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# CARTESIAN CLOSED CATEGORIES OF TOPOLOGICAL ALGEBRAS

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**Abstract:** It has been observed by several authors that cartesian closed topological categories ensure many nice properties for any associated topological algebras. It turns out that a topological algebra $\boldsymbol{XA}$ can be cartesian closed without its topological component $\boldsymbol{X}$ being so. However, the algebra component $\boldsymbol{A}$ has to be cartesian closed. It is proved, under the assumptions that $\boldsymbol{X}$ is a cartesian closed well-fibred topological construct and $\boldsymbol{A}$ is a cartesian closed category of algebras with unary operations satisfying a minor condition, $\boldsymbol{XA}$ is cartesian closed.

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## 1. Introduction

Cartesian closedness of topological categories is important in the theory of topological algebra (see, for example, [8] and [18]), especially for the construction of free topological algebras over topological objects (see [19]). It is a subject or a tool of investigation in so many papers that it is impossible to quote all of them. Besides mentioning some representatives, references [1], [3]-[5], -[21],

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we refer to the recent book [2, p. 407] and the bibliography contained therein. The present author has examined cartesian closed universal algebras in [10]. However, it seems cartesian closedness of a category of topological algebras has not been explored before.

In the present paper we observe that a category of topological algebras is cartesian closed only if the associated category of universal algebras is cartesian closed. Conversely, a category of topological algebras $\boldsymbol{XA}$, obtained from a cartesian closed well-fibred topological construct $\boldsymbol{X}$ and a cartesian closed subcategory $\boldsymbol{A}$ of $\boldsymbol{Alg(\Sigma)}$ (the category of universal algebras of a fixed type $\Sigma$ all of whose operations are unary) satisfying a minor condition, is cartesian closed.

## 2. Preliminaries

A category $\boldsymbol{A}$ is called *cartesian closed* iff it has finite products and for each $\boldsymbol{A}$-object $A$ the functor $(A \times -) : \boldsymbol{A} \to \boldsymbol{A}$ is co-adjoint (see, for example, [2, 27.1]). Equivalently, a category with finite products is cartesian closed iff for any two $\boldsymbol{A}$-objects $A$ and $B$ there exists an object $B^A$ (*power object*) and a $\boldsymbol{A}$-morphism $ev : A \times B^A \to B$ (*evaluation map*) such that for each $\boldsymbol{A}$-morphism $f : A \times C \to B$ there exists a unique $\boldsymbol{A}$-morphism $\bar{f} : C \to B^A$ (the *exponential morphism* for $f : A \times C \to B$) so that the diagram

$$\begin{tikzcd} A \times C \arrow[d, "id_A \times \bar{f}"'] \arrow[dr, "f"] & \\ A \times B^A \arrow[r, "ev"'] & B \end{tikzcd}$$

($id_A$ is the identity map on $A$) commutes.

A family $\Omega = (n_j)_{j \in J}$ of natural numbers indexed by some set $J$ is called a *type*. The index set $J$ is called the *order* of $\Omega$. In the following, we let a type $\Omega = (n_j)_{j \in J}$ be fixed. A pair $(|A|, (\omega_j)_{j \in J})$ of a set $|A|$ and a family $\omega_j : |A|^{n_j} \to |A|$ $(j \in J)$ of mappings is called an $\Omega$-*algebra* (see, for example, [6]). For the sake of simplicity, we write $A$ instead of the pair $(|A|, (\omega_j)_{j \in J})$ and $\omega_{j,A}$ for the $n_j$-*ary operation* $\omega_j$ on $A$. If the $\Omega$-algebra $A$ is clear from the context, we drop the suffix $A$ in denoting its $n_j$-ary $(j \in J)$ operation. Moreover, the symbol $\Sigma$ is used instead of $\Omega$ in case $n_j = 1$ for each $j \in J$. A mapping $f : |A| \to |B|$ between two $\Omega$-algebras $A$ and $B$ is said to be an $\Omega$-*homomorphism* iff for each $j \in J$, $f \circ \omega_{j,A} = \omega_{j,B} \circ f^n$, where $n = n_j$ and $f^n : |A|^n \to |B|^n$ is the mapping with the obvious definition $(a_1, \ldots, a_n) \to (fa_1, \ldots, fa_n)$.

The symbol $\boldsymbol{Alg(\Omega)}$ denotes the category whose objects are $\Omega$-algebras and whose morphisms are $\Omega$-homomorphisms.

Let $\boldsymbol{X}$ be a topological category and $\boldsymbol{A}$ be a subcategory of $\boldsymbol{Alg(\Omega)}$. By a *paired object* (from $\boldsymbol{X}$ and $\boldsymbol{A}$) is meant an ordered pair $(X, A)$, where $X$ and $A$ are objects in $\boldsymbol{X}$ and $\boldsymbol{A}$ respectively with the same underlying set such that, for each $j \in J$, the $n(= n_j)$-ary operation $\omega_{j,A} : |A|^n \to |A|$ on $A$ is an $\boldsymbol{X}$-morphism $\omega_{j,A} : X^n \to X$. In this case, we write $\omega_{j,X}$ for the $\boldsymbol{X}$-morphism from $X^n$ to $X$ whose underlying function is $\omega_{j,A}$. If $(X, A)$ and $(X', A')$ are two paired objects (from $\boldsymbol{X}$ and $\boldsymbol{A}$), then an $\boldsymbol{X}$-morphism $f : X \to X'$ that is also an $\boldsymbol{A}$-morphism $f : A \to A'$ is called a *paired morphism* (from $\boldsymbol{X}$ and $\boldsymbol{A}$) and is denoted by $f : (X, A) \to (X', A')$. The category of all paired objects (from $\boldsymbol{X}$ and $\boldsymbol{A}$) together with paired morphisms (from $\boldsymbol{X}$ and $\boldsymbol{A}$) is called the *paired category* (from $\boldsymbol{X}$ and $\boldsymbol{A}$). We denote this category by $\boldsymbol{XA}$.

We refer the reader to [2] for a definition of any concept that we use in this paper which is not defined here.

In this work, we assume that all subcategories are full and isomorphism closed. The fact that the most of the natural subcategories fall into this class justifies our assumption.

## 3. Cartesian Closedness of $\boldsymbol{XA}$

First of all we note that cartesian closedness of just one of $\boldsymbol{X}$ or $\boldsymbol{A}$ does not guarantee the cartesian closedness of $\boldsymbol{XA}$ even if $\boldsymbol{X}$ is topological and $\boldsymbol{A}$ is algebraic. Indeed, if $\boldsymbol{X}$ and $\boldsymbol{A}$ are, respectively, the categories $\boldsymbol{Set}$ and $\boldsymbol{Ab}$ (the category of abelian groups), then $\boldsymbol{XA} \approx \boldsymbol{Ab}$, which is not cartesian closed. (In $\boldsymbol{Ab}$, $Z_2 \times (Z_2 \oplus Z_2)$ is not isomorphic to $(Z_2 \times Z_2) \oplus (Z_2 \times Z_2)$.) If $\boldsymbol{X} := \boldsymbol{Top}$ and $\boldsymbol{A} := \boldsymbol{Set}$, then $\boldsymbol{XA} \approx \boldsymbol{Top}$, which is not cartesian closed (see [2, p. 409]).

On the other hand, $\boldsymbol{XA}$ can be cartesian closed without $\boldsymbol{X}$ being cartesian closed. For example, if $\boldsymbol{X} = \boldsymbol{Top}$ and $\boldsymbol{A}$ is the subcategory of $\boldsymbol{Grp}$ consisting of one object $\{e\}$ (trivial group), then $\boldsymbol{XA}$ is clearly cartesian closed as it contains only one object.

However if $\boldsymbol{XA}$ is cartesian closed, then $\boldsymbol{A}$ is cartesian closed. Indeed, the forgetful functor $\boldsymbol{XA} \to \boldsymbol{A}$, being topological, preserves cartesian closedness (see [2, 24.14]).

This turns our attention to cartesian closed categories of universal algebras. Cartesian closed varieties are studied by Johnstone ([14], [15]), however, a novel description using a concept called $T$-friendly (see [10]) is very convenient in this work. We will explain what is meant by a $T$-friendly category.

In the following discussion, we reserve the symbol $T$ for a $\Sigma$-algebra generated by a distinguished member $\mathbf{0}$ in $T$ (i.e., for any $t \in T$ there exist $j_1 \in J, \ldots, j_n \in J$ such that $\omega_{j_n,T} \circ \ldots \circ \omega_{j_1,T}(\mathbf{0}) = t$), equipped with a family $(\delta_j)_{j \in J}$ of $\Sigma$-homomorphisms $\delta_j : T \to T$

$$\begin{tikzcd}
T \arrow[r, "\omega_{k,T}"] \arrow[d, "\delta_j"'] & T \arrow[d, "\delta_j"] \\
T \arrow[r, "\omega_{k,T}"'] & T
\end{tikzcd}$$

satisfying

$$\delta_j(\mathbf{0}) = \omega_{j,T}(\mathbf{0}). \tag{3.0}$$

A $\Sigma$-algebra $A$ is said to be *$T$-friendly* iff for each $t \in T$ there exists a function $A_t : A \to A$, such that

$$A_{\mathbf{0}} = id_A, \tag{3.1}$$

$$\omega_{j,A} \circ A_t = A_{\omega_{j,T}(t)}, \tag{3.2}$$

and

$$A_t \circ \omega_{j,A} = A_{\delta_j(t)} \tag{3.3}$$

$$\begin{tikzcd}
& A \arrow[d, "\omega_{j,A}"] \arrow[dr, "A_{\delta_j(t)}"] & \\
A \arrow[ur, "A_t"] \arrow[r, "A_{\omega_{j,T}(t)}"'] & A \arrow[r, "A_t"'] & A
\end{tikzcd}$$

for all $j \in J$ and for all $t \in T$.

A subcategory $\boldsymbol{A}$ of $\boldsymbol{Alg(\Sigma)}$ is called *$T$-friendly* iff each $\boldsymbol{A}$-object is $T$-friendly. Consider the following examples.

**Example 1.** Equip the set $\mathbf{N}$ of nonnegative integers with the unary operation $\omega : \mathbf{N} \to \mathbf{N}$ that maps any integer $n$ to its successor $n+1$ and take $\delta = \omega$ (the distinguished member of $\mathbf{N}$ is the integer zero.) For any object $A$ in $\boldsymbol{Alg(1)}$ with a unary operation $u$, define $A_n := u^n$ to be the composition of $u$ to itself $n$ times. It is straight forward to see that $\boldsymbol{Alg(1)}$ is $\mathbf{N}$-friendly.

**Example 2.** Let $T$ be the set of all $n$-tuples ($n \geq 1$) of members of $J$ together with a distinguished point $\mathbf{0}$. For each $j \in J$, define

$$\omega_j(\mathbf{0}) = (j), \qquad \omega_j(j_1, \ldots, j_n) = (j_1, \ldots, j_n, j),$$

$$\delta_j(\mathbf{0}) = (j), \text{ and } \delta_j(j_1, \ldots, j_n) = (j, j_n, \ldots, j_1).$$

Then $T$ is a $\Sigma$-algebra with the unary operations $\omega_j$ ($T$ is the $\Sigma$-word algebra on the alphabet $\{\mathbf{0}\}$) and $\delta_j$'s are $\Sigma$-homomorphisms. For any $\Sigma$-algebra $A$, define

$$A_{\mathbf{0}} := \mathrm{id}_A$$

and

$$A_{(j_1,\ldots,j_n)} = \omega_{j_n,A} \circ \ldots \circ \omega_{j_1,A}.$$

With these definitions, $\boldsymbol{Alg(\Sigma)}$ is $T$-friendly.

**Example 3.** If $(J, +, \mathbf{0})$ is a monoid, then the subcategory $\boldsymbol{A}$ of $\boldsymbol{Alg(\Sigma)}$ consisting of all $J$-sets (A $\Sigma$-algebra $A$ is said to be a *$J$-set* iff $\omega_{\mathbf{0},A} = id_A$ and $\omega_{j,A} \circ \omega_{j',A} = \omega_{j+j',A}$ for any $j \in J$, $j' \in J$.) is $J$-friendly: Indeed, the unary operations $\omega_{j,J}(j \in J)$ on $J$ are given by $\omega_{j,J}(j') = j + j'$ ($j' \in J$) and the homomorphisms $\delta_j$ ($j \in J$) on $J$ are defined by $\delta_j(j') = j' + j$ ($j' \in J$). For any $\boldsymbol{A}$-object $A$ and any $j \in J$, $A_j$ is the $n_j$-ary operation $\omega_{j,A}$ on $A$.

If $A$ and $B$ are any two $\Sigma$-algebras, then we write $\hom_{\boldsymbol{A}}(A \times T, B)$ for the $\Sigma$-algebra whose underlying set is the set of all $\Sigma$-homomorphisms from $A \times T$ to $B$ and whose $n_j$-ary (unary) operation $\omega_j$ assigns the $\Sigma$-homomorphism $f \circ (id_A \times \delta_j)$ for any morphism $f : A \times T \to B$ in $\hom_{\boldsymbol{A}}(A \times T, B)$ , i.e.,

$$\begin{tikzcd} A \times T \arrow[d, "id_A \times \delta_j"'] \arrow[dr, "\omega_j f"] & \\ A \times T \arrow[r, "f"'] & B \end{tikzcd}$$

$$\omega_j f = f \circ (id_A \times \delta_j). \tag{3.4}$$

If $A$ is $T$-friendly and $a \in A$, then define $g_{A,a} : T \to A$ by

$$g_{A,a}(t) := A_t a \tag{3.5}$$

for any $t \in T$. It is easy to see that $g_{A,a}$ is a $\Sigma$-homomorphism and

$$g_{A,\omega_j a} = g_{A,a} \circ \delta_j \tag{3.6}$$

for any $a \in A$ and $j \in J$.

We are now ready to prove the main result of this paper.

**Proposition 1.** *Suppose $\boldsymbol{X}$ is a cartesian closed well-fibred topological construct and $\boldsymbol{A}$ is a full subcategory of $\boldsymbol{Alg(\Sigma)}$ with finite products which is $T$-friendly for some $\boldsymbol{A}$-object $T$ such that all subalgebras of $\hom_{\boldsymbol{A}}(A \times T, B)$ are $\boldsymbol{A}$-objects for any two $\boldsymbol{A}$-objects $A$ and $B$. Then $\boldsymbol{XA}$ is cartesian closed.*

*Proof.* We have to prove the existence of power objects and evaluation morphisms in the paired category $\boldsymbol{XA}$. Suppose $(X, A)$ and $(Y, B)$ are any two $\boldsymbol{XA}$-objects. Equip $T$ with the discrete $\boldsymbol{X}$-structure and denote this $\boldsymbol{X}$-object by $S$. Clearly $(S, T)$ is an $\boldsymbol{XA}$-object, $\delta_j : (S, T) \to (S, T)$ is an $\boldsymbol{XA}$-morphism for any $j \in J$, and $g_{C,c} : (S, T) \to (Z, C)$ (defined as in (3.5)) is an $\boldsymbol{XA}$-morphism for any $\boldsymbol{XA}$-object $(Z, C)$ and $c \in C$.

Let $A'$ be the set of all $\boldsymbol{XA}$-morphisms from $(X, A) \times (S, T)$ to $(Y, B)$. $A'$ is a subalgebra of $\hom_{\boldsymbol{A}}(A \times T, B)$ because for any $j \in J$ and $f \in A'$, $\omega_j f$, being equal to $f \circ (id_X \times \delta_j)$ (see (3.4)), is an $\boldsymbol{XA}$-morphism. Thus $A'$ is an $\boldsymbol{A}$-object by hypothesis. Define $ev : A \times A' \to B$ by

$$ev(a, f) := f(a, \mathbf{0}) \tag{3.7}$$

for any $a \in A$ and $f \in A'$. It is easy to see that $ev$ is an $\boldsymbol{A}$-morphism.

Index all $\boldsymbol{XA}$-morphisms from all possible $\boldsymbol{XA}$-objects of the form $(X, A)$ $\times (Z, C)$ into $(Y, B)$ by a class I and denote this family by

$$(f_i : (X, A) \times (X_i, A_i) \to (Y, B))_{i \in I}.$$

Suppose $f : (X, A) \times (Z, C) \to (Y, B)$ is an $\boldsymbol{XA}$-morphism. Then for any $c \in C$ the function $f \circ (id_A \times g_{C,c})$ is an $\boldsymbol{XA}$-morphism. Consequently, the function $\bar{f} : C \to A'$ that assigns to each member $c$ of $C$ the $\boldsymbol{XA}$-morphism $f \circ (id_A \times g_{C,c})$ is well defined. That is, $\bar{f} : C \to A'$ is a function given by

$$\bar{f}(c) := f \circ (id_A \times g_{C,c}) \tag{3.8}$$

for any $c \in C$. In fact, $\bar{f}$ is an $\boldsymbol{A}$-morphism and is unique with respect to this property $ev \circ (id_A \times \bar{f}) = f$.

Thus, for each $i \in I$, there exists a unique $\boldsymbol{A}$-morphism $\bar{f}_i : A_i \to A'$ such that the diagram

$$\begin{tikzcd}
A \times A_i \arrow[d, "id_A \times \bar{f}_i"'] \arrow[dr, "f_i"] & \\
A \times A' \arrow[r, "ev"'] & B
\end{tikzcd}$$

commutes. Let $X'$ be the $\boldsymbol{X}$-object with the same underlying set as $A'$ final with respect to $(\bar{f}_i : X_i \to X')_{i \in I}$. We show that $(X', A')$ is an $\boldsymbol{XA}$-object. Let $j \in J$. For any $i \in I$, since $\bar{f}_i$ is an $\boldsymbol{A}$-morphism, $\omega_{j,A'} \circ \bar{f}_i = \bar{f}_i \circ \omega_{j,A_i}$, which shows that $\omega_{j,A'} \circ \bar{f}_i$ is an $\boldsymbol{X}$-morphism. Since $X'$ has final structure with respect to $\bar{f}_i$'s, $\omega_{j,A'} : X' \to X'$ is an $\boldsymbol{X}$-morphism. Thus $(X', A')$ is an $\boldsymbol{XA}$-morphism.

We show that $(\bar{f}_i : X_i \to X', X')_{i\in I}$ is an epi sink in $\boldsymbol{X}$. Assume that $\alpha : X' \to Z$ and $\beta : X' \to Z$ are any two $\boldsymbol{X}$-morphisms such that $\alpha \circ \bar{f}_i = \beta \circ \bar{f}_i$ for each $i \in I$. We prove that $\alpha = \beta$. Let $f \in A'$. Then $f : (X, A) \times (S, T) \to (Y, B)$ is an $\boldsymbol{XA}$-morphism. Thus there exists an $i \in I$, such that $X_i = S$, $A_i = T$, and $f_i = f$. Since $g_{T,\mathbf{0}} = id_T$, $\bar{f}_i(\mathbf{0}) = f$ by (3.8). Thus

$$\alpha(f) = \alpha(\bar{f}_i(\mathbf{0})) = \beta(\bar{f}_i(\mathbf{0})) = \beta(f).$$

This being true for all $f \in A'$, $\alpha = \beta$. Thus the sink $(\bar{f}_i : X_i \to X', X')_{i\in I}$ is a final epi sink in $\boldsymbol{X}$. Since $\boldsymbol{X}$ is a cartesian closed well-fibred topological construct, the sink $(id_X \times \bar{f}_i : X \times X_i \to X \times X', X \times X')_{i\in I}$ is final in $\boldsymbol{X}$ (see [2, p. 416]). Since $ev \circ (id_X \times \bar{f}_i) = f_i$ is an $\boldsymbol{X}$-morphism for each $i \in I$, $ev : X \times X' \to Y$ is an $\boldsymbol{X}$-morphism and hence $ev : (X, A) \times (X', A') \to (Y, B)$ is an $\boldsymbol{XA}$-morphism. Since any $\boldsymbol{XA}$-morphism $f : (X, A) \times (Z, C) \to (Y, B)$ is some $f_i$ and $\bar{f}_i : (X_i, A_i) \to (X', A')$ is the unique $\boldsymbol{XA}$-morphism such that the diagram

$$\begin{tikzcd}
(X,A) \times (X_i, A_i) \arrow[d, "id_A \times \bar{f}_i"'] \arrow[dr, "f_i"] & \\
(X,A) \times (X', A') \arrow[r, "ev"'] & (Y,B)
\end{tikzcd}$$

commutes, we conclude that $ev : (X, A) \times (X', A') \to (Y, B)$ is an evaluation morphism in $\boldsymbol{XA}$ and $(X', A')$ is a power object in the category $\boldsymbol{XA}$. Thus $\boldsymbol{XA}$ is cartesian closed. $\square$

If $\boldsymbol{A}$ is a cartesian closed subcategory of $\boldsymbol{Alg(\Sigma)}$ which is $T$-friendly for some $\boldsymbol{A}$-object $T$, then $\boldsymbol{A}$ satisfies the hypothesis of the above proposition (see [10]) and hence $\boldsymbol{XA}$ is cartesian closed.

**Corollary 1.** *If $\boldsymbol{X}$ is a cartesian closed well-fibred topological category and $\boldsymbol{A}$ is $\boldsymbol{Alg(1)}$, $\boldsymbol{Alg(\Sigma)}$, or the category of $J$-sets (when $J$ is a monoid), then $\boldsymbol{XA}$ is cartesian closed.*

*Proof.* Obviously $\boldsymbol{Alg(1)}$ and $\boldsymbol{Alg(\Sigma)}$ satisfy the hypothesis of the above Proposition from the discussion after Equation (3.3). If $(J, +, \mathbf{0})$ is a monoid, A and $B$ are any two $J$-sets, and $C$ is any subalgebra of $\hom_{\boldsymbol{A}}(A \times J, B)$ , then

$$\omega_{j+j',C}f = f \circ (id_A \times \delta_{j+j'}) = f \circ (id_A \times (\delta_{j'} \circ \delta_j))$$
$$= f \circ (id_A \times \delta_{j'}) \circ (id_A \times \delta_j) = \omega_{j,C} \circ \omega_{j',C} f,$$

where $j \in J$, $j' \in J$, and $f \in C$. This shows that $C$ is also a $J$-set. Thus, the category of $J$-sets also meets the hypothesis of the above proposition. $\square$

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