,x,y,z)=(\mathcal{H}U(t,x,y))(z)$ for all $(t,x,y,z)\in\mathbb{G}\times\mathbb{K}_q(r)$, where $U(t,x,y)$ is the inverse Hermite transformation of $u(t,x,y,z)$. Consequently, $U(t,x,y)$ solves equation (1.1). Since $exp^{\diamond}\{B(t)\}=exp\{B(t)-\frac{1}{2}t^2\}$ (see Lemma 2.6.16 in [7]), from equations (3.9)-(3.15), we obtain that stochastic solutions of equation (1.1), as follows:

$$U_1(x,y,t)=-\frac{4ak\sec^2(\frac{\sqrt{a}}{2})\theta_1}{4-[1-\tanh(\frac{\sqrt{a}}{2})\theta_1]^2}-\frac{1}{2a}\{\frac{4ak\sec^2(\frac{\sqrt{a}}{2})\theta_1}{4-[1-\tanh(\frac{\sqrt{a}}{2})\theta_1]^2}\}^2,\quad a>0,\ c>0,\tag{3.18}$$

$$U_2(x,y,t)=\phi(t,y)+\frac{ak^2}{2}\sec^2(\frac{\sqrt{a}}{2}\theta_2),\quad a>0,\tag{3.19}$$

$$U_3(x,y,t)=\phi(t,y)+ak^2\frac{\sec(\sqrt{a}\theta_2)}{\sec(\sqrt{a}\theta_2)-1},\quad a>0,\ b>0,\tag{3.20}$$

$$U_4(x,y,t)=\phi(t,y)+ak^2\frac{\sec(\sqrt{a}\theta_2)}{\sec(\sqrt{a}\theta_2)+1},\quad a>0,\ b<0,\tag{3.21}$$

where

$$\theta_1=kx-ak^3t-\frac{kC}{2}y^2+K_1\exp[2C\int_0^t h(s)ds+2Cb_2(B(t)-\frac{t^2}{2})]y$$

$$-\frac{K_1^2}{k}\{\int_0^t [h(s)\diamondsuit \exp[4C\int_0^s h(\tau)d\tau + 4Cb_2(B(s)-\frac{s^2}{2})]]ds$$

$$+\int_0^t [b_2 \exp[4C\int_0^s h(\tau)d\tau + 4Cb_2(B(s)-\frac{s^2}{2})]]\delta B(t)\} + K_2, \quad (3.22)$$

$$\theta_2 = kx - \frac{kC}{2}y^2 + b_1 W(t)y - ak^3t - 6k[\int_0^t \alpha(s)ds + B(t)]$$

$$-\frac{1}{k}\int_0^t b_1[h(s)\diamondsuit W^{2\diamond}(s) + a_2 W^{3\diamond}(s)]ds + A_5, \quad (3.23)$$

$$\phi(t,z) = \frac{1}{6k}[2b_1 C(h(t) + a_2 W(t))\diamondsuit W(t) - b_1 W'(t)]y + \alpha(t) + b_4 W(t). \quad (3.24)$$

We have used the following relation in equation (3.22)

$$\int_R \Psi(t)\delta B(t) = \int_R \Psi(t)\diamond W(t)dt, \quad \Psi(t)\in L^2(R),$$

where $\int(\cdot)\delta B(t)$ is Skorohod integral.

4. Conclusion

1. We have discussed some stochastic exact soliton-like solutions of the generalized Wick-type stochastic KdV equation by using extended tanh method and Hermite transformation.

2. When Wick product $\diamondsuit$ is an ordinary product $\cdot$ in equation (1.1), we obtain equation (1.2).

3. Equation (1.1) can be regarded as the perturbation of equation (1.2).

4. Noting that there exists a unitary mapping between the Wiener white noise space and the Poisson white noise space, we can simply obtain the solution of the Poisson SPDE by applying this mapping to the solution of the corresponding Gaussian SPDE. A nice and concise account of this connection was given by Benth and Gjerde, see [1]. We can see this in Section 4.9 of [7] as well. Hence, we can attain stochastic soliton solutions as we do in this section if the coefficients $H_i(t)$ $(i = 1, 2)$ are Poisson white noise functions in equation (1.1).

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