**International Journal of Pure and Applied Mathematics**

**Volume 21 No. 3 2005, 135-136**

# ON THE ZERO-LOCUS OF REAL ANALYTIC FUNCTIONS ON DOMAINS OF TOPOLOGICAL VECTOR SPACES

E. Ballico

Department of Mathematics
University of Trento
380 50 Povo (Trento) - Via Sommarive, 14, ITALY
e-mail: ballico@science.unitn.it

**Abstract:** Let $V$ be a real Fréchet space without a continuos norm, $W$ a real Banach space, $U$ an open subset of $V$ and $f : U \to W$ a real analytic function. Then for every $P \in f^{-1}(0)$ there is no open subset $\Omega$ of $P$ in $U$ and a closed finite-dimension real submanifold $Z$ of $\Omega$ such that $f^{-1}(0) \cap \Omega \subseteq Z$. Furthermore, there is an open neighborhood $A$ of $P$ such for every integer $z \geq 1$, there is an a $z$-dimensional closed real submanifold $T_z$ of $A$ such that $P \in T_z \subset f^{-1}(0)$.

**AMS Subject Classification:** 32C05, 32D20, 46E99
**Key Words:** real analytic function, real analytic function in infinite-dimensional topological vector spaces, topological vector space without a continuous norm

## 1. Zero-Loci of Real Analytic Functions

In [1] we used [2] on the zero-loci of real analytic functions on open subsets of real Fréchet spaces without a continuos norm: no such zero-locus may have an isolated point. Using [2] and [4] we also proved that there is a Banach space with the same properties. Here we adapt [1], proof of Th. 1, to prove the following result.

**Theorem 1.** *Let $V$ be a real Fréchet space without a continuos norm, $W$ a real Banach space, $U$ an open subset of $V$ and $f : U \to W$ a real analytic*

Received: May 6, 2005

*function. Then for every $P \in f^{-1}(0)$ there is no open subset $\Omega$ of $P$ in $U$ and a closed finite-dimension real submanifold $Z$ of $\Omega$ such that $f^{-1}(0) \cap \Omega \subseteq Z$. Furthermore, there is an open neighborhood $A$ of $P$ such for every integer $z \geq 1$, there is an a $z$-dimensional closed real submanifold $T_z$ of $A$ such that $P \in T_z \subset f^{-1}(0)$.*

*Proof.* By [5], Th. 2.6.13, $V$ has a subspace $E$ isomorphic to $\mathbb{R}^{\mathbb{N}}$. Hence we reduce to the case $V = \mathbb{R}^{\mathbb{N}}$. Hence it is sufficient to check the extension of [3], Cor. 1, to the Banach valued case. □

**Remark 1.** Take $V$, $U$, $W$ and $f$ as in Theorem 1. Assume that $V$ has the stronger property “ every continuos seminorm ” vanishes on a finite codimensional linear subspace. Then for every $P \in f^{-1}(0)$ there is an open neighborhhod $B$ of $P$ in $U$ and a finite-codimensional closed real analytic submanifold $T$ of $B$ such that $P \in T \subseteq f^{-1}(0)$.

## Acknowledgements

The author was partially supported by MIUR and GNSAGA of INdAM (Italy).

## References

[1] E. Ballico, Real analytic functions with isolated zeroes on domains of topological vector spaces, Yokohama Math. J. 51 (2005), no. 1, 99–102.

[2] S. Dineen, Growth properties of pseudo-convex domains and domains of holomorphy in locally convex topological vector spaces, Math. Ann. 226 (1977), 229–236.

[3] A. Hirschowitz, Remarques sur les ouverts d’holomorphie d’un produit dénombrable de droites, Ann. Inst. Fourier 19 (1969), no. 1, 219–229.

[4] B. Josefson, A counterexample to the Levi problem, Proc. in Infinite Dimensional Holomorphy, Lect. Notes in Math. 364, pp. 168–177, Berlin - Heidelberg - New York, Springer, 1974.

[5] J. Pérez Carreras and J. Bonnet, Barrelled Locally Convex Spaces, North-Holland, Amsterdam - New York, 1987.