International Journal of Pure and Applied Mathematics
Volume 21 No. 1 2005, 117-126

SENSITIVITY ANALYSIS OF SOLUTIONS FOR PARAMETRIC GENERAL QUASIVARIATIONAL-LIKE INEQUALITIES

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Abstract: In this paper, we introduce a new class of parametric general quasivariational-like inequalities and establish the existence and sensitivity analysis of solutions for the parametric general quasivariational-like inequalities. Our results extend the corresponding results in [4]-[8], [11].

AMS Subject Classification: 47J20, 49J40
Key Words: sensitivity analysis, η-subdifferential mapping, parametric general quasivariational-like inequality

Received: April 26, 2005

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1. Introduction

Variational inequality theory has become a very powerful tool in pure and applied mathematics. In 1996, Adly [2] used the resolvent operator technique for maximal monotone mapping to study a general class of variational inclusions with single-valued mapping. Recently, Dafermos [3] studied the sensitivity property of solutions of a parametric variational inequality involving single-valued mapping in R^n. Afterwards, using the ideas of Dafermos, many researchers including Agarwal, Cho and Huang [1], Liu, Wang, Kang and Ume [8], Liu, Debnath, Kang and Ume [9], Yen and Lee [10] and others have established the sensitivity analysis of solutions of various types of variational inequalities and quasivariational inclusions in Hilbert spaces, respectively. At the same time, Ding and Luo [4] studied the quasivariational-like inequalities with η-subdifferential mapping on Hilbert spaces.

Inspired and motivated by the results [1], [4], [8]-[11], in this paper, we introduce and study a new class of parametric general quasivariational-like inequalities. We show its equivalence with a fixed point problem and establish the existence and sensitivity analysis of solutions for the parametric general quasivariational-like inequalities involving strongly monotone, Lipschitz continuous and η-subdifferential mappings. Our results extend and improve the corresponding results in [4]-[8], [11].

2. Preliminaries

Let H be a real Hilbert space with a norm $\|\cdot\|$ and inner product $\langle\cdot,\cdot\rangle$, respectively. Let P be a nonempty open subset of H in which the parameter λ takes values.

Definition 2.1. A mapping $g : H \times P \to H$ is said to be:

(1) γ-*strongly monotone* in the first argument if there exists a constant $\gamma > 0$ satisfying

$$\langle g(x,\lambda) - g(y,\lambda), x - y\rangle \geq \gamma\|x-y\|^2, \quad \forall x, y \in H,\ \lambda \in P;$$

(2) σ-*Lipschitz continuous* in the first argument if there exists a constant $\sigma > 0$ satisfying

$$\|g(x,\lambda) - g(y,\lambda)\| \leq \sigma\|x-y\|, \quad \forall x, y \in H,\ \lambda \in P;$$

(3) *continuous* (resp., *uniformly continuous* or *Lipschitz continuous*) in the second argument, if for each $x \in H$, $g(x,\cdot)$ is continuous (resp., uniformly continuous or Lipschitz continuous).

Definition 2.2. A mapping $\eta : H \times H \times P \to H$ is said to be:
(1) δ-*strongly monotone* if there exists a constant $\delta > 0$ satisfying

$$\langle \eta(x, y, \lambda), x - y\rangle \geq \delta \|x - y\|^2, \quad \forall x, y \in H, \lambda \in P;$$

(2) τ-*Lipschitz continuous* if there exists a constant $\tau > 0$ satisfying

$$\|\eta(x, y, \lambda)\| \leq \tau \|x - y\|, \quad \forall x, y \in H, \lambda \in P.$$

Definition 2.3. (see [4]) A functional $f : H \times H \to R \cup \{+\infty\}$ is said to be 0-*diagonally quasi-concave* (in short, 0-DQCV) in x if for any finite set $\{x_1, \cdots, x_n\} \subset H, \lambda \in P$ and for any $y = \sum_{i=1}^n l_i x_i$ with $l_i \geq 0$ and $\sum_{i=1}^n l_i = 1, \min_{1 \leq i \leq n} f(x_i, y) \leq 0$.

Definition 2.4. (see [4]) Let $\eta : H \times H \to H$ be a mapping. A proper functional $\phi : H \to R \cup \{+\infty\}$ is said to be η-*subdifferentiable* at a point $x \in H$ if there exists a point $f^* \in H$ such that

$$\phi(y) - \phi(x) \geq \langle f^*, \eta(y, x)\rangle, \quad \forall y \in H,$$

where f^* is called a η-subgradient of ϕ at x. The set of all η-subgradient of ϕ at x is denoted by $\Delta\phi(x)$. The mapping $\Delta\phi : H \to 2^H$ defined by

$$\Delta\phi(x) = \{f^* \in H : \phi(y) - \phi(x) \geq \langle f^*, \eta(y, x)\rangle, \forall y \in H\} \tag{2.1}$$

is said to be η-*subdifferential* of ϕ.

Definition 2.5. (see [4]) Let $\phi : H \to R \cup \{+\infty\}$ be a proper functional. For any given $x \in H$ and $\rho > 0$, if there exist a mapping $\eta : H \times H \to H$ and a unique point $u \in H$ such that

$$\langle u - x, \eta(y, u)\rangle \geq \rho\phi(u) - \rho\phi(y), \quad \forall y \in H,$$

then the mapping $x \mapsto u$, denoted by $J_\rho^{\Delta\varphi}(x)$, is said to be η-proximal mapping of ϕ.

By (2.1) and the definition of $J_\rho^{\Delta\varphi}(x)$, we have $x - u \in \rho\Delta\phi(u)$. It follows that

$$J_\rho^{\Delta\varphi}(x) = (I + \rho\Delta\phi)^{-1}(x),$$

where I is the identity mapping on H.

Let T, A, $g : H \times P \to H$ and $\eta : H \times H \times P \to H$ be a mappings and $\phi : H \times H \times P \to R \cup \{+\infty\}$ be a proper functional such that for each fixed $(y, \lambda) \in H \times P$, $\phi(\cdot, y, \lambda) : H \to H$ is lower semicontinuous and η-subdifferentiable on H

and $g(H\times\{\lambda\})\cap \operatorname{dom}\Delta\phi(\cdot,y,\lambda)\neq\emptyset$. For each $\lambda\in P$, we consider the following parametric general quasivariational-like inequality (PGQVLI):

Find $x\in H$ such that $g(x,\lambda)\in\operatorname{dom}\Delta\phi(\cdot,x,\lambda)$ and

$$\begin{aligned}&\langle T(x,\lambda)-A(x,\lambda),\eta(y,g(x,\lambda),\lambda)\rangle\\ \geq\ &\phi(g(x,\lambda),x,\lambda)-\phi(y,x,\lambda),\quad \forall y\in H.\end{aligned}\tag{2.2}$$

Special Cases

(A) If $T(x,\lambda)=T(x)$, $A(x,\lambda)=A(x)$, $\eta(y,g(x,\lambda),\ \lambda)=\eta(y,g(x))$, $\phi(y,x,\lambda)=\phi(y,x)$ and $\phi(g(x,\lambda),x,\lambda)=\phi(g(x),x)$ for all $(x,y,\lambda)\in H\times H\times P$, then the PGQVLI (2.2) is equivalent to find $x\in H$ such that $g(x)\in\operatorname{dom}\Delta\phi(\cdot,x)$ and

$$\langle T(x)-A(x),\eta(y,g(x))\rangle\geq\phi(g(x),x)-\phi(y,x),\quad \forall y\in H,\tag{2.3}$$

which was introduced and studied by Ding and Luo, [4].

(B) If $\phi(x,y,\lambda)=\phi(x)$ and $\eta(y,x,\lambda)=y-x$ for all $x,y\in H$, $\lambda\in P$, then the PGQVLI (2.2) reduces to the following variational inequality:

Find $x\in H$ such that $g(x)\in\operatorname{dom}\Delta\phi$ and

$$\langle T(x)-A(x),y-g(x)\rangle\geq\phi(g(x))-\phi(y),\quad \forall y\in H,\tag{2.4}$$

which was introduced and studied by Ding [5], Hassouni and Mouchfi [6] and Huang [7].

Lemma 2.1. (see [4]) *Let $\eta:H\times H\to H$ be δ-strongly monotone and τ-Lipschitz continuous such that $\eta(x,y)=-\eta(y,x)$ for all $x,y,\in H$ and for any given $x,u\in H$, the functional $h(y,u)=\langle x-u,\eta(y,u)\rangle$ is 0-DQCV in y. Let $\phi:H\to R$ be a lower semicontinuous η-subdifferentiable proper functional and $\rho>0$ be an arbitrary constant. Then the η-proximal mapping $J_\rho^{\Delta\phi}$ of ϕ is $\frac{\tau}{\delta}$-Lipschitz continuous.*

3. Existence and Sensitivity Analysis

We first transfer the PGQVLI (2.2) into a fixed point problem.

Theorem 3.1. *Let $\lambda\in P$ and $\rho>0$ be a constant. Then $x\in H$ is a solution of the* PGQVLI (2.2) *if and only if x satisfies the following relation:*

$$g(x,\lambda)=J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(x,\lambda)-\rho(T(x,\lambda)-A(x,\lambda))),\tag{3.1}$$

where $J_\rho^{\Delta\varphi(\cdot,x,\lambda)}(y) = (I+\rho\Delta\phi(\cdot,x,\lambda))^{-1}(y)$, $y \in H$, *is the* η*-proximal mapping of* $\phi(\cdot,x,\lambda)$.

Proof. $x \in H$ is a solution of the PGQVLI (2.2) if and only if

$$g(x,\lambda) = J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(x,\lambda) - \rho(T(x,\lambda) - A(x,\lambda))).$$

The equality holds if and only if

$$A(x,\lambda) - T(x,\lambda) \in \Delta\phi(g(x,\lambda),x,\lambda).$$

By the definition of η-subdifferential of $\phi(\cdot,x,\lambda)$, the above relation holds if and only if

$$\phi(y,x,\lambda) - \phi(g(x,\lambda),x,\lambda) \geq \langle A(x,\lambda) - T(x,\lambda), \eta(y,g(x,\lambda),\lambda)\rangle, \quad \forall y \in H.$$

That is,

$$\langle T(x,\lambda) - A(x,\lambda), \eta(y,g(x,\lambda),\lambda)\rangle \geq \phi(g(x,\lambda),x,\lambda) - \phi(y,x,\lambda), \quad \forall y \in H.$$

Hence, x is a solution of the PGQVLI (2.2). This completes the proof. □

Remark 3.1. Equation (3.1) can be written as

$$x = x - g(x,\lambda) + J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(x,\lambda) - \rho(T(x,\lambda) - A(x,\lambda))).$$

Now we prove the existence of a solution of the PGQVLI (2.2).

Theorem 3.2. *Let* $T : H \times P \to H$ *be* α*-strongly monotone and* β*-Lipschitz continuous in the first argument,* $A : H \times P \to H$ *be* γ*-Lipschitz continuous in the first argument and* $g : H \times P \to H$ *be* m*-strongly monotone and* σ*-Lipschitz continuous in the first argument. Let* $\eta : H \times H \times P \to H$ *be* δ*-strongly monotone and* τ*-Lipschitz continuous with* $\eta(x,y,\lambda) = -\eta(y,x,\lambda), \forall x,y \in H, \lambda \in P$, *and for each* $x,u \in H, \lambda \in P$ *the function* $h(y,u,\lambda) = \langle x-u, \eta(y,u,\lambda)\rangle$ *is* 0-DQCV *in* y. *Let* $\phi : H \times H \times P \to H$ *satisfy for each fixed* $(y,\lambda) \in H \times P$, $\phi(\cdot,y,\lambda) : H \to H$ *is a lower semicontinuous* η*-subdifferentiable proper functional with* $g(H \times \{\lambda\}) \cap \operatorname{dom} \Delta\phi(\cdot,y,\lambda) \neq \emptyset$. *Suppose that there exists a constant* μ *satisfying*

$$\|J_\rho^{\Delta\phi(\cdot,x,\lambda)}(z) - J_\rho^{\Delta\phi(\cdot,y,\lambda)}(z)\| \leq \mu\|x-y\|$$

for all $x,y,z, \in H$, $\lambda \in P$. *Let* $k = \mu + \frac{\tau+\delta}{\delta}\sqrt{1-2m+\sigma^2}$. *If there exists a constant* $\rho > 0$ *satisfying*

$$k + \frac{\rho\tau\gamma}{\delta} < 1 \tag{3.2}$$

and one of the following conditions:

$$\begin{aligned}
&|\beta| > |\gamma|,\\
&\tau\alpha > \delta\gamma(1-k) + \sqrt{(\beta^2-\gamma^2)(\tau^2-\delta^2(1-k)^2)},\\
&\left|\rho - \frac{(\tau\alpha+\delta\gamma(k-1))}{\tau(\beta^2-\gamma^2)}\right|\\
&\qquad < \frac{\sqrt{[\tau\alpha+\delta\gamma(k-1)]^2-(\beta^2-\gamma^2)[\tau^2-\delta^2(1-k)^2]}}{\tau(\beta^2-\gamma^2)};
\end{aligned} \tag{3.3}$$

$$\begin{aligned}
&|\beta| < |\gamma|,\\
&\left|\rho - \frac{(\tau\alpha+\delta\gamma(k-1))}{\tau(\beta^2-\gamma^2)}\right|\\
&\qquad > \frac{\sqrt{[\tau\alpha+\delta\gamma(k-1)]^2-(\beta^2-\gamma^2)[\tau^2-\delta^2(1-k)^2]}}{\tau(\gamma^2-\beta^2)}.
\end{aligned} \tag{3.4}$$

Then for each $\lambda \in P$, the PGQVLI (2.2) *has a unique solution in H.*

Proof. By Theorem 3.1, it is sufficient to show that there exists a unique $x \in H$ satisfying equation (3.1). Definite a mapping $F : H \times P \to H$ by

$$\begin{aligned}
&F(x,\lambda)\\
= \;& x - g(x,\lambda) + J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(x,\lambda)-\rho(T(x,\lambda)-A(x,\lambda))), \quad \forall x \in H.
\end{aligned} \tag{3.5}$$

For any $x, y \in H$, $\lambda \in p$, by the assumptions and Lemma 2.1, we have

$$\begin{aligned}
&\|F(x,\lambda)-F(y,\lambda)\|\\
\le\;& \|x-y-(g(x,\lambda)-g(y,\lambda))\|\\
&+\|J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(x,\lambda)-\rho(T(x,\lambda)-A(x,\lambda)))\\
&-J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(y,\lambda)-\rho(T(y,\lambda)-A(y,\lambda)))\|\\
&+\|J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(y,\lambda)-\rho(T(y,\lambda)-A(y,\lambda)))\\
&-J_\rho^{\Delta\phi(y,\lambda)}(g(y,\lambda)-\rho(T(y,\lambda)-A(y,\lambda)))\|\\
\le\;& \left(1+\frac{\tau}{\delta}\right)\|x-y-(g(x,\lambda)-g(y,\lambda))\|\\
&+\frac{\tau}{\delta}\|x-y-\rho(T(x,\lambda)-T(y,\lambda))\|\\
&+\frac{\rho\tau}{\delta}\|A(x,\lambda)-A(y,\lambda)\|+\mu\|x-y\|.
\end{aligned} \tag{3.6}$$

Since T, g are both strongly monotone and Lipschitz continuous in the first argument, we obtain that

$$\|x-y-(g(x,\lambda)-g(y,\lambda))\| \;\le\; \sqrt{1-2m+\sigma^2}\|x-y\|,$$

$$\|x-y-\rho(T(x,\lambda)-T(y,\lambda))\| \quad \leq \quad \sqrt{1-2\rho\alpha+\rho^2\beta^2}\|x-y\|.$$

By the Lipschitz continuity in the first argument of A, we get that

$$\|A(x,\lambda)-A(y,\lambda)\| \leq \gamma\|x-y\|.$$

From the above inequalities and (3.6), we derive that

$$\begin{aligned}
& \|F(x,\lambda)-F(y,\lambda)\| \\
\leq \;& [(1+\tau\delta^{-1})\sqrt{1-2m+\sigma^2} \\
& +\tau\delta^{-1}\sqrt{1-2\rho\alpha+\rho^2\beta^2}+\rho\tau\gamma\delta^{-1}+\mu]\|x-y\| \\
= \;& \theta\|x-y\|, \qquad (3.7)
\end{aligned}$$

where $\theta = k+\tau\delta^{-1}(\sqrt{1-2\rho\alpha+\rho^2\beta^2}+\gamma\rho)$. Note that (3.2) and one of (3.3) and (3.4) imply $\theta < 1$. Hence $F(\cdot,\lambda)$ is a contraction mapping and there exists a unique point $x \in H$ such that $x = F(x,\lambda)$, that is,

$$g(x,\lambda) = J_\rho^{\Delta\phi(\cdot,x,\lambda)}(g(x,\lambda)-\rho(T(x,\lambda)-A(x,\lambda))).$$

Therefore, $x \in H$ is a unique solution of the PGQVLI (2.2). This completes the proof. □

Remark 3.2. Theorem 3.2 generalizes Theorem 3.3 of Ding and Luo [4].

Now we analyze the sensitivity of solutions for the parametric general quasivariational-like inequality (2.2).

Theorem 3.3. *Let the conditions of Theorem 3.2 be satisfied. Assume that g, T, A are continuous (resp., uniformly continuous or Lipschitz continuous) in the second argument. Suppose that there exists ζ satisfying*

$$\|J_\rho^{\Delta\phi(\cdot,x,\lambda)}(z)-J_\rho^{\Delta\phi(\cdot,x,\tau)}(z)\| \leq \zeta\|\lambda-\tau\|, \quad \forall x,z \in H,\ \lambda,\tau \in P. \qquad (3.8)$$

If there exists a constant $\rho > 0$ satisfying (3.2) and one of (3.3) and (3.4), then the solutions of the PGQVLI (2.2) are continuous (resp., uniformly continuous or Lipschitz continuous).

Proof. Let F be defined by (3.3). It follows from Theorem 3.2 that for each $\lambda \in P$, there exists a unique $x \in H$ denoted by $x(\lambda)$ such that it is the solution of the PGQVLI (2.2). Thus we have

$$x(\lambda) = F(x(\lambda),\lambda), x(\overline{\lambda}) = F(x(\overline{\lambda}),\overline{\lambda}), \quad \forall\lambda,\overline{\lambda} \in P,$$

and

$$\begin{aligned} & \|x(\lambda) - x(\overline{\lambda})\| \\ = & \|F(x(\lambda),\lambda) - F(x(\overline{\lambda}),\overline{\lambda})\| \\ \leq & \|F(x(\lambda),\lambda) - F(x(\lambda),\overline{\lambda})\| + \|F(x(\lambda),\overline{\lambda}) - F(x(\overline{\lambda}),\overline{\lambda})\|. \end{aligned} \tag{3.9}$$

It follows from (3.8) that

$$\begin{aligned} & \|F(x(\lambda),\lambda) - F(x(\lambda),\overline{\lambda})\| \\ = & \|x(\lambda) - g(x(\lambda),\lambda) \\ & + J_\rho^{\Delta\phi(\cdot,x(\lambda),\lambda)}(g(x(\lambda),\lambda) - \rho(T(x(\lambda),\lambda) - A(x(\lambda),\lambda)) \\ & - x(\lambda) + g(x(\lambda),\overline{\lambda}) \\ & - J_\rho^{\Delta\phi(\cdot,x(\lambda),\overline{\lambda})}(g(x(\lambda),\overline{\lambda}) - \rho(T(x(\lambda),\overline{\lambda}) - A(x(\lambda),\overline{\lambda})))\| \\ \leq & \|g(x(\lambda),\lambda) - g(x(\lambda),\overline{\lambda})\| \\ & + \|J_\rho^{\Delta\phi(\cdot,x(\lambda),\lambda)}(g(x(\lambda),\lambda) - \rho(T(x(\lambda),\lambda) - A(x(\lambda),\lambda))) \\ & - J_\rho^{\Delta\phi(\cdot,x(\lambda),\lambda)}(g(x(\lambda),\overline{\lambda}) - \rho(T(x(\lambda),\overline{\lambda}) - A(x(\lambda),\overline{\lambda})))\| \\ & + \|J_\rho^{\Delta\phi(\cdot,x(\lambda),\lambda)}(g(x(\lambda),\overline{\lambda}) - \rho(T(x(\lambda),\overline{\lambda}) - A(x(\lambda),\overline{\lambda}))) \\ & - J_\rho^{\Delta\phi(\cdot,x(\lambda),\overline{\lambda})}(g(x(\lambda),\overline{\lambda}) - \rho(T(x(\lambda),\overline{\lambda}) - A(x(\lambda),\overline{\lambda})))\| \\ \leq & \left(1 + \frac{\tau}{\delta}\right) \|g(x(\lambda),\lambda) - g(x(\lambda),\overline{\lambda})\| \\ & + \frac{\rho\tau}{\delta}(\|T(x(\lambda),\lambda) - T(x(\lambda),\overline{\lambda})\| \\ & + \|A(x(\lambda),\lambda) - A(x(\lambda),\overline{\lambda})\|) + \zeta\|\lambda - \overline{\lambda}\|. \end{aligned} \tag{3.10}$$

From (3.7) we know that

$$\|F(x(\lambda),\overline{\lambda}) - F(x(\overline{\lambda}),\overline{\lambda})\| \leq \theta\|x(\lambda) - x(\overline{\lambda})\|. \tag{3.11}$$

Combining (3.9)-(3.11) we conclude that

$$\begin{aligned} & \|x(\lambda) - x(\overline{\lambda})\| \\ \leq & \frac{1}{1-\theta}[\left(1 + \frac{\tau}{\delta}\right) \|g(x(\lambda),\lambda) - g(x(\lambda),\overline{\lambda})\| \\ & + \frac{\rho\tau}{\delta}(\|T(x(\lambda),\lambda) - T(x(\lambda),\overline{\lambda})\| \\ & + \|A(x(\lambda),\lambda) - A(x(\lambda),\overline{\lambda})\|) + \zeta\|\lambda - \overline{\lambda}\|]. \end{aligned} \tag{3.12}$$

Note that g, T, A are continuous (resp., uniformly continuous or Lipschitz continuous) in the second argument. It follows from (3.12) that the solutions of the PGQVLI (2.2) are continuous (resp., uniformly continuous or Lipschitz continuous). This completes the proof. □

Remark 3.3. Theorem 3.3 extends and improves Theorem 3.3 of Ding and Luo [4], Theorem 2.1 of Liu, Wang, Kang and Ume [8], and Theorem 3.1 of Zhu, Feng, Liu and Kang [11].

Acknowledgements

This work was supported by the Science Research Foundation of Educational Department of Liaoning Province (2004C063).

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