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STABILITY OF SWITCHED LINEAR NEUTRAL DELAY SYSTEMS

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Abstract: The concept for switched neutral delay system is introduced for the first time in this paper. The asymptotical stability problems for switched linear neutral delay systems are considered. By using single-Lyapunov technique and convex combination condition, some sufficient conditions of asymptotical stability and the corresponding switching laws are given in form of linear matrix inequalities (LMIs). The simulation results demonstrate the effectiveness of the design method.

AMS Subject Classification: 34K40, 34K20
Key Words: switched system, neutral delay system, asymptotical stability

1. Introduction

Many engineering systems can be described by using neutral delay systems, such as processes including steam or water pipes, heater exchangers [6], lossless transmission lines and full wave equivalent circus (PEECS), see [1]. In [10], four dimensional linear neutral delay system was derived for the current and voltage in a lossless transmission line. Also, the neutral system often appears in the study of automatic control, population dynamics and so on [9]. Because the neutral delay system is of both theoretical and practical interests, so in recent years, considerable attention has been focused on it, especially in stability analysis and controller synthesis.

On the other hand, switched system is an important kind in hybrid systems, which attracted great attention recently (see [5], [11], [13], [7], [2]). If every subsystem in switched system is neutral delay system, then this system is

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referred to as switched neutral delay system. To the best authors' knowledge, up to now, the report for the switched neutral delay system has not been seen yet. It is natural to introduce the concept of switched neutral delay system. There are a lot of models in real life. For example, when we consider the relationship between the current and the voltage, several lossless transmission lines switched by a certain switching law can construct a model of switched neutral delay system. For switching may destabilize a switched system even if all individual subsystems are stable; on the other hand, it may be possible to stabilize a switched system by means of suitably constrained switching even if all individual subsystems are unstable. So it is important to study the stability for switched neutral delay system.

This paper focuses on the stability for switched linear neutral delay system, and sufficient conditions of stability are presented by using single-Lyapunov in terms of linear matrix inequalities. In this paper, the method used to divide the switching reign distinguishes itself from the method before. Switching reign was divided in R^n in previous woks. While in this paper, switching reign is divided in R^{2n}. At last, a numerical example is given to illustrate the effectiveness of the proposed method.

2. System Description and Problem Formulation

Consider a class of switched linear neutral delay system as following

$$\begin{aligned} &\frac{d}{dt}[x(t) - A_d x(t-h)] = A_{\sigma(t)} x(t) + A_h x(t-h); \\ &x_{t_0}(\theta) = x(t_0+\theta) = \varphi(\theta); \quad \theta \in [-h, 0]\,, \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is state, A_h, A_d are constant matrices, $A_{\sigma(t)} \in R^{n\times n}, \sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ is a piecewise function dependent on the time t or state x. $\varphi(\theta)$ is continuous vector-valued initial function, $h > 0$ is constant time delay.

Problem. Design switching law $\sigma(t) : [0, +\infty) \to M$ such that switched linear neutral delay system (2.1) is asymptotically stable by using single-Lyapunov function technique.

3. The Asymptotically Stability for Switched Linear Neutral Delay System

Let $\gamma_{\alpha_1,\alpha_2,\cdots,\alpha_m}(A_1, A_2, \cdots, A_m)$ denotes the family convex combination of the matrices $A_1, A_2, \cdots, A_m$, i.e.,

$$\gamma_{\alpha_1,\alpha_2,\cdots,\alpha_m}(A_1, A_2, \cdots, A_m) \\ = \{\alpha_1 A_1 + \alpha_2 A_2 + \cdots + \alpha_m A_m \mid \alpha_1, \alpha_2, \cdots, \alpha_m \in [0,1], \sum_{i=1}^{m} \alpha_i = 1\}\,.$$

Theorem 3.1. *If there exists* $\overline{A} \in \gamma_{\alpha_1,\alpha_2,\cdots,\alpha_m}(A_1, A_2, \cdots, A_m)$*, positive matrices* $P > 0, S > o$ *such that the LMI*

$$\Pi = \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{12}^T & -\Delta 22 \end{pmatrix}, \tag{3.1}$$

where:

$$\Delta_{11} = P\overline{A} + \overline{A}^T P + S\,, \quad \Delta_{12} = (P\overline{A} + S)A_d + PA_h\,, \quad \Delta_{22} = S - A_d^T S A_d$$

holds, then for any delay $h > 0$ *there exists a switching law* $\sigma(t) : [0, +\infty) \to M$*, which makes the system (2.1) be asymptotically stable.*

Proof. According to $\overline{A} \in \gamma_{\alpha_1,\alpha_2,\cdots,\alpha_m}(A_1, A_2, \cdots, A_m)$, there exist $\alpha_1, \alpha_2, \cdots, \alpha_m \in [0,1]$, such that

$$\sum_{i=1}^{m} \alpha_i = 1, \quad \overline{A} = \sum_{i=1}^{m} \alpha_i A_i\,. \tag{3.2}$$

Substituting (3.2) into (3.1), results in

$$\sum_{i=1}^{m} \alpha_i \Pi_i < 0, \quad \text{where} \quad \Pi_i = \begin{pmatrix} \Delta_{i11} & \Delta_{i12} \\ \Delta_{i12}^T & -\Delta 22 \end{pmatrix},$$

$$\Delta_{i11} = PA_i + A_i^T P + S\,, \quad \Delta_{i12} = (PA_i + S)A_d + PA_h\,.$$

Assuming $\forall z, y \in R_n$ and $(z, y) \in R_{2n}/\{0\}$, then we have

$$(z^T, y^T) \sum_{i=1}^{m} \alpha_i \Pi_i (z^T, y^t)^T < 0.$$

Now, divide the space R_{2n}. Let

$$\Omega_i = \{(z^T, y^T)^T \mid (z^T, y^T)\Pi_i(z^T, y^t)^T < 0\},$$

then

$$\bigcup_{i=1}^{m} \Omega_i = R_{2n}/\{0\}.$$

Construct switching reign,

$$\ddot{\Omega}_1 = \Omega_1, \cdots, \ddot{\Omega}_i = \Omega_i - \bigcup_{j=1}^{i-1} \ddot{\Omega}_j, \cdots, \quad \ddot{\Omega}_m = \Omega_m - \bigcup_{j=1}^{m-1} \ddot{\Omega}_j.$$

It is obvious

$$\bigcup_{i=1}^{m-1} \ddot{\Omega}_i = R_{2n}/\{0\}, \quad \text{and} \quad \ddot{\Omega}_i \bigcap \ddot{\Omega}_j = \emptyset, \quad i \neq j.$$

Define a difference operator $\wp$ as

$$\wp(\varphi) = \varphi(0) - A_d\varphi(-h).$$

Construct switching law

$$\sigma(t) = i, \quad \text{when} \quad (\wp, x_h) \in \ddot{\Omega}_i, \quad i \in M. \tag{3.3}$$

Next, by using single-Lyapunov function technique we will proof system (2.1) satisfies conditions of theory of single-Lyapunov.

Consider the following Lyapunov-Krasovskii candidate function,

$$V(x_t, t) = \wp^T(x(t))P\wp(x(t)) + \int_{-h}^{0} x^T(t+\theta)Sx(t+\theta)d\theta\,. \tag{3.4}$$

It is easy to proof that there exist $c_1 > 0, c_2 > 0$ such that

$$c_1 \parallel \wp(\varphi) \parallel^2 \leq V(\varphi) \leq c_2 \sup_{-\tau\leq\theta\leq 0} \parallel \varphi(\theta) \parallel^2, \tag{3.5}$$

where $c_1 = \lambda_{max}(P)$, $c_2 = 4\lambda_{max}(P) + h\lambda_{max}(S)$.

For convenience, denote $x_h = x(t-h), x(t) = x$. From (3.1), we have

$$-\Delta_{22} = -(S - A_d^T S A_d) < 0,$$

i.e.

$$A_d^T S A_d - S < 0.$$

So, $\wp$ is stable.

When $(\wp^T, x_h^T)^T \in \ddot{\Omega}_i$, we have

$$(\wp^T, x_h^T)^T \Pi_i (\wp^T, x_h^T) < 0.$$

Differentiating $V(x_t, t)$ along solutions of (2.1) results in

$$\begin{aligned}\dot{V}(x_t,t) &= 2\wp^T P\dot{\wp} + x^T Sx - x_h^T Sx_h = 2\wp^T P(A_i x + A_h x_h) + x^T Sx - x_h^T Sx_h \\ &\leq 2\wp^T PA_i(\wp + A_d x_h) + 2\wp^T PA_h x_h + x^T Sx - x_h^T Sx_h \\ &= \wp^T(PA_i + A_i^T P)\wp + 2\wp^T(PA_iA_d + PA_h)x_h + (\wp + A_d x_h)^T S(\wp + A_d x_h) - x_h^T Sx_h \\ &= \wp^T(PA_i + A_i^T P + S)\wp + 2\wp^T[(PA_i + S)A_d + PA_h]x_h - x_h^T[S - A_d^T SA_d]x_h \\ &= (\wp^T, x_h^T)\Pi_i(\wp^T, x_h^T)^T < 0.\end{aligned}$$

Then by using single-Lyapunov function technique and Theorem 7.1 (see [3], p. 297), we can conclude that the system (2.1) is asymptotically stable under switching law (3.3).

Remark 3.2. If $\alpha_j = 1 (j \in M)$, then j-th subsystem is stable. If we let $\sigma(t) \equiv j$, then switched linear system becomes an usual neutral delay system.

Remark 3.3. The stability of difference operator can be found in [4], [3]. The stability of difference operator and

$$\dot{V}(x_t,t) \leq (\wp^T, x_h^T)\Pi_i(\wp^T, x_h^T)^T < 0$$

can make $V(x_t, t)$ monotonically decreasing.

Remark 3.4. The initial values for non-delay system are scalars or constant vector, while the initial values for delay systems are vector functions. So if let denote time distance from i_j-th to i_{j+1}-th subsystem, when $l < h$, then the initial-valued conditions for i_{j+1}-th subsystem not only depend on the state of i_j-th subsystem, but also depend on the state of subsystem before i_j-th. But from Theorem 7.2 (see [3], p. 26), we know, if initial condition is continuous vector-valued function, then there is a unique solution of the equation (3.6) though φ.

$$\frac{d}{dt}[x(t) - A_d x(t-h)] = Ax(t) + A_h x(t-h). \tag{3.6}$$

So switching can proceed for the system (2.1). But for the system (3.7), switching may be impossible.

$$\dot{x}(t) - A_d\dot{x}(t-h)] = A_{\sigma(t)}x(t) + A_h x(t-h). \tag{3.7}$$

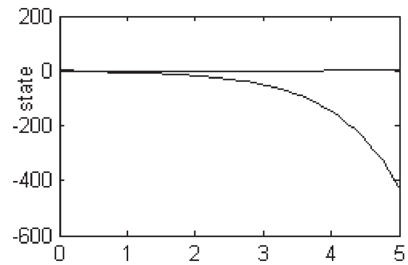

Figure 1: The state responses of subsystem (2.1). Y-coordinate is state, X-coordinate is time (sec.)

According to Theorem 7.1 (see [3], p. 25), the initial condition must be strengthened to be continuously differentiable on $[-h, 0]$, then the solution of system (3.7) can exist. While the solution will have discontinuous derivatives at $t = nh, n \in N$. So if switched neutral delay system switches in the vicinity of these time, the initial value may contain a discontinuous derivative and next subsystem may have no solution.

Remark 3.5. In this paper, the method used to divide the switching reign distinguishes itself from the method before. Switching reign was divided in R^n in previous woks such as [5], [8]. While in this paper, switching reign is divided in R^{2n}. If we continue using the method before, because there will exist $PAA_dA_d^TAP$ in conclusion, then it is impossible to divide the switching reign by means of the conditions of matrix convex combination.

4. Simulation

Consider the following switched neutral delay system

$$\frac{d}{dt}[x(t) - A_d x(t-h)] = A_i x(t) + A_h x(t-h)\,,$$
$$x_{t_0}(\theta) = \varphi(\theta)\,, \quad \theta \in [-h, 0] \quad (i = 1, 2)\,, \quad (4.1)$$

where

$$A_1 = \begin{pmatrix} -5 & -0.5 \\ 0 & 1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -0.5 & 0 \\ -1 & -5.5 \end{pmatrix},$$

$$A_h = \begin{pmatrix} -0.5 & 0 \\ 0 & -0.8 \end{pmatrix}, \quad A_d = \begin{pmatrix} -0.1 & 0.5 \\ 0.1 & 0 \end{pmatrix}.$$

For system (4.1), the states trajectories of subsystem (2.1) and (3.1) are shown in Figure 1 and Figure 2 receptively with initial condition $\varphi(\theta) = (3, -2)$. It is obvious the two subsystems are all unstable. If we take $\alpha_1 = \alpha_2 = 0.5$, we can get

$$\overline{A} = \begin{pmatrix} -2.25 & -0.25 \\ -0.5 & -2.25 \end{pmatrix}.$$

Solve LMI (3.1), we get

$$P = \begin{pmatrix} 35.034 & -5.6492 \\ -5.6492 & 35.2457 \end{pmatrix}, \quad S = \begin{pmatrix} 71.8540 & 6.9325 \\ 6.9325 & 86.4741 \end{pmatrix}.$$

So

$$\Omega_i = \{(\wp^T, x_k^T)^T \mid (\wp^T, x_k^T)\Pi_i(\wp^T, x_k^t)^T < 0\} \quad (i = 1, 2)\,,$$

$$\Pi_1 = \begin{pmatrix} -278.4802 & 12.0127 & -8.8087 & -47.1372 \\ 12.0127 & 162.6147 & 11.7612 & -10.6072 \\ -8.8087 & 11.7612 & -70.4094 & -10.1785 \\ -47.1372 & -10.6072 & -10.1785 & -68.5106 \end{pmatrix},$$

$$\Pi_2 = \begin{pmatrix} 118.1859 & -0.0670 & -23.2184 & 52.0294 \\ -0.0670 & -301.2280 & -4.7993 & -43.7654 \\ -23.2184 & -4.7993 & -70.4094 & -10.1785 \\ 52.0294 & -43.7654 & -10.1785 & -68.5106 \end{pmatrix},$$

then

$$\ddot{\Omega}_1 = \Omega_1, \quad \ddot{\Omega}_2 = \Omega_2 - \Omega_1, \quad \bigcup_{i=1}^{2} \ddot{\Omega}_i = R^4/\{0\}$$

and

$$\ddot{\Omega}_1 \bigcap \ddot{\Omega}_2 = \emptyset\,.$$

Construct switching law

$$\sigma(t) = i, \text{ when } (\wp, x_h) \in \ddot{\Omega}_i, \quad i \in \{1, 2\}. \tag{4.2}$$

If we take initial condition $\varphi(\theta) = (3, -2)$, as Figure 3 shows, obviously, the state responses of switched system (4.1) under switching law (4.2) are stable.

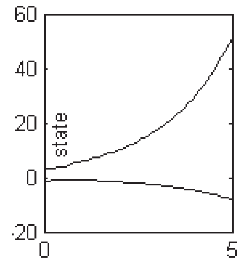

Figure 2: The state responses of subsystem (3.1). Y-coordinate is state, X-coordinate is time (sec.)

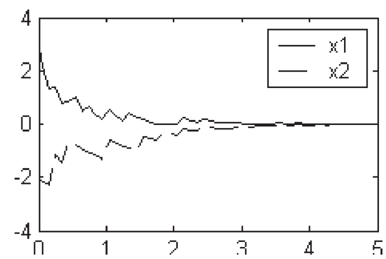

Figure 3: The state responses of switched system (4.1) under switching law (4.2). Y-coordinate is state, X-coordinate is time (sec.)

5. Conclusion

The concept for switched neutral delay system is introduced in this paper. The stability for switched linear neutral delay system is addressed. Sufficient conditions of stability are presented by using single-Lyapunov function technique in form of LMI. Simulation illustrates the effectiveness of the proposed method.

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