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AN AXIOMATIC CONSTRUCTION OF DECORATED SEMISTABLE VECTOR BUNDLES ON SMOOTH CURVES: GALOIS UNRAMIFIED COVERINGS

E. Ballico

Department of Mathematics
University of Trento
380 50 Povo (Trento) - Via Sommarive, 14, ITALY
e-mail: ballico@science.unitn.it

Abstract: Let $f : X \to Y$ a Galois unramified covering of smooth and projective curve and $\Phi(X)$, $\Phi(Y)$ " compatible " categories of decorated vector bundles on X and Y. Here we give an axiomatic framework to say when the push forward preserves stability.

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1. Semistable Decorated Vector Bundles and Galois Unramified Coverings

Let X be a smooth and connected projective curve. Let $\Phi(X)$ denote any additivecategory of decorated vector bundles on X (e.g. holomorphic triples or categories coming from quivers ([1], [3], [4], [5]) and μ a real valued additive function $\mu : \Phi(X) \to \mathbb{R}$ used to define a notion of μ-stability and μ semistablility for objects of $\Phi(X)$. Condider the following properties that the pair $(\Phi(X), \mu)$ may have:

(a) Let

$$0 \to T' \to T'' \to 0$$

be any exact sequence of elements of $\Phi(X)$. The following conditions are equivalent:

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(i) $\mu(T') \leq \mu(T)$;

(ii) $\mu(T) \leq \mu(T'')$;

(iii) $\mu(T') \leq \mu(T'')$.

(b) $\mu(g^*(T)) = \mu(T)$ for every $g \in \mathrm{Aut}(X)$ and any $T \in \Phi(X)$.

(c) let T_i, $1 \leq i \leq s$, be a finite subset of $\Phi(X)$ such that each T_i is μ-semistable and $\mu(T_i) = \mu(T_j)$ for all i, j; then $T_1 \oplus \cdots \oplus T_s$ is μ-semistable and $\mu(T_1 \oplus \cdots \oplus T_s) = s \cdot \mu(T_1)$.

Remark 1. Assume that $\Phi(X)$ satisfies (a) and (c). As in the vector bundle case (see [3], Prop. 2.1) we get that the category of μ-semistable objects of $\Phi(X)$ is an abelian category.

Let X, Y be smooth and connected projective curves equipped with pairs $\Phi(X), \mu)$ and $(\Phi(Y), \nu)$ of decorated vector bundles with a slope function. Let $f : X \to Y$ a Galois unramified covering with G as its Galois group. Set $m := \sharp(G)$. We mill say that $(\Phi(X), \mu)$ and $(\Phi(Y), \nu)$ are f-compatible if the following conditions are satisfied:

(i) $f_*(T) \in \Phi(Y)$ and $\nu(f_ast(T)) = \mu(T)$ for all $T \in \Phi(X)$.

(ii) $f^*(T) \in \Phi(X)$ and $\mu(f^*(T)) = m \cdot \nu(T)$ for all $T \in \Phi(Y)$.

The first proof given in [3] of [3], Th. 3.1, gives the following result.

Theorem 1. *Let $f : X \to Y$ a Galois unramified covering between smooth and projective curves and $(\Phi(X), \mu)$, $(\Phi(Y), \nu)$ f-compatible categories of decorated sheaves. Assume that $(\Phi(X), \mu)$ satisfies properties (a), (b) and (c), and that $(\Phi(Y), \nu)$ satisfies properties (a) and (c). Let G be the Galois group of f. Fix any $T \in \Phi(X)$. Then:*

(i) T is μ-semistable if and only if $f_*(T)$ is ν-semistable

(ii) $f_*(T)$ is ν-stable if and only if T is μ-stable and no two of the decorated bundles $g^*(T)$, $h^*(T)$, $g, h \in G$, $h \neq g$, are isomorphic.

We work over an algebraically closed field $\mathbb{K}$ such that $\mathrm{char}(\mathbb{K}) = 0$.

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