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NOTE ON FUZZY ENTROPY OF VAGUE SETS

Zhang Cheng-Yi[1 §], Shi Yu-Qiang[2]

[1]Department of Computer Science and Mathematics
Hainan Normal University
Haikou, Hainan, 571158, P.R. CHINA

[2]Department of Computer Science and Technology
Qiangzou University
Wuzhishan, Hainan, 572200, P.R. CHINA

Abstract: In this paper, we discussed the action the translation function from vague set to fuzzy set given by references [1], [2], and pointed out that a vague set is a special L-fuzzy set. We also studied the fuzzy entropy of vague sets in references [1], [2]. Based on the background of vague information, the fuzzy positive entropy and fuzzy negative entropy of vague set are defined. Their properties are discussed.

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1. Introduction

In 1965, Zadeh [6] proposed the theory of Fuzzy sets, and proposed the concept of Fuzzy entropy in 1969, which described the fuzziness and amount of information. Generally specking, a fuzzy set A of the universe of discourse X is a set of ordered pairs $\{x_t, \mu_A(x_t)|t \in T\}$, where, μ_A is the membership function of fuzzy set A. The most main character of fuzzy set is that membership function gives every object a value of [0,1] as it is grade of membership. The grade of membership has the following characteristic: this single value combines the evidence for and the evidence against, it is impossible to stand for any one of

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§Correspondence author

them, and let alone both of them. In order to solve the problem that fuzzy set cannot stand for and deal with the fuzzy information and data, Gau and Buchrer proposed the theory of vague set. In the article [1], [2]. Mr. Li Fan offers translation function from vague set to fuzzy set, and defines the fuzzy entropy of vague set on the basis of translation function. we discussed the action the translation function from vague set to fuzzy set given by references [1], [2], and pointed out that a vague set is a special L-fuzzy set. We also studied the fuzzy entropy of vague sets in references [1], [2], and holds the view that this translation function and fuzzy entropy of fuzzy set arising there from still cannot reflect the immanent information handling. Based on the background of vague information handling, the fuzzy positive entropy and fuzzy negative entropy of vague set are given, and their properties are discussed.

2. Preliminaries

Let X be the universe of discourse, a vague set A in X is characterized by the truth-membership function t_A and the false-membership function f_A:

$$t_A:\ x \to [0,1] \qquad f_A:\ x \to [0,1],$$

where $t_A(x)$ is a lower bound on the grade of membership of x from the evidence for $x \in X$, $f_A(x)$ is a lower bound on the negation of membership of x from evidence against for x, and $t_A(x)+f_AJ(x) \leq 1$. Then a vague A can be written as:

$$A = \{(x, t_A(x), f_A(x))|x \in X\}$$

and $< t_A(x), 1 - f_A(x) >$ is called value value of x in A denoted also x.

Note. For closed interval [a,b], we write vague value as $< t_A(x), 1-f_A(x) >$, differ to notation $[t_A(x), 1 - f_A(x)]$ used in [1].

If $x =< t_A(x), 1 - f_A(x) >$ is a vague value of a vague set A, then $x^{'} =< t_A(x), 1 - f_A(x) >$ is defined by the complement of x.

Let A be a vague set in X, X is a finite set, then A can be written as:

$$A = \sum_{i=1}^{n} < t_A(x_i), 1 - f_A(x_i) > /x_i\,, \quad x_i \in X\,.$$

If X is a continuous set, then A can be written as:

$$A = \int_x < t_A(x), 1 - f_A(x) > /x\,, \quad x \in X\,.$$

In the following, we write power set of U as $P(U)$, $V(U)$ denotes total vague sets in U. Obviously, we have: $P(U) \subseteq F(U) \subseteq V(U)$.

In paper [1], [2], Li Fan given a translation function from vague to fuzzy set, and discussed its some properties. Further, fuzzy entropy is generalized from fuzzy set to vague.

Definition 1. (Definition 7 in [1]) Let

$$R_\alpha(t_A(x), f_A(x)) = (1-\alpha)t_A(x) + \alpha(1 - f_A(x)) \quad (\alpha \in [0,1])$$

in $R : [0,1] \times [0,1]$ is translation function from vague $A = \{< t_A(x), 1 - f_A(x) > |x \in U\}$ to fuzzy set $F = \{\mu_F(x)|x \in U\}$, after translation membership function of fuzzy set is $\mu_F(x) = (1-\alpha)t_A(x) + \alpha(1 - f_A(x))$.

Definition 2. (Definition 8 in [1]) $E : V(U) \to [0,1]$ is fuzzy entropy of vague set V(U), if following conditions are satisfied:

(E_1) $E(A) = 0$, if and only if $A \in F(U)$.

(E_2) $E(A) = 1$, if and only if $\forall x \in U, t_A(x) = 0 = f_A(x)$.

(E_3) $E(A) = E(\overline{A})$, $\forall A \in V(U)$.

(E_4) $E(A) = 0$, if $\forall x \in U$, has $t_A \leq t_B(x)$, $1 - f_A(x) \geq 1 - f_B(x)$, then

$$E(A) \geq E(B), \ \forall A, B \in V(U).$$

Pointed to: for arbitrary function $F_5 : S \to [0,1]$ (where $S = \{(x,y)|x, y \in [0,1]; x + y = 1\}$) we can construct fuzzy entropy of a vague set if the following conditions are satisfied:

(F_1) $F_5(x,y) = 1$, if and only if $x + y = 1$.

(F_2) $F_5(x,y) = 0$, if and only if $x + y = 0$.

(F_3) $F_5(x,y) = F_5(y,x)$.

(F_4) if $x_1 \leq x_2$, $y_1 \leq y_2$, then $F_5(x_1,y_1) = F_5(x_2,y_2)$.

For example $F_5(x,y) = x+y$; $F_5(x,y) = (x+y)^n$; $F_5(x,y) = (x+y)e^{1-(x+y)}$ etc.

Theorem 1. (see [1]) *Assuming* $\mu = \{x_1, x_2, ...x_n\}, A \in V(U), E : V(U) \to [0,1]$.

If $E(A) = \frac{1}{n}\sum_{n=1}^{n}(1 - F_5(t_A(x_i)))$, *where,* F_5 *satisfied the conditions* $(F_1) - (F_4)$*, then* $E(A)$ *is fuzzy entropy of vague set.*

3. About Translation Function from Vague Sets to Fuzzy Sets

Let total vague sets construct a set be $V(U)$ in U, total fuzzy sets construct a set be $F(U)$ in U. Obviously, hereinbefore translation function R ascertained from $V(U)$ to $F(U)$ mapping R_α:$V(U) \to F(U)$, $A \to R_\alpha(A)$ is a surjection. But, the mapping is not a injection.

Example 1. Let $A = \{< 0.4, 0.8 > |x \in U\}$, $B = \{< 0.3, 0.9 > |x \in U\}$, then

$$R_\alpha(A)(x) = 0.4(1-\alpha) + 0.8\alpha,\ R_\alpha(B)(x) = 0.3(1-\alpha) = 0.9\alpha.$$

Choosing $\alpha = (0.4 - 0.3)/(0.6 - 0.4) = 0.5$, then $R_\alpha(A) = R_\alpha(B)$. We give two different vague sets, make certain α, let them transform the same fuzzy set. At the same time, we take $F_5(x,y) = x + y$, then $E(A) = \frac{1}{n}\sum_{i=1}^{n}(1 - (t_A(x_i) + f_A(x_i)))$. For A, B of example, $E(A) \neq E(B)$ (the same is true of other $F_5(x,y)$).

Example 2. Let $A = \{t_A(x), 1 - f_A(x) > |x \in U\}$. For arbitrary $\alpha \neq 0$, let $B = \{< t_A(x) + \varepsilon_1, 1 - f_A(x) + \varepsilon_2 > |x \in U\}$, where $\varepsilon_2 = (1-\alpha)\varepsilon_1/\alpha$. Then $R_\alpha(A)(x) = R_\alpha(B)(x)$. Showing for arbitrary $\alpha \neq 0$ and vague set A, we can ascertain vague set B, and caused A, B transformed into the same fuzzy set. Meanwhile, take $F_5(x,y) = x+y$, then $E(A) = \frac{1}{n}\sum_{i=1}^{n}(1-(t_A(x_i)+f_A(x_i)))$. For A, B of the example, $E(A) \neq E(B)$ (the same is true of other $F_5(x,y)$).

Thus, action of translation function R creates problems: two vague sets of different fuzzy entropy transform the same fuzzy set for R, obviously, the fuzzy set take on same fuzzy entropy, showing: a lot of useful information is discarded in the translation. Obviously, this is what we do not wish to get. Meanwhile, the translation does not agree with the original idea that that paper [1] first introduced into vague set deal with general fuzzy set can not deal with uncertain information and data. The fact, He Yingyu and Wang Guojun have pointed out in paper [3]: The so-called vague set is a special L-fuzzy set.

4. Discussion about Fuzzy Entropy

Firstly, according to the definition of fuzzy entropy in papers [1] and [2], there is a misprint in papers [1] and [2], namely in Theorem 1,

$$E(A) = \sum i = 1^n(1 - F_5(t_A(x_i), f_A(x_i)))$$

must be

$$E(A) = \frac{1}{n}\sum i = 1^n(1 - F_5(t_A(x_i), f_A(x_i)))\,.$$

Secondly, according to the definition of fuzzy entropy in paper [1], if $t_A(x_i) + f_A(x_i) = t_B(x_i) + f_B(x_i)$ $(\forall x_i \in U)$, then: $E(A) = E(B)$ For example: let $A =$

$\{< 0.2, 0.3 > |x \in U\}$, $B = \{< 0.8, 0.9 > |x \in U\}$, Then $E(A) = E(B) = 0.1$. A vague set which has different grade of truth-membership and false-membership can make the same fuzzy entropy. Therefore, only characterized uncertain grade of element belonging to vague and the uncertain grade is ascertained by sum of truth-membership function and false-membership function of a vague set, and cannot differentiate there is different grade of truth-membership, but the same sum of grade of truth-membership and false-membership.

If we do not consider the dissimilarity, we get $\frac{1}{n}\sum_{i=1}^{n}\pi_A(x_i) = \frac{1}{n}[1-t_A(x_i)-f_A(x_i)]$ as fuzzy entropy of A, namely most shortly definition.

A vague set is a special fuzzy set, when vague set degenerates general fuzzy set, it should be consistent with fuzzy entropy of general fuzzy set, but not when $E(A) = 0$.

5. Redefining of Fuzzy Entropy

Because grade of element x belonging to or not belonging A is characterized by $t_A(x), f_A(x)$ in vague A, respectively, the corresponding fuzzy entropy is characterized by them respectively. We give the definition of fuzzy post entropy and fuzzy negative entropy using Shannon function given below:

Definition 3. Supposing $U = \{x_1, x_2, ...x_n\},\ A \in V(U)$. We call:

(1) $E_O : V(U) \to [0, 1], A \to E_O(A) = \frac{2}{n}\sum_{i=1}^{n} -t_A(x_i)\log_2 t_A(x_i)$ is fuzzy positive entropy of vague A;

(1) $E_N : V(U) \to [0, 1], A \to E_N(A) = \frac{2}{n}\sum_{i=1}^{n} -f_A(x_i)\log_2 f_A(x_i)$ is fuzzy positive entropy of vague A; $E : V(U) \to [0, 1],\ A \to E(A) = \frac{1}{2}(E_O(A) + E_N(A)) = \frac{1}{n}\sum_{i=1}^{n}[-t_A(x_i)\log_2 t_A(x_i) + (-f_A(x_i)\log_2 f_A(x_i))]$ if fuzzy entropy of vague A.

Remark 1. Fuzzy post entropy of vague A characterized uncertain grade of element x belong to vague A in U, E_O is less, uncertain grade of element x belong to vague set A is less;

Fuzzy negative entropy of vague set A characterized uncertain grade of element x not belong to vague A. E_N is less, uncertain grade of element x not belong to vague set A is less; Fuzzy entropy of vague set A characterized uncertain grade between relation element x in U and vague set A. E is bigger, we know less about the relation between x and A.

Remark 2. when vague set A degenerated into general fuzzy set, fuzzy entropy of set A and fuzzy entropy of general fuzzy entropy in paper [4] is consistent, and if $A(x) = 0.5, V_x \in U$, fuzzy entropy of A gets the maximum value 1.

This is consistent with intuition.

Remark 3. By the way in paper [4], formula (2.2) $h(u) := -u\ln(u) - (1-u)\ln(1-u)$ should be modified into $h(u) := -u\log_2 u - (1-u)\ln_2(1-u)$. Otherwise, when $A(x) = 0.5, V_x \in U$, fuzzy entropy of A does not get the maximum value 1.

Example 3. Let $A = \{< 0.2, 0.3 > |x \in U\},\ B = \{< 0.8, 0.9 > |x \in U\}$, then:

$$E_O(A) = \frac{2}{n}\sum_{i=1}^{n} -0.2\log_2(0.2) = -0.4\log_2(0.2);$$

$$E_N(A) = \frac{2}{n}\sum_{i=1}^{n} -0.7\log_2(0.7) = -1.4\log_2(0.7);$$

$$E_O(B) = \frac{2}{n}\sum_{i=1}^{n} -0.8\log_2(0.8) = -1.6\log_2(0.8);$$

$$E_N(B) = \frac{2}{n}\sum_{i=1}^{n} -0.1\log_2(0.1) = -0.2\log_2(0.1).$$

Obviously, $E(A) \neq E(B)$.

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