International Journal of Pure and Applied Mathematics
Volume 21 No. 2 2005, 139-149

SUBSPACE BASED CHANNEL ESTIMATION FOR DOWNLINK W-CDMA

Vidhyacharan Bhaskar[1 §], Kondo Hloindo Adjallah[2], Laurie L. Joiner[3]

[1]Departement Genie des Systemes d'Information
et de Telecommunication
Universite de Technologie de Troyes
12 Rue Marie Curie, Troyes Cedex, 10010, FRANCE
E-mail: Vidhyacharan.Bhaskar@utt.fr

[2]Institute of Computer Science and Engineering of Troyes
Universite de Technologie de Troyes
12 Rue Marie Curie, Troyes Cedex, 10010, FRANCE

[3]Department of Electrical and Computer Engineering
University of Alabama in Huntsville
Huntsville, AL 35899, USA

Abstract: In wideband code division multiple access (W-CDMA) systems, inter-symbol interference (ISI) introduced by multipath propagation can be significant due to large multipath delays. In the presence of multipath, each signal is subjected to frequency-selective fading and the orthogonality condition is distorted due to increased cross-correlation between the signature waveforms of the different users. In this paper, we study a subspace-based signature waveform estimation for downlink W-CDMA systems that provides estimates of the multiuser channels by exploiting the second-order statistics of the transmitted signal at the base station. We employ multiple antennas at the receiver to estimate the signature waveforms for overloaded systems. A zero-forcing equalizer and a matched-filter detector are employed to estimate the received signal constellations. Simulations are performed to evaluate the effectiveness of the algorithm.

Received: November 22, 2004

§Correspondence author

AMS Subject Classification: 26A33
Key Words: signature waveform, signal subspace, noise subspace, capacity

1. Introduction

Wideband Code Division Multiple Access (W-CDMA) techniques have the potential to offer higher capacity than narrowband CDMA (N-CDMA) systems in that they meet the bandwidth flexibility requirements of multimedia services. This third generation wireless technology supports voice, data, and multimedia services, which will have distinct bandwidth and QoS requirements.

In downlink CDMA systems, the information signals of the user of interest are detected at the receiver by cross-correlation of the received signal with the spreading code sequence of the user of interest [4]. Downlink (base-station to mobile station) W-CDMA signals possess some intrinsic structures that are not available in uplink.

In W-CDMA, when there is multipath, the effective signature signals, known as the composite signature waveforms, are the convolution of these signals with the unknown channel impulse response. Achieving orthogonality between channel-dependent composite signature waveforms is impossible. Each signal in W-CDMA is not only subject to multiple-access interference (MAI), but also to the adverse inter-chip interference (ICI) which affects the signature waveforms used at the transmitter [3]. The ISI introduced by multipath propagation can be significant due to the large multipath delay spread.

In [1], a subspace-based algorithm is developed for an uplink CDMA system which eliminates the use of a training sequence. Torlak and Xu in [5] developed a blind channel estimation algorithm for asynchronous CDMA systems to accommodate an overloaded system as an extension to [1], but with high computational complexity. A blind channel estimation algorithm is developed in [2] which provides closed-form estimates of the signature waveforms for an uplink synchronous CDMA system.

In [6], the problem of channel estimation and signal detection is considered for a downlink synchronous CDMA system. The transmitted signal from several users at the base station passes through a common multipath channel. However, the subspace based approach considered in [6] is sub-optimal, because its mean-square error is significantly larger than the Cramer-Rao lower bound.

In this paper, we discuss the performance of downlink signal reception for W-CDMA signals by exploiting its structural information. We present a new method to construct the data matrix. The problem of downlink signal reception

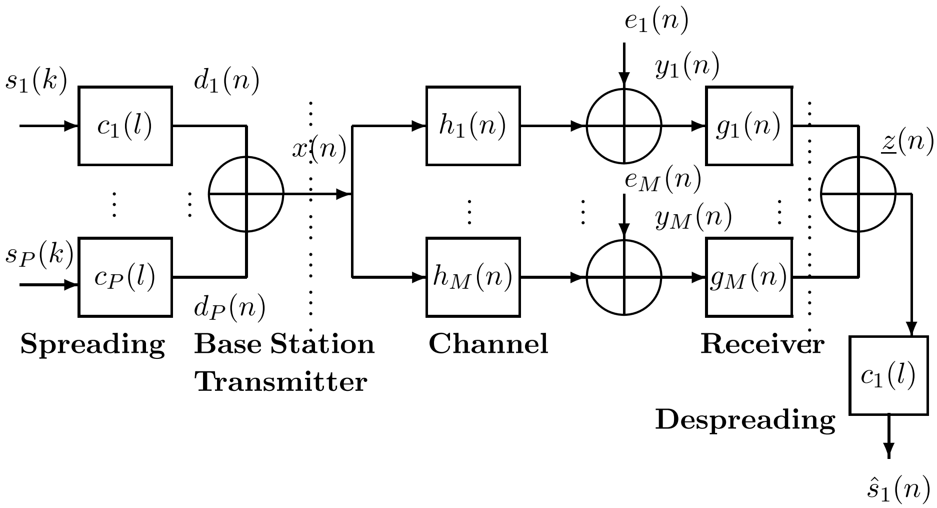

Figure 1: Block diagram of a M-channel downlink W-CDMA system with P users

is discussed for single and multiple receiver antennas.

2. Channel Model and Description

Figure 1 shows a multiple receiving antenna downlink channel model of a W-CDMA system. The W-CDMA system specifications are as given below:

- P is the number of users,
- N is the number of information symbols from the users,
- L is the channel length, i.e., the duration of the impulse response is LT_c, where T_c is the chip duration,
- L_c is the spreading code length,
- T_s is the symbol duration ($= L_cT_c$),
- M is the number of sub-channels, and
- $c_i(l)$ is the short and periodic-spreading code for the i-th user.

The spreading codes are assumed to be orthogonal to each other. Because of this inherent orthogonality, downlink signal reception is usually performed in two steps, see [3]:

- Channel equalization which compensates the "common" multipath channel effect and restores the orthogonality, and
- Despreading operation which extracts the desired signal.

The rest of the W-CDMA specifications are given below:

- $d_i(n)$ is the modulated chip sequence of the i-th user,

$d_i(n) = \sum_{k=-\infty}^{\infty} s_i(k) c_i(n - kL_c)$,

- $x(n)$ is the transmitted signal at the base station transmitter and is given by

$$x(n) = \sum_{i=1}^{P} d_i(n) = \sum_{i=1}^{P} \sum_{k=-\infty}^{\infty} s_i(k) c_i(n - kL_c). \tag{1}$$

- $h_m(n)$ is the m-th single-input multiple-output (SIMO) channel impulse response,
- $e_m(n)$ is the thermal noise at the m-th sub-channel,
- $y_m(n)$ is the received signal at the m-th sub-channel, and
- $g_m(n)$ is the channel equalizer of the m-th multipath sub-channel $h_m(n)$.

The received signal (without noise) of the m-th sub-channel is

$$y_m(n) = \sum_{u=0}^{L-1} h_m(u) x(n - u). \tag{2}$$

Substituting $x(n)$ from (1) into (2), we have

$$y_m(n) = \sum_{u=0}^{L-1} h_m(u) \sum_{i=1}^{P} \sum_{k=-\infty}^{\infty} s_i(k) c_i(n - u - kL_c). \tag{3}$$

We assume that the channel impulse responses, $h_m(n), \forall\, m = 1, 2, ..., M$ are unknown to the receiver. The "effective signature waveform" of the i-th user through the m-th channel, $w_{i,m}(n)$, is obtained by convolving the spreading code of the i-th user with the impulse response of the m-th channel,

$$w_{i,m}(n) = \sum_{u=0}^{L-1} h_m(u)\, c_i(n - u)\,, \quad \forall\, n = 1, 2, ..., L + L_c - 1. \tag{4}$$

The effective signature waveforms are channel-dependent and thus unknown to the receiver. Let $h_m(n)$ have support in $n \in [0, LT_c]$. So, $w_{i,m}(n)$ has support in $n \in [0, (L + L_c - 1)T_c]$.

3. Wideband CDMA Data Formulation

N observation vectors are collected into a data matrix $\mathbf{Y}_m$, and a subspace decomposition is performed on $\mathbf{Y}_m$. For simplicity, we ignore noise in our discussion. The accommodation of noise will be addressed in subspace decomposition and algorithm implementation. The data matrix is composed of distinct samples in the N observation vectors, each sample comprising a collection of $(L_c - L + 1)$ chips unaffected by ISI. Here, N corresponds to the number of data samples.

The contribution of this paper is how to construct the data matrix and how the method exploits the data structure. The data matrix is

$$\mathbf{Y}_m(n) = \begin{pmatrix} y_m((N-n)L_c + L) & y_m((N-n+1)L_c + L) & \dots & y_m((2N-n-1)L_c + L) \\ y_m((N-n)L_c + L + 1) & y_m((N-n+1)L_c + L + 1) & \dots & y_m((2N-n-1)L_c + L + 1) \\ \vdots & \vdots & \ddots & \vdots \\ y_m((N-n+1)L_c) & y_m((N-n+2)L_c) & \dots & y_m((2N-n)L_c) \end{pmatrix}. \tag{5}$$

Writing (4) in matrix form,

$$\begin{aligned} \mathbf{W}_{i,m} &= \begin{bmatrix} c_i(L) & c_i(L-1) & \dots & c_i(1) \\ c_i(L+1) & c_i(L) & \dots & c_i(2) \\ \vdots & \vdots & \ddots & \vdots \\ c_i(L_c) & c_i(L_c-1) & \dots & c_i(L_c-L+1) \end{bmatrix} \begin{bmatrix} h_m(0) \\ h_m(1) \\ \vdots \\ h_m(L-1) \end{bmatrix} \\ &= \mathbf{C}_i \mathbf{h}_m. \end{aligned} \tag{6}$$

Given N data vectors, we have

$$\begin{aligned} \mathbf{Y}_m &= [\mathbf{Y}_m(N)\ \mathbf{Y}_m(N-1)\ \dots\ \mathbf{Y}_m(1)]^T \\ &= [\mathbf{W}_{1,m}\ \mathbf{W}_{2,m}\ \dots \mathbf{W}_{P,m}] \begin{bmatrix} s_1(1) & s_1(2) & \dots & s_1(N) \\ s_2(1) & s_2(2) & \dots & s_2(N) \\ \vdots & \vdots & \ddots & \vdots \\ s_P(1) & s_P(2) & \dots & s_P(N) \end{bmatrix} \end{aligned}$$

$$= \mathbf{W}_m\mathbf{S} = \begin{bmatrix} \mathbf{W}_{1,m} & \mathbf{W}_{2,m} & \ldots & \mathbf{W}_{P,m} \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 \\ \mathbf{S}_2 \\ \vdots \\ \mathbf{S}_P \end{bmatrix}, \quad (7)$$

where the information symbols of the i-th user are $\mathbf{S}_i = [s_i(1)\ s_i(2)\ \ldots\ s_i(N)]$, and $[.]^T$ refers to the transpose of a matrix.

4. Channel Estimation

The signature waveform matrix of the i-th user, m-th channel is given as $\mathbf{W}_{i,m} = \mathbf{C}_i\mathbf{h}_m$. This indicates that the $(L_c - L + 1)$ vector $\mathbf{W}_{i,m}$ is uniquely determined by the L-vector, $\mathbf{h}_m$. Therefore, the estimation of the signature vectors is equivalent to the determination of the channel vectors.

4.1. Subspace Concept

Given the data matrix in (7), a subspace decomposition can be performed on the (data + noise) matrix, $\mathbf{Y}'_m = \mathbf{Y}_m + \mathbf{E}_m$ by a singular value decomposition (SVD) [2]

$$\begin{aligned} \mathbf{Y}_m + \mathbf{E}_m &= \mathbf{W}_m\mathbf{S} + \mathbf{E}_m \\ &= \begin{pmatrix} \mathbf{U}_s & \mathbf{U}_0 \end{pmatrix} \begin{pmatrix} \sum_s & 0 \\ 0 & \sum_o \end{pmatrix} \begin{pmatrix} \mathbf{V}_s^H \\ \mathbf{V}_0^H \end{pmatrix}, \end{aligned} \quad (8)$$

where the vectors in $\mathbf{U}_s$ associated with the P non-zero singular values, span the *signal subspace* defined by the columns of $\mathbf{W}_m$; while the vectors in $\mathbf{U}_0$ associated with the zero singular values, span the *noise subspace*

$$\mathbf{U}_0 = [\mathbf{U}_1\ \mathbf{U}_2\ \ldots\ \mathbf{U}_{L_c-L+1-P}]\,. \quad (9)$$

The dimensions of $\mathbf{U}_s$ and $\mathbf{U}_0$ are $(L_c-L+1)\times P$ and $(L_c-L+1)\times(L_c-L+1-P)$ respectively. Since the noise subspace is orthogonal to the signal subspace, we have

$$\mathbf{U}_0 \perp \mathbf{W}_m \;\Rightarrow\; \mathbf{U}_0^H\mathbf{W}_{i,m} = 0\,, \quad \forall\, i = 1, 2, ..., P, \quad (10)$$

where $(.)^H$ denotes the Hermitian of a matrix.

4.2. Proposed Algorithm

The problem addressed in this paper is to estimate the signature vectors $\{\mathbf{W}_{i,m}\}$ from $\mathbf{Y}'_m$ without the knowledge of $\mathbf{S}$. Hence, a connection is made between the noise subspace and the signature vectors. Substituting $\mathbf{W}_{i,m} = \mathbf{C}_i \mathbf{h}_m$ (for a fixed m) into (10) yields

$$\mathbf{U}_0^H \mathbf{C}_i \mathbf{h}_m = 0\,, \quad \forall i = 1, 2, ..., P. \tag{11}$$

Thus, the channel vectors can be determined from the noise subspace of the (data + noise) matrix, $\mathbf{Y}'_m$. The above equation set has $(L_c - L + 1 - P)$ equations and L unknowns. If the number of equations is greater than the number of unknowns, we have an over-determined system of linear equations. This will happen if $(L_c - L + 1 - P) \geq L$, or $P \leq L_c - 2L + 1$. Equation (11) has an unique trivial solution $\hat{\mathbf{h}}_m$ subject to $||\mathbf{h}_m|| = 1$.

An outline of the algorithm is given below:

- Construct the data plus noise matrix $\mathbf{Y}'_m$ for $m = 1, 2, ..., M$.
- Apply SVD to $\mathbf{Y}'_m$, $\forall\ m = 1, 2, ..., M$ to obtain the orthogonal subspace $\mathbf{U}_0$.
- For each user, estimate the m-th channel vector $\hat{\mathbf{h}}_m$ by solving the linear equation set in (11).
- Reconstruct the signature vectors $\{\mathbf{W}_{i,m}\}$ and signature waveform matrix $\{\mathbf{W}_m\}$ from (6).

The channel estimate of the m-th sub-channel is given by

$$\hat{\mathbf{h}}_m = \arg \min_{\|\mathbf{h}_m\|=1} \mathbf{h}_m^H \mathbf{c}_i^H \left[\sum_{l=1}^{L_c-L+1-P} \mathbf{U}_l \mathbf{U}_l^H \right] \mathbf{c}_i \mathbf{h}_m, \tag{12}$$

where

$$\mathbf{c}_i = \begin{bmatrix} \mathbf{C}_i(1) \\ \mathbf{C}_i(2) \end{bmatrix}, \quad \mathbf{C}_i(j) \text{ is } (L_c - L + 1) \times L, \quad j = 1, 2\,, \tag{13}$$

$$\mathbf{U}_l = \begin{bmatrix} \mathbf{u}_1^{(l)} & \mathbf{0} \\ \mathbf{0} & \mathbf{u}_1^{(l)} \end{bmatrix}. \tag{14}$$

Here, $\mathbf{u}_1^{(l)}$ is a $(L_c - L + 1) \times 1$ matrix, and $\mathbf{0}$ is a zero-matrix of order $(L_c - L + 1) \times 1$.

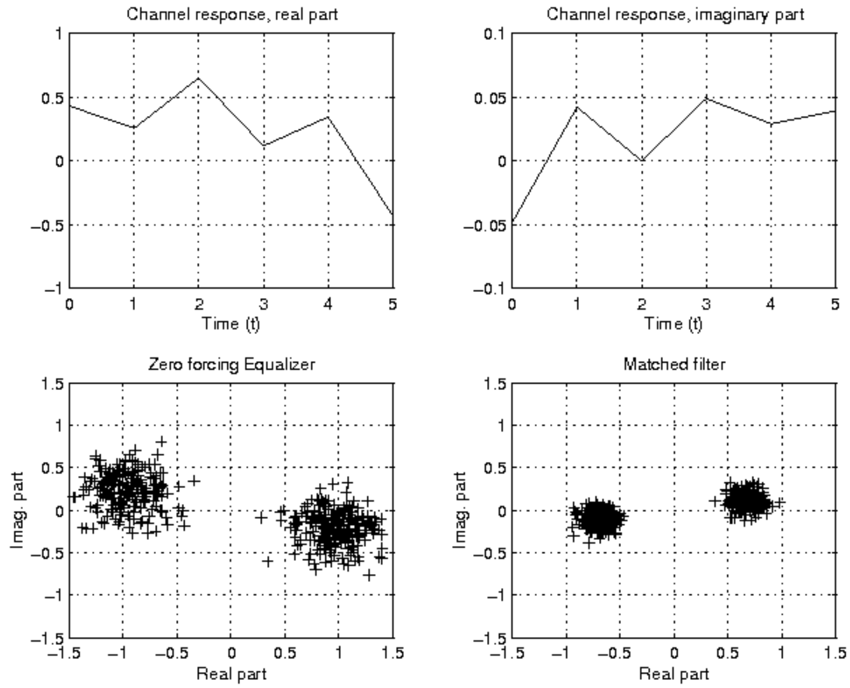

Figure 2: Channel and signal constellations for user 1

4.3. Single Receiver Antenna – Simulation Results

We performed simulation of a single-receiver W-CDMA system with specifications, $M = 1$, $L_c = 31$, $P = 11$, $A = 1$, $L = 6$, where A is the amplitude of the information signal. Gold sequences are used as pseudo random sequences for each user. The number of data vectors applied for each user is 1000. All users transmit their information symbols with corresponding pseudo random codes to spread their information symbols. The spreaded signals are added to form the transmitted signal at the base station transmitter. Each sub-channel depicts the several paths taken by the transmitted signal to the receiver. Thermal noise with AWGN noise variance 1 is added at the receiver. After the channel vectors, $\{\mathbf{h}_m\}$ are estimated, a zero-forcing equalizer (ZFE) is employed to recover the original signal for each user. As an alternative, we also constructed a matched filter (MF) to recover the original signal for each user. The channel responses from the subspace algorithm and the received signal constellations from the two methods (matched filter receiver and zero forcing equalizer) are provided for comparison in Figure 2. Figure 2 illustrates the estimated chan-

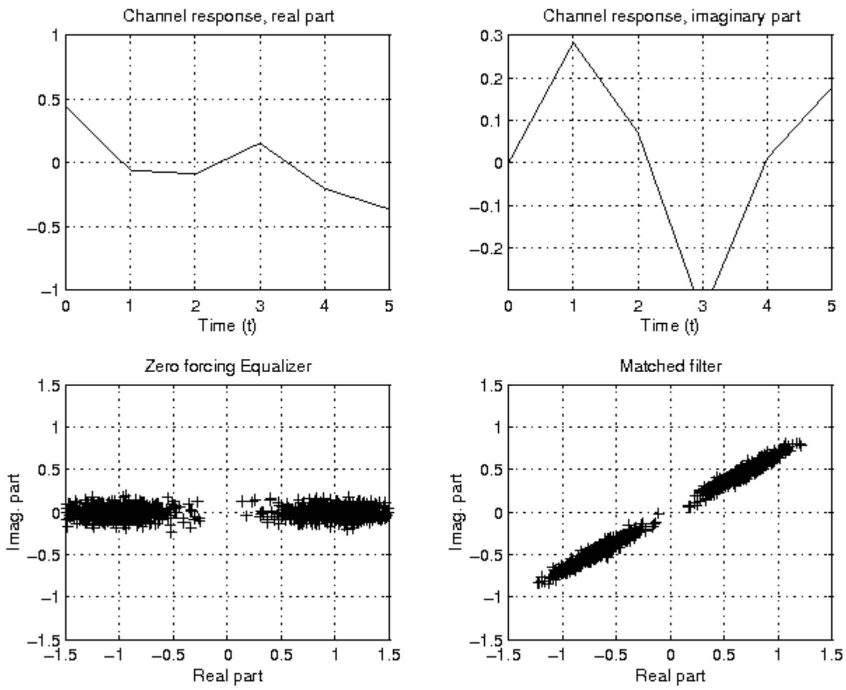

Figure 3: Channel and signal constellations for user 32 in an overloaded system

nel responses used in the simulations and processing results for user 0. The proposed method (ZFE) outperforms the MF method because the real parts of the received signal constellations obtained by the ZFE method are closer to one than those obtained by the MF method.

4.4. Multiple Receiving Antennas – Channel Estimation

For an overloaded system ($P > L_c$), we need to accommodate additional orthogonal vectors to guarantee (11) to be consistent. Assume B receiving antennas at the mobile user terminal and denote the superscript as the antenna index. The data vectors (matrices), and the channel vectors at the receiving antennas can be stacked in the following fashion:

$$\mathbf{Y}_m = \begin{pmatrix} \mathbf{Y}_m^1 \\ \vdots \\ \mathbf{Y}_m^B \end{pmatrix} = \sum_{i=1}^{P} \begin{pmatrix} \mathbf{W}_{i,m}^1 \\ \vdots \\ \mathbf{W}_{i,m}^B \end{pmatrix} \mathbf{S}_i. \qquad (15)$$

Clearly $\mathbf{Y}_m = \mathbf{W}_m\mathbf{S}$ still holds good, and so does the subspace relation between $\mathbf{Y}_m$ and $\mathbf{W}_m$. However, the number of orthogonal vectors in $\mathbf{U}_0$ has been substantially increased to $B(L_c - L + 1) - P$. It is readily seen that $\{\mathbf{W}_{i,m}\}$ is given by the solution of the following linear equations through a new kernel matrix:

$$\mathbf{U}_0^H \begin{pmatrix} \mathbf{C}_i & 0 & \dots & 0 \\ 0 & \mathbf{C}_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{C}_i \end{pmatrix} \mathbf{H}_m = 0\,, \quad \forall\, i = 1, 2, \dots, P. \tag{16}$$

The signature vector $\mathbf{W}_{i,m}$ is determined from the BL-channel vector $\mathbf{h}_m$, whose estimate can be formulated as

$$\hat{\mathbf{h}}_m = arg \min_{\|\mathbf{h}_m\|=1} \mathbf{h}_m^H \begin{pmatrix} \mathbf{c}_i & 0 & \dots & 0 \\ 0 & \mathbf{c}_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{c}_i \end{pmatrix}^H \left(\sum_{l=1}^{B(L_c-L+1)-P} \mathbf{U}_l \mathbf{U}_l^H \right) \times \begin{pmatrix} \mathbf{c}_i & 0 & \dots & 0 \\ 0 & \mathbf{c}_i & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{c}_i \end{pmatrix} \mathbf{h}_m. \tag{17}$$

4.5. Multiple Receiver Antennas – Simulation Results

Two receiver ($B = 2$) antennas are employed in a 32-user system shown in Figure 3. The signature vector estimate is determined from the BL-channel vector estimate $\hat{\mathbf{h}}_m$. The maximum number of signature waveforms that can be estimated using B receiver antennas is $B(L_c-2L+1)$ since there are $(B(L_c-L+1) - P)$ equations in BL unknowns. Figure 3 illustrates the channel responses used in the simulations and processing results for user 32.

5. Conclusions

In this paper, a subspace-based algorithm for signature waveform estimation is discussed for downlink W-CDMA systems. The algorithm exploits the fact that the signature waveform is the intersection of the signal subspace and the kernel subspace, thus allowing it to be uniquely determined without the knowledge of the user's input signals. A zero forcing equalizer and a matched filter receiver are used to present the received signal constellations for different users. For overloaded systems, we employ multiple receiving antennas to include additional orthogonal vectors required to represent the signal constellations. Extensive computer simulations are performed to estimate the channel response and received signal constellations.

References

[1] S.E. Bensley, B. Aazhang, Subspace-based channel estimation for code division multiple access communication systems, *IEEE Transactions on Communications*, **44**, No. 8 (August 1996), 1009-1020.

[2] H. Liu, A subspace method for signature waveform estimation in synchronous CDMA systems, *IEEE Transactions on Communications*, **44**, No. 10 (October 1996), 1346-1354.

[3] H. Liu, *Signal Processing Applications in CDMA Communications*, MA, Artech House (2000).

[4] J.G. Proakis, *Digital Communications*, Forth Edition, NY, McGraw Hill (2001).

[5] M. Torlak, G. Xu, Blind multiuser channel estimation in asynchronous CDMA systems, *IEEE Transactions on Signal Processing*, **45**, No. 1 (January 1997), 137-147.

[6] A. Weiss, B. Friedlander, Synchronous DS-CDMA downlink with frequency selective fading, *IEEE Transactions on Signal Processing*, **47**, No. 1 (January 1999), 158-167.