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KINEMATICS OF THE TURN OF WHEEL VEHICLE WITH ACCOUNT OF THE DEFORMATIONS OF THE TIRES AND THE INFLUENCE OF THE SUSPENSION

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Abstract: Generalized relations for determination of the parameters of the turn of wheel vehicle with account of the deformation of the tires and the influence of the suspension are suggested. As a particular case the well-known from the theory of automobile, tractor and truck formulae are obtained.

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The kinematics of the turn of wheel vehicle with rigid wheels and with wheels which possessing the behavior transversal sweeping in the theory of the automobile, tractor and truck is sufficiently thorough investigated. Simultaneously the rates of deformations of the tires and the changes in the machine as result of the influence of the kinematical parameters of the suspension are not taken into account.

The purpose of the resent work is an analysis of the kinematics of the turn of the wheel vehicle to be made, taking into account the deformations of the tires and the changes of the position of the axles and the hull under the kinematical parameters of the suspension.

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On Figure 1 the projection in the plain of the road π of wheel vehicle 4×2 with dependent back suspension is shown. The belonging to the respective element is denoted by indexes "$_1$" and "$_2$". The indexes "$_l$" and "$_r$" denote the belonging to the left or right suspension. B_k are the intersection points of the straights of the greatest slope of the longitudinal plains of symmetry of the wheels, crossing through the point O_k, with the plain of the road. B denotes the position of the centers of the contact trace after deformation of the wheels: longitudinal – η_w and transversal – ξ_w, measured in the plain of the road (index "$_w$"); with respect to the longitudinal plain of symmetry of the wheels. C_w is the projection of the gravity center of the vehicle on the plain of the road. Right-oriented coordinate system $Oxyz$ (fixed, with initial point in the intersection of the left blade axe with the plain of the road in rated position) is the same as in [1] and [2]. Further all coordinates are expressed relatively to this coordinate system.

It is accepted that the position of the wheels and the hull relatively to the coordinate system $Oxyz$ is known which defines the coordinates $\{x_C, y_C, 0\}$ of the gravity center C of the machine.

D_{1lw} and D_{1rw} are the projections in the plain π of the points D_{1l} and D_1r which are the intersection between the axes of the wheels with their blade axes (they determine the defined in [1] distance $D_{1l}D_{1r}$). D_{1w} is the denoted projection in the plain π of the middle point D_1 of the vector $\vec{\ell_s} = D_{1l}\vec{D}_{1r}$. Point D_{2w} represents the projection in the plain π of the middle point D_1 of the vector $\vec{\ell_s} = O_{k2l}\vec{O}_{k2r}$ which defines the back axle. The coordinates of the points D_1 and D_2 could be calculated by the following relations:

$$O\vec{D}_1 = O\vec{D}_{1l} + 0.5\ell_s\,, \quad O\vec{D}_1 = O\vec{O}_{k2l} + 0.5O_{k2l}O\vec{O}_{k2r}\,. \tag{1}$$

The coordinates of the points D_{1w} and D_{2w} could be defined by the help of vectors $O\vec{D}_{1w}$ and $O\vec{D}_{2w}$

$$O\vec{D}_{1w} = \{OD_{1x}, OD_{1y}, 0\}\,, \quad O\vec{D}_{2w} = \{OD_{2x}, OD_{2y}, 0\}\,. \tag{2}$$

The vector $\vec{d}$ and its projection $\vec{d_w}$, respectively

$$\vec{d} = O\vec{D}_1 - O\vec{D}_2\,, \quad \vec{d_w} = D_{2w}\vec{D}_{1w} = O\vec{D}_{1w} - O\vec{D}_{2w}\,, \tag{3}$$

defines the longitudinal axe of the driving system. The angle of rotation in respect to the rate position is defined by

$$\psi_{nw} = \arctan\left(d_y/d_x\right). \tag{4}$$

The vertical plain trough the vectors $\vec{d}$ and $\vec{d_w}$ crosses the road plain in a straight line d_w, whose equation is

$$d_w = y - (d_y/d_x)x + (d_y/d_x)x_{D2} - y_{D2} = 0\,. \tag{5}$$

The angles θ'_{1b}, θ'_{1bw} and θ'_2 are the angles of rotation in the road plain π of the right and left dirigible wheel and the back axle under the influence of the kinematical parameters of the suspension. The other symbols are the same as in [2].

In the theory of the automobile, tractor and truck the Ackerman statement that for any vehicle with rigid wheels the normal to the plains of rolling of the wheels must intersect in one point.

The equations of the straights a_{1l} and a_{1r}, which are the intersection of the plain of the left, respectively right, dirigible wheel with the plain π (Figure 1) and the straights through the points B_{k1l} and B_{k1r} perpendicular to them b_{1l} and b_{1r} are, [1]:

$$\begin{aligned} a_{1l(r)} = y + (N_{1l(r)x}/N_{1l(r)y})x - (N_{1l(r)x}/N_{1l(r)y})x_{Bk1l(r)} - y_{Bk1l(r)} \\ = 0\,, \end{aligned} \tag{6}$$

$$\begin{aligned} b_{1l(r)} = y - (N_{1l(r)x}/N_{1l(r)y})x + (N_{1l(r)y}/N_{1l(r)x})x_{Bk1l(r)} - y_{Bk1l(r)} \\ = 0\,, \end{aligned} \tag{7}$$

where $N_{1l(r)x}$, $N_{1l(r)y}$ are the coordinates of the vectors of the axes of the wheels $\vec{N}_l$ and $\vec{N}_r$; $x_{bt1l(r)}$ and $y_{bt1l(r)}$ are the coordinates of the points B_{k1l} and B_{k1r}.

The line c_{2k}, which pass through the points B_{k2l} and B_{k2r}, has the following governing vector and equation:

$$\begin{aligned} &\vec{c_{2k}} = O\vec{B}_{k2r} - O\vec{B}_{k2l}\,, \\ &c_{2k} \equiv y - (c_{2ky/c_{2kx}})x + (c_{2ky}/c_{2kx})x_{bk2l} - y_{bk2l} = 0\,. \end{aligned} \tag{8}$$

By dependent suspension of the back axle this straight coincides with the projection of its axe in the plain π. θ'_2 is the angle defined by the rotation of the back axle. From (7) it follows that it is

$$\theta'_2 = \arctan(c_{2kx}/c_{2ky})\,. \tag{9}$$

The straights b_{1l} and b_{1r} intersect each other in the point M_k, whose coordinates are

$$\begin{aligned} x_{Mk} &= \frac{A_{1r} - A_{1l}}{(N_{1ry}/N_{1rx}) - (N_{1ly}/N_{1lx})}\,, \\ y_{Mk} &= \frac{N_{1ry}}{N_{1rx}} \frac{A_{1r} - A_{1l}}{(N_{1ry}/N_{1rx}) - (N_{1ly}/N_{1lx})} - A_{1r}\,, \end{aligned} \tag{10}$$

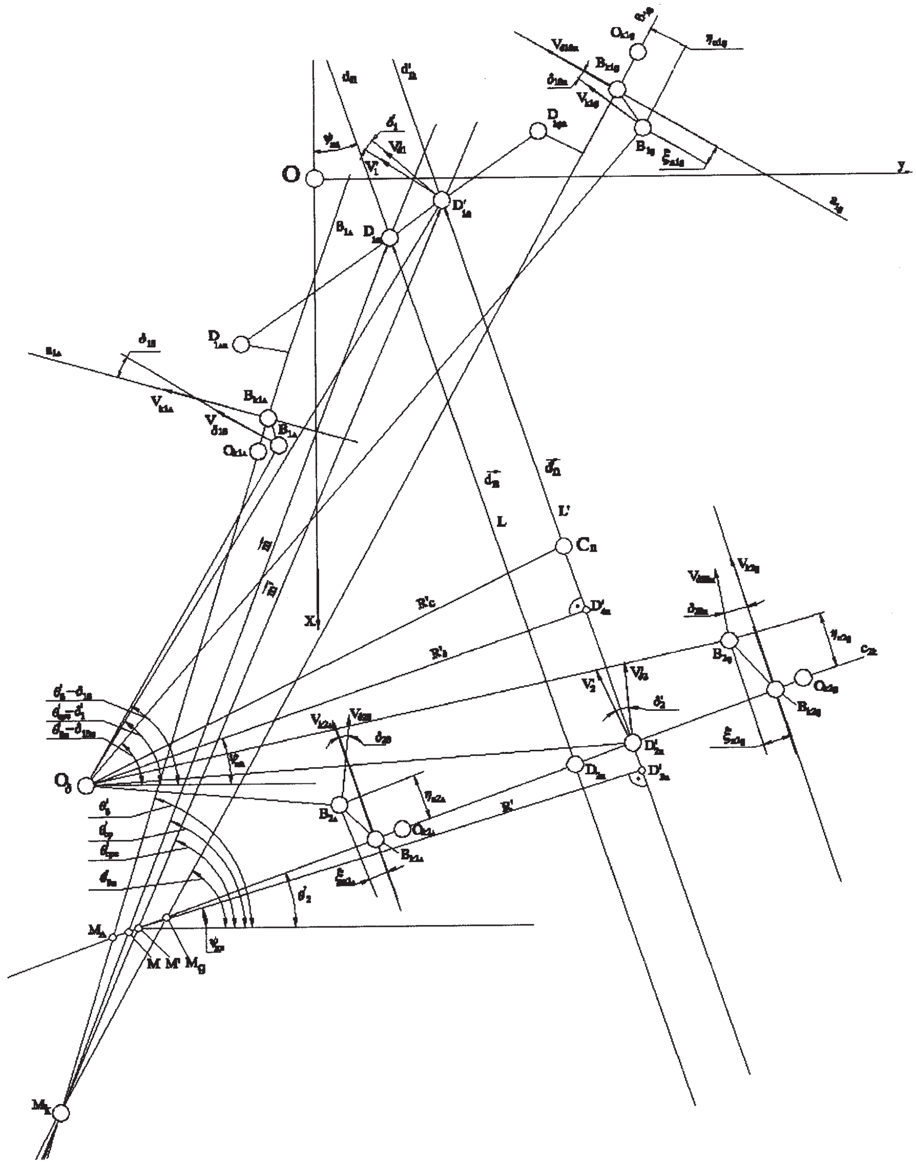

Figure 1

where $A_{1l} = \frac{N_{1ly}}{N_{1lx}} x_{bk1l} - y_{bk1l}$, $A_{1r} = \frac{N_{1ry}}{N_{1rx}} x_{bk1r} - y_{bk1r}$.

From the assertion of Ackerman it follows that the point M_k should lie on the straight c_{2k}. It is known that the steering wheel trapezium cannot fulfill this requirement. Therefore the intersection points of b_{1l} and b_{1r} with the straight c_{2k}, M_l and M_r have coordinates

$$\begin{aligned} x_{Ml} &= \frac{A_{2k} - A_{1l}}{(c_{2ky}/c_{2kx}) - (N_{1ly}/N_{1lx})}\,, \\ y_{Ml} &= \frac{c_{2ky}}{c_{2kx}} \frac{A_{2k} - A_{1l}}{(c_{2ky}/c_{2kx}) - (N_{1ly}/N_{1lx})} - A_2\,, \end{aligned} \tag{11}$$

$$\begin{aligned} x_{Mr} &= \frac{A_{2k} - A_{1r}}{(c_{2ky}/c_{2kx}) - (N_{1ry}/N_{1rx})}\,, \\ y_{Mr} &= \frac{c_{2ky}}{c_{2kx}} \frac{A_{2k} - A_{1r}}{(c_{2ky}/c_{2kx}) - (N_{1ry}/N_{1rx})} - A_2\,, \end{aligned} \tag{12}$$

where $A_{2k} = \frac{c_{2wy}}{c_{2wx}} x_{bw2l} - y_{bw2l}$.

The line m, passing through point M_k and D_{1w}, has the governing vector and equation as follows:

$$\begin{aligned} &\vec{m} = M_k\vec{D}_{1w} = O\vec{D}_{1w} - O\vec{M}_k\,, \\ &m \equiv y - (m_y/m_x)x + (m_y/m_x)x_{Mk} - y_{Mk} = 0\,. \end{aligned} \tag{13}$$

It crosses c_{2k} in M and its coordinates are

$$\begin{aligned} x_M &= \frac{A_{2k} - A_m}{(C_{2ky}/C_{2kx}) - (m_y/m_x)}\,, \\ y_M &= \frac{c_{2ky}}{c_{2kx}} \frac{A_{2k} - A_m}{(c_{2ky}/c_{2kx}) - (m_y/m_x)} - A_{2k}\,, \end{aligned} \tag{14}$$

where $A_m = \frac{m_y}{m_x} x_{Mk} - y_{Mk}$.

As it was mentioned above, the Ackerman assertion is fulfilled when the points M_k, M_l and M_r coincide with the point M.

The straight line m defines the average angle of turning $\theta_{av,m}$ of the dirigible wheels in the road plain, when the deformations of the tires are not taken into account. So,

$$\theta'_{av,m} = -\arctan\left(m_x/m_y\right). \tag{15}$$

For determination of the kinematical parameters of the turn when the changes of the position of the axles and hull are taken into account, a straight line d'_w through the projection of the gravity center C_w on the plain of the road parallel to the straight d_w is constructed with governing vector and equation, respectively

$$\vec{d'_w} = \{d_x, d_y, 0\}\,,\ d'_w \equiv y - (d_y/d_x)x + (d_y/d_x)x_c - y_c = 0\,. \tag{16}$$

The straight d'_w crosses ℓ_{sw}, which represent the projection on the road plain of the straight ℓ_s. ℓ_s crosses through the points D_{1l} and D_{1r}, its governing vector is $\vec{\ell}_s$ and its equation is

$$\ell_{sw} \equiv y - (\ell_{sy}/\ell_{sx})x + (\ell_{sy}/\ell_{sx})x_{D1l} - y_{D1l} = 0\,, \tag{17}$$

The coordinates of the point D'_{1w} are

$$\begin{aligned} x_{D'1w} &= \frac{A_{d'w} - A_{sw}}{(d_y/d_x) - (\ell_{sy}/\ell_{sx})}\,, \\ y_{D'1w} &= \frac{d_y}{d_x}\frac{A_{d'w} - A_{sw}}{(d_y/d_x) - (\ell_{sy}/\ell_{sx})} - A_{d'w}\,, \end{aligned} \tag{18}$$

where $A_{d'w} = \frac{d_y}{d_x}x_c - y_c$; $A_{sw} = \frac{\ell_{sy}}{\ell_{sx}}x_{D1lw} - y_{D1lw}$.

The coordinates of the cross point D'_{2w} of straights d'_w and c_{2k} are

$$\begin{aligned} x_{D'2w} &= \frac{A_{2k} - A_{d'w}}{(c_{2ky}/c_{2kx}) - (d_y/d_x)}\,, \\ y_{D'2w} &= \frac{c_{2ky}}{c_{2kx}}\frac{A_{2k} - A_{d'w}}{(c_{2ky}/c_{2kx}) - (d_y/d_x)} - A_{2k}\,, \end{aligned} \tag{19}$$

The vector $\vec{m}'$, which coordinates are

$$\vec{m}' = M_k\vec{D}'_{1w} = O\vec{D}'_{1w} - O\vec{M}_k \tag{20}$$

is a governing vector of the straight m', given by equation

$$m' \equiv y - (m'_y/m'_x)x + (m'_y/m'_x)x_{Mk} - y_{Mk} = 0\,. \tag{21}$$

The straight m' crosses c_{2k} in point M', the coordinates of which are

$$\begin{aligned} x_{M'} &= \frac{A_{2k} - A_{m'}}{(c_{2y}/c_{2x}) - (m'_y/m'_x)}\,, \\ y_{M'} &= \frac{c_{2y}}{c_{2x}}\frac{A_{2k} - A_{m'}}{(c_{2y}/c_{2x}) - (m'_y/m'_x)} - A_{2k}\,, \end{aligned} \tag{22}$$

where $A_{m'} = \frac{m'_y}{m'_x}x_{Mk} - y_{Mk}$.

The angle of turning of the vector $\vec{m}'$ is $\theta'_{m'} = -\arctan\frac{m'_x}{m'_y}$. If a comparison with the angle $\theta'_{av,m}$ (equation (15)) is made, in the analysis of the kinematics of the turn with account of the kinematics of suspension $\theta'_{m'}$ has the meaning of average angle of turning of the dirigible wheels in the road plain $\theta_{av,c}$, i.e.

$$\theta'_{av,c} = \theta'_{m'} = -\arctan{(m'_x)}/m'_y\,. \tag{23}$$

The radius of the turn of vehicle with rigid tires, with account of the kinematics of suspension is

$$R' = M'D'_{3w}\,. \tag{24}$$

It follows from the triangle $\triangle M'D'_{3w}D'_{1w}$ that

$$\begin{aligned} R' &= D'_{1w}D'_{3w}/\tan(\angle D'_{3w}M'D'_{1w}) \\ &= (D'_{2w}D'_{1w} + D'_{2w}D'_{3w})/\tan(\angle D'_{3w}M'D'_{1w})\,. \end{aligned} \tag{25}$$

From the triangle $\triangle M'D'_{2w}D'_{2w}$, it follows

$$\begin{aligned} &\angle D'_{2w}M'D'_{3w} = \theta'_2 - \psi_{nw}\,, \\ &D'_{2w}D'_{3w} = M'D'_{2w}\sin(\theta'_2 - \psi_{nw}) = R'_2\sin(\theta'_2 - \psi_{nw})\,, \end{aligned} \tag{26}$$

where the length $M'D'_{2w}$ of the vector $M'\vec{D}'_{2w}$, defined by the vector equation

$$M'\vec{D}'_{2w} = O\vec{D}'_{2w} - O\vec{M}'\,, \tag{27}$$

is denoted by R'_2, i.e.

$$R'_2 = |M'D'_{2w}|\,. \tag{28}$$

The length of the vector

$$D'_{2w}\vec{D}'_{1w} = O\vec{D}'_{1w} - O\vec{D}'_{2w} \tag{29}$$

could be denoted by L' (the distance between the axles - longitudinal basis), i.e.

$$L' = |D'_{2w}\vec{D}'_{1w}|\,. \tag{30}$$

According to Figure 1, the angle

$$\angle D'_{3w}M'D'_{1w} = \theta'_{av,c} - \psi_{nw} \tag{31}$$

Finally, for the radius of the turn of wheel vehicle with rigid tires with account of the kinematics of the suspension it is obtained the relation

$$R' = \frac{L' + R'_2\sin(\theta'_2 - \psi_{nw})}{\tan(\theta'_{cw,c} - \psi_{nw})}\,. \tag{32}$$

The equation obtained represents generalized relation for determination of the radius of turn of wheel vehicle with rigid tires with account of the kinematics of the suspension. Is in (32) we replace $\theta'_2 = 0$ and $\psi_{nw} = 0$ and neglect the kinematics of the suspension, as particular case, the known from the theory formula will be obtained

$$R' = R = \frac{1}{\tan\theta'_{aw}}\,, \tag{33}$$

where R is the theoretical radius of turn; L is longitudinal basis; θ'_{av} is the average angle of turning of the dirigible wheels.

As consequence of the elastic properties of the tires, the centers of the contact traces (the points B_{1l}, B_{1r}, B_{2l} and B_{2r}) do not coincide with the points B_{w1l}, B_{w1r}, B_{w2l} and B_{w2r} and the wheels are moving with angles of transversal sweeping δ_{1b}, δ_{1bw}, δ_{2b} and δ_{2bw}. If the angles of deviating of the speeds $\vec{v}'_1$ and $\vec{v}'_2$ of points D'_{1w} and D'_{2w} of a vehicle with rigid wheels from the direction of the vector $\vec{d}'_w$ are

$$\begin{aligned}\angle(\vec{v'_1}, \vec{d'_w}) &= \theta'_{av,c} - \psi_{nw}\,,\\ \angle(\vec{v'_2}, \vec{d'_w}) &= \theta'_2 - \psi_{nw}\,,\end{aligned} \tag{34}$$

than the angles of deviating of the vectors $\vec{v}'_{\delta 1}$ and $\vec{v}'_{\delta 2}$ of the same points with a sweeping are

$$\begin{aligned}\angle(\vec{v'_{\delta 1}}, \vec{d'_w}) &= \theta'_{av,c} - \psi_{nw} - \delta'_1\,,\\ \angle(\vec{v'_{\delta 2}}, \vec{d'_w}) &= \theta'_2 - \psi_{nw} - \delta'_2\,.\end{aligned} \tag{35}$$

Here δ'_1 and δ'_2 are the angles of transversal sweeping of the points D'_{1w} and D'_{2w} from the front axle.

Because of the transversal sweeping, the center of the turn moves from point M' to O_δ, which is the cross point of the normal to the speeds of the wheels and axles with transversal sweeping $\vec{v}_{\delta 1b}$, $\vec{v}_{\delta 1bw}$, $\vec{v}_{\delta 2b}$, $\vec{v}_{\delta 2bw}$, $\vec{v}_{\delta 1}$ and $\vec{v}_{\delta 2}$.

The radius of a turn with transversal sweeping (Figure 1) is

$$R'_\delta = O_\delta D'_{4w}\,. \tag{36}$$

From Figure 1 it follows

$$D'_{4w}D'_{1w} + D'_{2w}D'_{4w} = D'_{2w}D'_{1w} = L'\,.$$

Than from the triangles $\triangle O_\delta D'_{4w}D'_{1w}$ and $\triangle O_\delta D'_{4w}D'_{2w}$ the segments could be determined

$$\begin{aligned}D'_{4w}D'_{1w} &= R_\delta \tan\left(\angle D'_{4w}O_\delta D'_{1w}\right),\\ D'_{2w}D'_{4w} &= R_\delta \tan\left(\angle D'_{4w}O_\delta D'_{2w}\right).\end{aligned} \tag{38}$$

Hence for the segment $D'_{2w}D'_{1w} = L'$ it could be written

$$L' = R_\delta \left[\tan\left(\angle D'_{4w}O_\delta D'_{1w}\right) + \tan\left(\angle D'_{4w}O_\delta D'_{2w}\right)\right] . \tag{39}$$

The angles $D'_{4w}O_\delta D'_{1w}$ and $D'_{4w}O_\delta D'_{2w}$ can be defined by the relations

$$\begin{aligned} \angle D'_{4w}O_\delta D'_{1w} &= \theta'_{av,c} - \psi_{nw} - \delta'_1 , \\ \angle D'_{4w}O_\delta D'_{2w} &= \theta'_2 - \psi_{nw} - \delta'_2 . \end{aligned} \tag{40}$$

Having in mind the direction (clockwise is negative) for angle $D'_{4w}O_\delta D'_{2w}$ is obtained

$$\angle D'_{4w}O_\delta D'_{2w} = -(\theta'_2 - \psi_{nw} - \delta'_2) = \delta'_2 - \theta'_2 + \psi_{nw} . \tag{41}$$

Finally, it is obtained for the radius of turn with transversal sweeping the following equation

$$R'_\delta = \frac{L'}{\tan\left(\theta'_{av,c} - \psi_{nw} - \delta'_1\right) + \tan\left(\delta'_2 - \theta'_2 - \psi_{nw}\right)} . \tag{42}$$

The obtained generalized relation (42) describes the influence of the kinematics of suspension of wheel vehicle on the radius of a turn with transversal sweeping of the tires. If we replace $\psi_{nw} = 0$; $\theta'_2 = 0$, we receive the well-known from the theory of wheel vehicles equation

$$R'_\delta = R_\delta = \frac{L}{\tan\left(\theta'_{cw} - \delta_1\right) + \tan\delta_2} , \tag{43}$$

where δ_1 and δ_2 are the angles of transversal sweeping for the front and back axle, respectively.

The coordinates of the point D'_{4w} can be defined from the vector equation

$$O\vec{D}'_{4w} = O\vec{D}'_{2w} + D'_{2w}\vec{D}'_{4w} , \tag{44}$$

where the coordinates of the vector $D'_{2w}\vec{D}'_{4w}$ are

$$D'_{2w}\vec{D}'_{4w} = \{D'_{2w}D'_{4w}(d'_{nx}/|\vec{d'_w}|), D'_{2w}D'_{4w}(d'_{ny}/|\vec{d'_w}|), 0\} . \tag{45}$$

Hence, for the coordinates of the real center of the turn O_δ is obtained

$$O\vec{O}_\delta = O\vec{D}'_{4w} - O_\delta\vec{D}'_{4w} , \tag{46}$$

where the coordinates of the vector $O_\delta \vec{D}'_{4w}$ are

$$O_\delta \vec{D}'_{4w} = \{-R'_\delta \sin\psi_{nw}, R'_\delta \cos\psi_{nw}, 0\}\,. \qquad (47)$$

If we denote the distance $D'_{2w}C_w$ from point D'_{2w} of the projection of the back axle on the road plain to the projection of the gravity center C_w, measured in the direction of the straight d'_w by b', it could be written

$$\vec{D'_{2w}C_w} = O\vec{C}_w - O\vec{D}'_{2w}\,. \qquad (48)$$

Finally,

$$b' = |D'_{2w}C_w|\,. \qquad (49)$$

For the distance R'_c from the real center of the turn O_δ to the point C_w we can write

$$R'_c = \sqrt{{R'_\delta}^2 + [b' - R'_\delta \tan(\delta'_2 - \theta'_2 + \psi_{nw})]^2}\,. \qquad (50)$$

In particular, analogously to the given above, the following formula can be obtained

$$R'_c = \sqrt{{R'_\delta}^2 + [b - R_\delta \tan\delta_2]^2}\,, \qquad (51)$$

which is known from the theory of the wheel vehicles. Here b is the distance between the back axle and the gravity center.

The following vectors are connected with the coordinates of the point O_δ (equation (46)), the coordinates of the centers of the contact traces (B_{1l}, B_{1r}, B_{2l}, B_{2r}) and the points D'_{1w} and D'_{2w}

$$\begin{aligned} O_\delta\vec{B}_{1l} &= O\vec{B}_{1l} - O\vec{O}_\delta = O\vec{B}_{k1l} + \vec{B_{k1l}B_{1l}} - O\vec{O}_\delta\,,\\ O_\delta\vec{B}_{1r} &= O\vec{B}_{1r} - O\vec{O}_\delta = O\vec{B}_{k1r} + \vec{B_{k1r}B_{1r}} - O\vec{O}_\delta\,, \end{aligned} \qquad (52)$$

$$\begin{aligned} O_\delta\vec{B}_{2l} &= O\vec{B}_{2l} - O\vec{O}_\delta = O\vec{B}_{k2l} + \vec{B_{k2l}B_{2l}} - O\vec{O}_\delta\,,\\ O_\delta\vec{B}_{2r} &= O\vec{B}_{2r} - O\vec{O}_\delta = O\vec{B}_{k2r} + \vec{B_{k2r}B_{2r}} - O\vec{O}_\delta\,, \end{aligned} \qquad (53)$$

$$\begin{aligned} O_\delta\vec{D}'_{1w} &= O\vec{D}'_{1w} - O\vec{O}_\delta\,,\\ O_\delta\vec{D}'_{2w} &= O\vec{D}'_{2w} - O\vec{O}_\delta\,. \end{aligned} \qquad (54)$$

The vectors of deformation of the tires for the presented on Figure 1 wheel vehicle with back leading and front dirigible wheels are:

$$\begin{aligned} \vec{B_{k1l(r)}B_{1l(r)}} = \{&(\xi_{w1l(r)} \sin\theta'_{1l(r)} + \eta_{w1l(r)} \cos\theta'_{1lr}),\\ &(-xi_{w1l(r)} \cos\theta'_{1l(r)} + \eta_{w1l(r)} \sin\theta'_{1l(r)}), 0\}\,, \end{aligned} \qquad (55)$$

$$B_{k2l(r)}\vec{B}_{2l(r)} = \{(\xi_{w2l(r)} \sin\theta_2' + \eta_{w2l(r)} \cos\theta_2'), \\ (-\xi_{w2l(r)} \cos\theta_2' + \eta_{w2l(r)} \sin\theta_2'), 0\}\,. \quad (56)$$

Then, it follows for the angles of transversal sweeping of the tires δ_{1b}, δ_{1bw}, δ_{2b}, δ_{2bw} and the points D'_{1w} and D'_{2w} from the axles δ'_1 and δ'_2

$$\begin{aligned} \delta_{1b} &= \theta'_b - \arctan\left(-O_\delta B_{1lx}/O_\delta B_{1ly}\right), \\ \delta_{1bw} &= \theta'_{bw} - \arctan\left(-O_\delta B_{1rx}/O_\delta B_{1ry}\right), \end{aligned} \quad (57)$$

$$\begin{aligned} \delta_{2b} &= \theta'_2 - \arctan\left(-O_\delta B_{2lx}/O_\delta B_{2ly}\right), \\ \delta_{2bw} &= \theta'_2 - \arctan\left(-O_\delta B_{2rx}/O_\delta B_{2ry}\right), \end{aligned} \quad (58)$$

$$\begin{aligned} \delta'_1 &= \theta'_{cw} - \arctan\left(-O_\delta D'_{1lx}/O_\delta D'_{1ly}\right), \\ \delta'_2 &= \theta'_2 - \arctan\left(-O_\delta D'_{2wx}/O_\delta D_{2wy}\right). \end{aligned} \quad (59)$$

Finally, the following conclusions could be made:

1. The rate of the deformations of the tires and the changes due to the influence of the kinematical parameters of the suspension are not taken into account in the theory of wheel vehicles.

2. The suggested generalized relations allow determination of the kinematical parameters of the turn of wheel vehicles taking into account the deformations of the tires as well as the changes due to the influence of the kinematical parameters of the suspension. The known relations from the theory of wheel vehicles could be obtained from them as particular cases.

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