International Journal of Pure and Applied Mathematics
Volume 23 No. 2 2005, 283-289

A LOCAL POTENTIAL WITH ANALYTICAL DISTORTED WAVE APPROXIMATION MODEL FOR ELASTIC SCATTERING OF PIONS FROM A NUCLEUS

Reza Safari[1]
Department of Nuclear Physics
Faculty of Physics
Tabriz University
Tabriz, IRAN
e-mail: r-safari@tabrizu.ac.ir

Abstract: The ordinary equivalent local form of Kisslinger [7] type optical potential together with an analytical distorted wave approximation (ADWA) is explained to discuss the elastic scattering of pions with intermediate energies from a nucleus. Calculation of angular distribution of the scattering of negative pion on ${}^{208}_{82}Pb$ using Born approximation gives an acceptable fit to the data. The possible values for the potential parameters and ADWA parameters are obtained by comparison of data with the theoretical formula.

AMS Subject Classification: 92E10
Key Words: nuclear scattering, ordinary nuclear scattering, nuclear reactions, meson reactions

1. Introduction

We have discussed previously the pion-nucleus elastic scattering for low energy region [11]. The study of such physical interaction with incident energies around the range of the first pion nucleon resonance $\Delta(1232)$ has been interesting subject for many theoretical and experimental nuclear physicians over the past decade. Theoretical studies are mostly based on microscopic impulse approximation which is in substantial contrast to another approach based upon

Received: September 27, 2005

T_π(MeV)	$b_0(fm)$	$b_1(fm)$	$c_0(fm^3)$	$c_1(fm^3)$	$C_0(fm^6)$	$C_1(fm^6)$
162	0.179;0.117	-0.12;0.004	0.039;0.554	0.276;1.630	0.043.620	2.227;0.471
180	-0.085;0.045	-0.124;0.007	0.146;0.639	0.082;0.159	0.030;2.138	0.172;4.514
291	-0.043;0.135	-0.118;0.020	-0.101;0.651	-0.056;0.121	0.728;0.469	-0.599;10183

Table 1: Local potential parameter values calculated in this work with $\zeta = 1$. Note: In each case the left number is the real part and the right number is the imaginary part

phenomenological local potential models. The reason for the importance of this approach is that both s and p-wave components are presented in pion-nucleus scattering. Here we explain the pion-nucleus elastic scattering for the energy range where $l = 0, 1$ partial waves are in resonance by an equivalent local form of a Kissliger optical potential (KOP) [6, 11] using the method of Born approximation(BA). But pion-nucleon two body interaction becomes stronger at intermediate energies for larger distances from the center of the nucleus which causes pions not to penetrate deeper in to the nucleus. Therefore the elastic scattering cross-section would have a considerable increase which is not consistent with the values obtained from KOP. This needs to increase the strength of the potential. In fact related to the spin and isospin degrees of freedom the scattering amplitude is related to complex phase shifts at intermediate energies. The real part of the phase shifts is measured by an observer which give the elastic component of the scattering. On the other hand the imaginary part is responsible for inelastic scattering. The imaginary part indicates that this potential behaves like a source or a well for incident flux. These all indicate that the KOP must be altered from the original form for medium energies. For these reasons we use the distorted wave impulse approximation method (DWAM) [9] in our calculation and so an approximated plane waves are used instead of the usual distorted wave functions. Therefore a sort of analytic distorted wave approximation [2, 5, 1] should be applied which has been successfully used in calculation of other particles from nuclei. A brief review of the theoretical calculation of optical potential (OP) is described in Section 2. In Section 3 we present elastic scattering formalism using the ADWA model. Section 4 being devoted to discussion of results and conclusions.

T_π(MeV)	α	β	A_1	B_1	A_2	B_2	$k(fm^{-1})$	$k_{eff}(fm^{-1})$	P_1	P_2
162	1.35	-0.02	4	0.08	0.7	0.6	1.353	1.8268	1.3247	1.1623
180	1.33	-0.03	4.5	0.08	0.08	0.6	1.454	1.9355	1.3440	1.1720
291	1.2	-0.185	6.2	0.08	0.7	0.6	2.060	2.5014	1.4632	1.2316

Table 2: ADWA and kinematic parameter values calculated in this work

2. The Theory

The actual KOP for pion scattering from a nucleus [7] is:

$$U_k(r) = \frac{(\hbar c)^2}{2\omega}.(q(r) + \nabla.\alpha(r)\nabla)\,. \tag{1}$$

The local transformed form of this potential is calculated from a Shrodinger like equation which is obtained from a Kein Gordan equation by using the Krell-Ericson transformation [8]:

$$U_L = \frac{(\hbar c)2}{2\omega}[\frac{q}{1-\alpha} - \frac{k^2\alpha}{1-\alpha}[\frac{\nabla^2\alpha}{1-\alpha} + \frac{(\nabla\alpha)^2}{4(1-\alpha)^2}]] + \frac{\alpha V_c}{1-\alpha}\,. \tag{2}$$

The s and p-wave coefficients of the potential which are denoted by $\alpha(r)$ and $\beta(r)$ respectively take the following equations.

$$q(r) = -4\pi P_1(b_0\rho(r) - e_\pi b_1 \Delta\rho(r)) + \Delta q(r)\,, \tag{3}$$

$$\alpha(r) = \frac{\alpha_1(r)}{1 + \frac{1}{3}\zeta\alpha_1(r)} + \alpha_2(r)\,, \tag{4}$$

where

$$\alpha_1(r) = 4\pi\frac{(c_0\rho(r) - e_\pi c_1\Delta\rho(r))}{P_1}\,, \tag{5}$$

$$\alpha_2(r) = 4\pi\frac{(C_0\rho_{np}(r) - e_\pi C_1\rho(r)\Delta\rho(r))}{P_2}\,, \tag{6}$$

and

$$\rho(r) = \rho_n(r) + \rho_p(r)\,, \tag{7}$$

$$\Delta\rho(r) = \rho_n(r) - \rho_p(r)\,, \tag{8}$$

$$\rho_{np} = 4\rho_n(r)\rho_p(r)\,. \tag{9}$$

Here e_π takes $+/-$ sign relative to the $+/-$ charge state of pion and $\rho_p(r)$ and $\rho_n(r)$ are proton and neutron density distributions of target nucleus respectively. P_1 and P_2 are kinematic constants which depend on pion energy. The

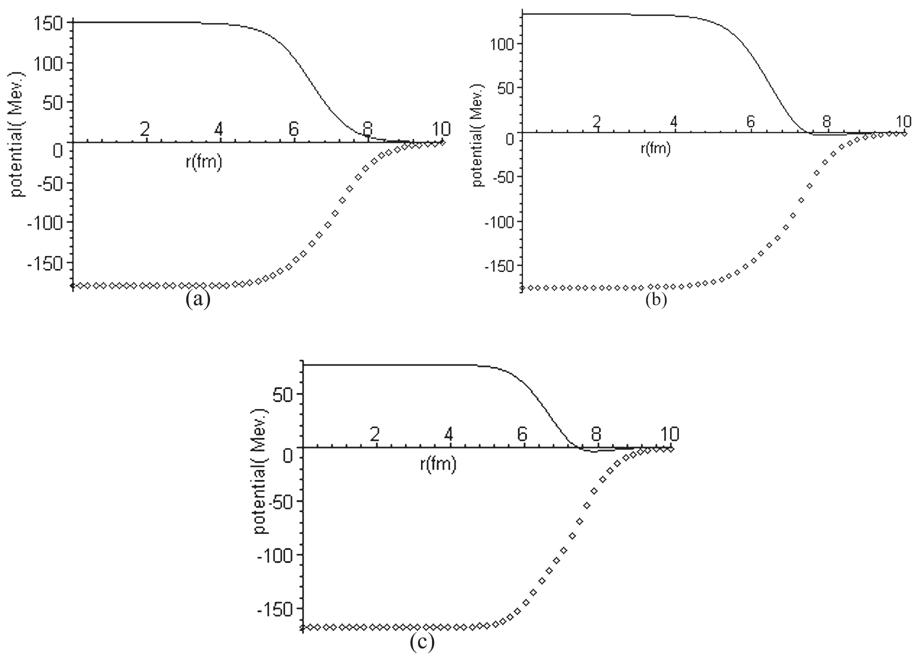

Figure 1: The real and imaginary parts of equivalent local potential, U_L, for $\pi^- +^{208}_{82} Pb$ at a : $162(MeV)$,b : $180(MeV)$, c : $291(MeV)$ energies. The continuous curves are the real part and the square shape curves are the imaginary part of the potential

value of the potential parameters c_0, c_1, b_0, b_1, C_0, and C_1 are presented in Table 1. These parameters are energy dependent and their values are obtained by fitting the theoretical angular distribution to the experimental data values for different energy values. In equation (2) we have ignored the second-order s-wave contributions and take $\zeta = 1$. The second order Coloumb interaction also has been omitted from these calculations. Here we have used two-parameter Fermi distribution for proton and neutron:

$$\rho_i = \frac{\rho_{0i}}{[1 + \exp(r - \frac{c_i}{a_i})]} \,. \tag{10}$$

Here i stands for proton and neutron, $\rho_{0i} = 0.0815(\text{fm})^{-3}$, $c_n = 6.4\text{fm}$, $c_p = 6.54\text{fm}$, $a_p = a_n = 0.545\text{fm}$. The RMS radius calculated by equation (10) is 5.545fm. To explain the necessary strength of the local potential and also to include the spin and isospin degrees of freedom one can argue that KOP should

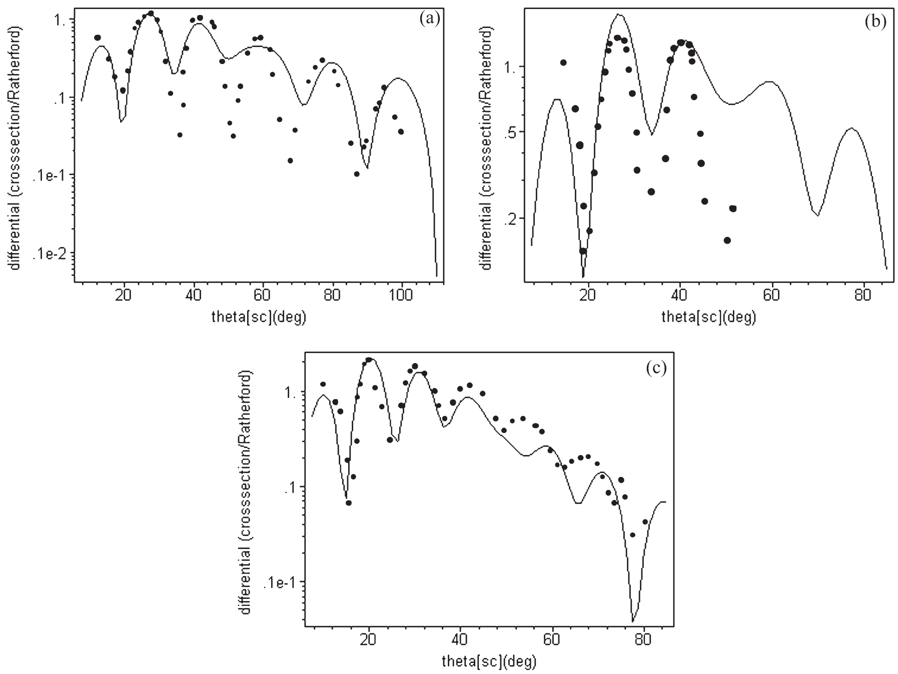

Figure 2: Elastic scattering differential cross-section of negative pion, continuous curves, from $^{208}_{82}Pb$ target using equation (14) at a : $162(MeV)$,b : $180(MeV)$, c : $291(MeV)$ energies. Data points are from references [10], [4], and [3] respectively

have a deformed shape as

$$V_{dop} = \frac{(\hbar c)^2}{2\omega}(a_0 q(r) + a_1 \nabla.\alpha\nabla)\,, \tag{11}$$

where a_0, a_1 are r dependent parameters. It is straightforward to show that these parameters are related to the phase shifts of different spin-isospin states. They also vary with energy and scattering angle. To explain these features we replace the incident wave function calculated from the OP with a distorted plane waves:

$$\Psi_i^d(\mathbf{r}) = N \exp(i(\alpha + i\beta)\mathbf{k}.\mathbf{r})\,. \tag{12}$$

Here α and β take energy dependent values and N is a function of r and scattering angle which may take the following form

$$N = (A_1 r^{B_1} - A_2 r^{B_2} i)\sin\theta_{sc}\,. \tag{13}$$

These parameters α, β, A_i, and B_i should also be obtained from a fit of the differential cross-section data to the theoretical formula resulted from the scattering amplitude:

$$f(\theta_{sc}) = -\frac{\mu}{2\pi\hbar^2} \int \exp(-i\mathbf{k_f}.\mathbf{r}) V_{op} \Psi_i^d(\mathbf{r})\,. \tag{14}$$

3. Results and Conclusion

Real and imaginary parts of the kisslinger optical potential for three energies are presented in Figure 1. The strength of the repulsive real part has a substantial decrease with increasing energy and the attractive imaginary part also becomes weaker when energy is increased. An attractive minimum in the real part is gradually produced with increasing energy. We have calculated the angular distribution for each energy by using equation (14) numerically by taking 400 points of integration. The results are compared with data in Figure 2. These results show that although there is differences between the theoretical estimates and data, but altogether, using the local form of KOP with the ADWA provides a quite successful theory of elastic scattering of negatively charged pions from $^{208}_{82}Pb$ target. This can be extended to the other target nuclei. Theoretical results are sensitive to the parameters of the optical potential which are obtained by comparing the theoretical and experimental differential cross-sections. In this case the optimum values for the parameters of the analytic function of the ADWA are also calculated. We think that both of the α and β parameters need to show smooth variations with the pion energy. A clear fact is obvious that the incident and emerging wave functions would provide an overlap function that resembles the overlap of the two actual distorted waves in the spatial region of integration. In doing so it is clear that these wave functions should provide a qualitatively good theoretical background for the study of inelastic scattering of pions from nuclei.

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