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ON LOWER AND UPPER δ-SEMICONTINUOUS MULTIFUNCTIONS

Erdal Ekici

Department of Mathematics
Canakkale Onsekiz Mart University
Terzioglu Campus
Canakkale, 17020, TURKEY
e-mail: eekici@comu.edu.tr

Abstract: In this paper, the author define a multifunction $F : X \to Y$ to be upper (lower) δ-semicontinuous if $F^+(V)$ $(F^-(V))$ is δ-semiopen in X for every open set V of Y. The author obtain some characterizations and several properties and relationships concerning upper (lower) δ-semicontinuous multifunctions.

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1. Introduction

Park, Lee and Son [10] introduced and investigated the notion of δ-semiopen sets in topological spaces. The purpose of the present paper is to introduce a new form of continuous multifunctions and to obtain properties of such multifunctions.

In this paper, spaces X and Y mean topological spaces. Let A be a subset of a space X. For a subset A of X, $\mathrm{cl}\,(A)$ and $\mathrm{int}\,(A)$ represent the closure of A and the interior of A, respectively. Throughout this paper, $F : X \to Y$ presents a multifunction. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X : F(x) \subset B\}$ and $F^-(B) = \{x \in X : F(x) \cap B \neq \emptyset\}$, see [3]. For a multifunction $F : X \to Y$, the graph multifunction $G_F : X \to X \times Y$

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is defined as follows $G_F(x) = \{x\} \times F(x)$ for every $x \in X$ and the subset $\{\{x\} \times F(x) : x \in X\} \subset X \times Y$ is called the multigraph of F and is denoted by $G(F)$.

A subset A of a space X is said to be regular open if $A = \mathrm{int}\,(\mathrm{cl}\,(A))$, see [12]. The δ-interior [13] of a subset A of (X, τ) is the union of all regular open sets of X contained in A is denoted by $\delta - \mathrm{int}\,(A)$. A subset A is called δ-open, see [13], if $A = \delta - \mathrm{int}\,(A)$. The complement of δ-open set is called δ-closed. The δ-closure of a subset A is defined by $\{x \in X : A \cap \mathrm{int}\,(\mathrm{cl}\,(U)) \neq \emptyset,\ U \in \tau$ and $x \in U\}$ and is denoted by $\delta - \mathrm{cl}\,(A)$.

A subset A of a space X is said to be δ-semiopen, see [10] (resp. semi-open [7], α-open [9], preopen [8], β-open, see [1]) if $A \subset \mathrm{cl}\,(\delta - \mathrm{int}\,(A))$ (resp. $A \subset \mathrm{cl}\,(\mathrm{int}\,(A))$, $A \subset \mathrm{int}\,(\mathrm{cl}\,(\mathrm{int}\,(A)))$, $A \subset \mathrm{int}\,(\mathrm{cl}\,(A))$, $A \subset \mathrm{cl}\,(\mathrm{int}\,(\mathrm{cl}\,(A))))$. The family of all δ-semiopen (resp. semi-open) sets of X (containing a point $x \in X$) is denoted by $\delta SO(X)$ (resp. $SO(X)$) (resp. $\delta SO(X, x)$, $SO(X, x)$). The family of all δ-open sets of X is denoted by $\delta O(X)$. The complement of a δ-semiopen set is said to be δ-semiclosed. The intersection of all δ-semiclosed sets (resp. semi-open, α-open, preopen, β-open) of X containing A is called the δ-semiclosure, see [10] (resp. semi-closure [4], α-closure [9], preclosure [5], β-closure, see [1]) of A and is denoted by $\delta - \mathrm{scl}\,(A)$ (resp. $s - \mathrm{cl}\,(A)$, $\alpha - \mathrm{cl}\,(A)$, $p - \mathrm{cl}\,(A)$, $\beta - \mathrm{cl}\,(A)$). The union of all δ-semiopen sets of X contained A is called δ-semi-interior of A and is denoted by $\delta - s\mathrm{int}\,(A)$. A subset U of X is called a δ-semineighborhood of a point $x \in X$ if there exists a δ-semiopen set V such that $x \in V \subset U$.

Definition 1. A multifunction $F : X \to Y$ is said to be (1) upper quasi-continuous [11] at $x \in X$ if for each open set V of Y containing $F(x)$, there exists $U \in SO(X, x)$ such that $F(U) \subset V$, (2) lower quasi-continuous [11] at $x \in X$ if for each open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists $U \in SO(X, x)$ such that $F(u) \cap V \neq \emptyset$ for every $u \in U$.

2. δ-Semicontinuous Multifunctions

Now, properties of the class of δ-semicontinuous multifunctions are investigated.

Definition 2. A multifunction $F : X \to Y$ is said to be:

(1) Lower δ-semicontinuous at $x \in X$ if for each open set V such that $x \in F^{-}(V)$, there exists a δ-semiopen set U containing x such that $U \subset F^{-}(V)$.

(2) Upper δ-semicontinuous at $x \in X$ if for each open set V such that $x \in F^{+}(V)$, there exists a δ-semiopen set U containing x such that $U \subset F^{+}(V)$.

(3) Lower (upper) δ-semicontinuous if F has this property at each point of X.

Theorem 3. *The following statements are equivalent for a multifunction* $F : X \to Y$.

a) F *is upper* δ*-semicontinuous;*

b) $F^+(V)$ *is a* δ*-semiopen set for any open set* $V \subset Y$*;*

c) $F^-(K)$ *is a* δ*-semiclosed set for any closed set* $K \subset Y$*;*

d) for each $x \in X$ *and for each open set* V *such that* $F(x) \subset V$*, there exists a* δ*-semiopen set* U *containing* x *such that if* $y \in U$*, then* $F(y) \subset V$*;*

e) for each point x *of* X *and each neighbourhood* V *of* $F(x)$*,* $F^+(V)$ *is a* δ*-semineighbourhood of* x*;*

f) for each point x *of* X *and each neighbourhood* V *of* $F(x)$*, there exists a* δ*-semineighbourhood* U *of* x *such that* $F(U) \subset V$.

Proof. (a)⇔(b). Let $V \subset Y$ be a open set and let $x \in F^+(V)$. From (a), there exists a δ-semiopen set U containing x such that $U \subset F^+(V)$. It follows that $F^+(V)$ is a δ-semiopen set.

The converse can be obtained similarly from the definition of δ-semiopen set.

(b)⇔(c), (a)⇔(d). Obvious.

(b)⇒(e). Let $x \in X$ and V be a neighbourhood of $F(x)$. Then there exists an open set G of Y such that $F(x) \subset G \subset V$. Therefore, we obtain $x \in F^+(G) \subset F^+(V)$. Since $F^+(G) \in \delta SO(X)$, $F^+(V)$ is a δ-semineighbourhood of x.

(e)⇒(f). Let $x \in X$ and V be a neighbourhood of $F(x)$. By (e), $F^+(V)$ is a δ-semineighbourhood of x. Take $U = F^+(V)$. Then $F(U) \subset V$.

(f)⇒(a). Let $x \in X$ and V be any open set of Y such that $F(x) \subset V$. Then V is a neighbourhood of $F(x)$. By (f), there exists a δ-semineighbourhood U of x such that $F(U) \subset V$. Therefore, there exists $G \in \delta SO(X)$ such that $x \in G \subset U$ and hence $F(G) \subset F(U) \subset V$. We obtain that F is upper δ-semicontinuos. □

Remark 4. For a multifunction $F : X \to Y$, the following diagram hold:

$$\delta\text{-semicontinuous} \;\Rightarrow\; \text{quasi-continuous}$$

The following example shows that the above implication is not reversible.

Example 5. Let $X = \{a, b, c\}$. Let τ be a topology on X given by $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$. Define the multifunction $F : X \to X$ by $F(a) = \{a\}$, $F(b) = \{b\}$, $F(c) = \{c\}$. Then F is lower (upper) quasi-continuous multifunction but F is not lower (upper) δ-semicontinuous.

Theorem 6. *Let $F : X \to Y$ be a multifunction. Then the following statements are equivalent:*

a) F is lower δ-semicontinuous multifunction;

b) $F^-(V)$ is a δ-semiopen set for any open set $V \subset Y$;

c) $F^+(K)$ is a δ-semiclosed set for any closed set $K \subset Y$;

d) for each $x \in X$ and for each open set V such that $F(x) \cap V \neq \emptyset$, there exists a δ-semiopen set U containing x such that if $y \in U$, then $F(y) \cap V \neq \emptyset$.

Proof. It can be obtained similarly as the previous theorem. □

Theorem 7. *Let $F : X \to Y$ be a multifunction from a topological space (X, τ) to a topological space (Y, υ) and let $F(X)$ be endowed with subspace topology. If F is upper δ-semicontinuous multifunction, then $F : X \to F(X)$ is upper δ-semicontinuous multifunction.*

Proof. Since F is upper δ-semicontinuous, $F^+(V \cap F(X)) = F^+(V) \cap F^+(F(X)) = F^+(V)$ is δ-semiopen for each open subset V of Y. Hence $F : X \to F(X)$ is upper δ-semicontinuous multifunction. □

Definition 8. Let (X, τ) be a topological space. The collection of all regular open sets forms a base for a topology τ^*. It is called the semiregularization. In case when $\tau = \tau^*$, the space (X, τ) is called semi-regular [12].

Definition 9. A subset A of a topological space X is said to be:

(1) α-paracompact [14] if every cover of A by open sets of X is refined by cover of A which consists of open sets of X and locally finite in X;

(2) α-regular [6] if for each $a \in A$ and each open set U of X containing a, there exists an open set G of X such that $a \in G \subset \mathrm{cl}\,(G) \subset U$.

Lemma 10. (see [6]) *If A is an α-regular α-paracompact set of a topological space X and U is an open neighbourhood of A, then there exists an open set G of X such that $A \subset G \subset \mathrm{cl}\,(G) \subset U$.*

Definition 11. For a multifunction $F : X \to Y$, a multifunction $\mathrm{cl}\,(F) : X \to Y$ [2] is defined as $\mathrm{cl}\,(F)(x) = \mathrm{cl}\,(F(x))$ for each point $x \in X$. Multifunctions $\alpha\text{-cl}\,(F)$, $\mathrm{scl}\,(F)$, $\delta\text{-scl}\,(F)$, $\mathrm{pcl}\,(F)$, $\beta\text{-cl}\,(F)$ can be defined similarly.

Lemma 12. *If $F : X \to Y$ is a multifunction such that $F(x)$ is α-regular α-paracompact for each $x \in X$ and Y is semi-regular, then for each open set V of Y, $G^+(V) = F^+(V)$, where G denotes $\alpha\text{-cl}\,(F)$, $\mathrm{scl}\,(F)$, $\delta\text{-scl}\,(F)$, $\mathrm{pcl}\,(F)$, or $\beta\text{-cl}\,(F)$.*

Theorem 13. *Let $F : X \to Y$ be a multifunction such that $F(x)$ is α-regular α-paracompact for each $x \in X$ and Y is semi-regular. Then the following are equivalent:*

(1) F is upper δ-semicontinuous;
(2) $\delta\text{-scl}\,(F)$ is upper δ-semicontinuous;
(3) $\beta\text{-cl}\,(F)$ is upper δ-semicontinuous;
(4) $\text{scl}\,(F)$ is upper δ-semicontinuous;
(5) $\text{pcl}\,(F)$ is upper δ-semicontinuous;
(6) $\alpha\text{-cl}\,(F)$ is upper δ-semicontinuous.

Proof. We take $G = \alpha\text{-cl}\,(F)$, $\text{scl}\,(F)$, $\delta\text{-scl}\,(F)$, $\text{pcl}\,(F)$, or $\beta\text{-cl}\,(F)$. Suppose that F is upper δ-semicontinuous. Let $x \in X$ and V be any open set of Y containing $G(x)$. By the previous lemma, we have $x \in G^+(V) = F^+(V)$ and hence there exists $U \in \delta SO(X, x)$ such that $F(U) \subset V$. Since $F(u)$ is α-regular α-paracompact for each $u \in U$, there exists an open set W such that $F(u) \subset W \subset \text{cl}\,(W) \subset V$, hence $G(u) \subset \text{cl}\,(W) \subset V$ for each $u \in U$. Therefore, we obtain $G(U) \subset V$. This shows that G is upper δ-semicontinuous.

Conversely, suppose that G is upper δ-semicontinuous. Let $x \in X$ and V be any open set of Y containing $F(x)$. By the previous lemma, we have $x \in G^+(V) = F^+(V)$ and hence $G(x) \subset V$. There exists $U \in \delta SO(X, x)$ such that $G(U) \subset V$. Therefore, we obtain $U \subset G^+(V) = F^+(V)$ and hence $F(U) \subset V$. This shows that f is upper δ-semicontinuous. □

Lemma 14. *If $F : X \to Y$ is a multifunction and Y is semi-regular, then for each open set V of Y, $G^-(V) = F^-(V)$, where G denotes $\alpha\text{-cl}\,(F)$, $\text{scl}\,(F)$, $\delta\text{-scl}\,(F)$, $\text{pcl}\,(F)$, or $\beta\text{-cl}\,(F)$.*

Theorem 15. *For a multifunction $F : X \to Y$, the following are equivalent:*
(1) F is lower δ-semicontinuous;
(2) $\delta\text{-scl}\,(F)$ is lower δ-semicontinuous;
(3) $\beta\text{-cl}\,(F)$ is lower δ-semicontinuous;
(4) $\text{scl}\,(F)$ is lower δ-semicontinuous;
(5) $\text{pcl}\,(F)$ is lower δ-semicontinuous;
(6) $\alpha\text{-cl}\,(F)$ is lower δ-semicontinuous.

Theorem 16. *Let $F : X \to Y$ be a multifunction from a topological space (X, τ) to a topological space (Y, υ). F is lower δ-semicontinuous multifunction if and only if $F(\delta\text{-scl}\,(A)) \subset \text{cl}\,(F(A))$ for each $A \subset X$.*

Proof. Suppose that F is lower δ-semicontinuous and $A \subset X$. Since $\text{cl}\,(F(A))$ is a closed set, it follows that $F^+(\text{cl}\,(F(A)))$ is a δ-semiclosed set in X from Theorem 6. Since $A \subset F^+(\text{cl}\,(F(A)))$, then $\delta\text{-scl}\,(A) \subset \delta - \text{scl}\,(F^+(\text{cl}\,(F(A)))) = F^+(\text{cl}\,(F(A)))$. Thus, we obtain that $F(\delta - \text{scl}\,(A)) \subset F(F^+(\text{cl}\,(F(A)))) \subset \text{cl}\,(F(A))$.

Conversely, suppose that $F(\delta - \mathrm{scl}\,(A)) \subset \mathrm{cl}\,(F(A))$ for each $A \subset X$. Let K be any closed set in Y. Then $F(\delta - \mathrm{scl}\,(F^+(K))) \subset \mathrm{cl}\,(F(F^+(K)))$ and $\mathrm{cl}\,(F(F^+(K))) \subset \mathrm{cl}\,(K) = K$. Hence, δ-$\mathrm{scl}\,(F^+(K)) \subset F^+(K)$ which shows that F is lower δ-semicontinuous multifunction. □

Theorem 17. *Let $F : X \to Y$ be a multifunction from a topological space (X, τ) to a topological space (Y, υ). F is lower δ-semicontinuous multifunction if and only if δ-$\mathrm{scl}\,(F^+(B)) \subset F^+(\mathrm{cl}\,(B))$ for each $B \subset Y$.*

Proof. Suppose that F is lower δ-semicontinuous multifunction and $B \subset Y$. Then $F^+(\mathrm{cl}\,(B))$ is δ-semiclosed in X and $F^+(\mathrm{cl}\,(B)) = \delta - \mathrm{scl}\,(F^+(\mathrm{cl}\,(B)))$. Hence, δ-$\mathrm{scl}\,(F^+(B)) \subset F^+(\mathrm{cl}\,(B))$.

Conversely, let K be any closed set in Y. Then δ-$\mathrm{scl}\,(F^+(K)) \subset F^+(\mathrm{cl}\,(K)) = F^+(K) \subset \delta - \mathrm{scl}\,(F^+(K))$. Thus, $F^+(K) = \delta - \mathrm{scl}\,(F^+(K))$ which shows that F is lower δ-semicontinuous multifunction. □

Definition 18. A space X is said to be δ-semi-compact if every δ-semiopen cover of X has a finite subcover.

Theorem 19. *Let $F : X \to Y$ be an upper δ-semicontinuous surjective multifunction such that $F(x)$ is compact for each $x \in X$. If X is a δ-semi-compact space, then Y is compact.*

Proof. Let $\{V_\lambda : \lambda \in \Lambda\}$ be a open cover of Y. Since $F(x)$ is compact for each $x \in X$, there exists a finite subset $\Lambda(x)$ of Λ such that $F(x) \subset \cup\{V_\lambda : \lambda \in \Lambda(x)\}$. Put $V(x) = \cup\{V_\lambda : \lambda \in \Lambda(x)\}$. Since F is upper δ-semicontinuous, there exists a δ-semiopen set $U(x)$ of X containing x such that $F(U(x)) \subset V(x)$. Then the family $\{U(x) : x \in X\}$ is a δ-semiopen cover of X and since X is δ-semi-compact, there exists a finite number of points, say, x_1, x_2, x_3, ...,x_n in X such that $X = \cup\{U(x_i) : i = 1, 2, 3, ..., n\}$. Hence we have $Y = F(X) = F(\overset{n}{\underset{i=1}{\cup}} U(x_i)) = \overset{n}{\underset{i=1}{\cup}} F(U(x_i)) \subset \overset{n}{\underset{i=1}{\cup}} V(x_i) = \overset{n}{\underset{i=1}{\cup}} \underset{\lambda \in \Lambda(x_i)}{\cup} V_\lambda$. This shows that Y is compact. □

Theorem 20. *Let (X, τ), (Y, υ), (Z, ω) be topological spaces and let $F : X \to Y$ and $G : Y \to Z$ be multifunctions. If $F : X \to Y$ is upper (lower) δ-semicontinuous multifunction and $G : Y \to Z$ is upper (lower) semicontinuous multifunction, then $G \circ F : X \to Z$ is a upper (lower) δ-semicontinuous multifunction.*

Proof. Let $V \subset Z$ be any open set. From the definition of $G \circ F$, we have $(G \circ F)^+(V) = F^+(G^+(V))$ $((G \circ F)^-(V) = F^-(G^-(V)))$. Since G is upper (lower) semicontinuous multifunction, it follows that $G^+(V)$ ($G^-(V)$) is an open set. Since F is upper (lower) δ-semicontinuous multifunction, it follows that $F^+(G^+(V))$ ($F^-(G^-(V))$) is a δ-semiopen set. It shows that $G \circ F$ is a upper (lower) δ-semicontinuous multifunction. □

Theorem 21. *Let $F : X \to Y$ be a multifunction from a semi-regular topological space (X, τ) to a topological space (Y, υ). Then F is upper (lower) δ-semicontinuous multifunction if and only if F is upper (lower) quasi-continuous.*

Lemma 22. *Let A and X_0 be subsets of a space (X, τ). If $A \in \delta SO(X)$ and $X_0 \in \delta O(X)$, then $A \cap X_0 \in \delta SO(X_0)$.*

Lemma 23. *Let $A \subset X_0 \subset X$. If $X_0 \in \delta O(X)$ and $A \in \delta SO(X_0)$, then $A \in \delta SO(X)$.*

Theorem 24. *Let $\{U_\lambda : \lambda \in \Lambda\}$ be a δ-open cover of a space X. Then a multifunction $F : X \to Y$ is upper δ-semicontinuous (resp. lower δ-semicontinuous) if and only if the restriction $F\,|_{U_\lambda}: U_\lambda \to Y$ is upper δ-semicontinuous (resp. lower δ-semicontinuous) for each $\lambda \in \Lambda$.*

Proof. ($\Rightarrow$) Let $\lambda \in \Lambda$ and V be any open set of Y. Since F is upper δ-semicontinuous, $F^+(V)$ is δ-semiopen in X. By Lemma 22, $(F\,|_{U_\lambda})^+(V) = F^+(V) \cap U_\lambda$ is δ-semiopen in U_λ and hence $F\,|_{U_\lambda}$ is upper δ-semicontinuous.

($\Leftarrow$) Let V be any open set of Y. Since $F\,|_{U_\lambda}$ is upper δ-semicontinuous for each $\lambda \in \Lambda$, $(F\,|_{U_\lambda})^+(V) = F^+(V) \cap U_\lambda$ is δ-semiopen in U_λ. By Lemma 23, $(F\,|_{U_\lambda})^+(V)$ is δ-semiopen in X for each $\lambda \in \Lambda$. We obtain that $F^+(V) = \bigcup_{\lambda \in \Lambda} (F\,|_{U_\lambda})^+(V)$ is δ-semiopen in X. Hence F is upper δ-semicontinuous. □

Theorem 25. *If Y is normal space and $F_i : X_i \to Y$ is upper δ-semicontinuous multifunction such that F_i is point closed for $i = 1, 2$, then a set $K = \{(x_1, x_2) \in X_1 \times X_2 : F_1(x_1) \cap F_2(x_2) \neq \emptyset\}$ is δ-semiclosed set in $X_1 \times X_2$.*

Proof. Let $(x_1, x_2) \in (X_1 \times X_2) \backslash K$. Then $F_1(x_1) \cap F_2(x_2) = \emptyset$. Since Y is normal and F_i is point closed for $i = 1, 2$, there exist disjoint open sets V_1, V_2 such that $F_i(x_i) \subset V_i$ for $i = 1, 2$. Since F_i is upper δ-semicontinuous, $F_i^+(V_i)$ is δ-semiopen for $i = 1, 2$. Put $U = F_1^+(V_1) \times F_2^+(V_2)$, then U is δ-semiopen and $(x_1, x_2) \in U \subset (X_1 \times X_2) \backslash K$. This shows that $(X_1 \times X_2) \backslash K$ is δ-semiopen and hence K is δ-semiclosed in $X_1 \times X_2$. □

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