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SINGLE MACHINE SCHEDULING PROBLEM WITH LINEAR DETERIORATION UNDER GROUP TECHNOLOGY

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Abstract: This paper considers the single machine scheduling problem under the condition that the job processing time is a linear deterioration function of its starting time. Under the group technology assumption, the total weighted completion time minimization problem is polynomial time solvable.

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1. Introduction

In classical scheduling theory it is assumed that the processing time of a job is constant. However, there are many situations where the processing time of the job depends on its starting time. Such deterioration appears, e.g., in scheduling maintenance jobs or cleaning assignments, where any delay in processing

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a job is penalized and often implies additional time for accomplishing the job. Browne and Yechiali [2] considered the single machine stochastic scheduling problem with general linear deterioration. Mosheiov [6] considered the problem that all jobs are characterized by a common positive basic processing time. Using this basic assumption, Mosheiov proved that the optimal schedule to minimize flowtime is symmetric and has a V-shaped property with respect to the increasing rates. Mosheiov [7] considered the single machine scheduling problem with simple linear deterioration. Wang et al [10] considered the single machine scheduling makespan problems under linear deterioration. Zhao et al [12] considered a special type of the actual processing time. They proved that the single machine scheduling problem of minimizing makespan, sum of weighted completion times, maximum lateness and maximum cost is polynomially solvable, respectively. Chen [3] and Mosheiov [8] considered scheduling deteriorating jobs in a *multi-machine* setting. They assumed a linear deterioration and parallel identical machines. Guo and Wang [5] considered a single machine scheduling problem under the condition that the job processing time is a linear deterioration function of its starting time. Under the group technology assumption, they proved the makespan minimization problem can be solved in polynomial time. Extensive surveys of different models and problems concerning deteriorating jobs can be found in Alidaee and Womer [1], and Cheng et al [3].

In this paper we study a single machine scheduling problem with a linear deterioration function under the group technology assumption. The objective function is to minimize the total weighted completion time. For the classical work on group technology scheduling problems, the reader is referred to Potts and Van Wassenhove [9], and Webster and Baker [11].

2. Statement of the Problem

Assume that there are n jobs $[J_1, J_2, \ldots, J_n]$ which are grouped into f groups, and these n jobs are to be processed on a single machine. Each of which is available at time zero, jobs are processed one by one in groups on the machine and a setup time is required if the machine switches from one group to another, we assume that the setup times are sequence independent, and the processing of a job may not be interrupted. We let n_i be the number of jobs belonging to group G_i. Thus, $n_1 + n_2 + \ldots + n_f = n$. Let $J_{i,j}$ denote the j-th job in group G_i, $i = 1, 2, \ldots, f; j = 1, 2, \ldots, n_i$. In addition, $p_{ij}(t) = a_{ij}(a + bt)$ denotes the actual processing time of job J_{ij} in group G_i if its starting time is t, where a_{ij}

denotes the normal processing time of job J_{ij}, a and b are positive constants. Let s_i denote the setup time required to process a job in group G_i following a job in some other group, w_{ij} and C_{ij} denote the weight and completion time of job J_{ij} in group G_i. The objective is to minimize total weighed completion time. Using the three field notation of scheduling, the problem can be denoted as

$$1|S, GT, a_{ij}(a+bt)|\sum w_j C_j, \tag{1}$$

where S denotes setup times vector, GT denotes group technology.

3. Main Results

Lemma 1. (see Zhao et al [12]) *The schedule obtained by the non-decreasing order of* $\frac{a_j}{w_j(1+ba_j)}$ *is optimal for the problem* $1|a_j(a+bt)|\sum w_j C_j$.

From Lemma 1, we can easily obtain the following theorem.

Theorem 1. *For the problem* $1|S, GT, a_{ij}(a+bt)|\sum w_j C_j$, *if the schedule of groups is given, then the optimal schedule that jobs within each group can be obtained by scheduling the jobs in non-decreasing order of* $\frac{a_{ij}}{w_{ij}(1+ba_{ij})}$, $i = 1, 2, \ldots, f; j = 1, 2, \ldots, n_i$.

So without loss of generality, we suppose the schedule of each group is given according the order of Theorem 1. Now we consider the order of groups. Once the optimal order of the jobs within each group, every group can be seen a compound job. Now we consider two groups of jobs.

Let $G_1 : J_{11}, J_{12}, \ldots, J_{1n_1}$; $G_2 : J_{21}, J_{22}, \ldots, J_{2n_2}$. Without loss of generality, stem from Theorem 1, we assume that

$$G_1 : \frac{a_{11}}{w_{11}(1+ba_{11})} \leq \frac{a_{12}}{w_{12}(1+ba_{12})} \leq \ldots \leq \frac{a_{1n_1}}{w_{1n_1}(1+ba_{1n_1})};$$

$$G_2 : \frac{a_{21}}{w_{21}(1+ba_{21})} \leq \frac{a_{22}}{w_{22}(1+ba_{22})} \leq \ldots \leq \frac{a_{2n_2}}{w_{2n_2}(1+ba_{2n_2})}.$$

Theorem 2. *For the problem* $1|S, GT, a_{ij}(a+bt)|\sum w_j C_j$, *if*

$$\rho(G_1) \leq \rho(G_2),$$

where

$$\rho(G_i) = \frac{(s_i + \frac{a}{b})\prod_{j=1}^{n_i}(1+ba_{ij}) - \frac{a}{b}}{\sum_{j=1}^{n_i} w_{ij}\prod_{k=1}^{j}(1+ba_{ik})}, \quad i = 1, 2,$$

then it is optimal to process the group G_1 *before the group* G_2.

Proof. If the order is G_1, G_2. Then

$$\begin{aligned}
C_{11} &= s_1 + a_{11}(a + bs_1) = (s_1 + \frac{a}{b})(1 + ba_{11}) - \frac{a}{b},\\
C_{12} &= C_{1,1} + a_{12}(a + bC_{1,1})\\
&= (s_1 + \frac{a}{b})(1 + ba_{11})(1 + ba_{12}) - \frac{a}{b},\\
&\cdots\cdots\\
C_{1,n_1} &= (s_1 + \frac{a}{b})\prod_{k=1}^{n_1}(1 + ba_{1k}) - \frac{a}{b};\\
C_{21} &= (C_{1,n_1} + s_2 + \frac{a}{b})(1 + ba_{21}) - \frac{a}{b},\\
C_{22} &= (C_{1,n_1} + s_2 + \frac{a}{b})(1 + ba_{21})(1 + ba_{22}) - \frac{a}{b},\\
&\cdots\cdots\\
C_{2,n_2} &= (C_{1,n_1} + s_2 + \frac{a}{b})\prod_{k=1}^{n_2}(1 + ba_{2k}) - \frac{1}{b}.
\end{aligned}$$

$$\sum_{i=1}^{2}\sum_{j=1}^{n_i} w_{ij}C_{ij}(G_1, G_2) = (s_1 + \frac{a}{b})\sum_{j=1}^{n_1} w_{1j}\prod_{k=1}^{j}(1 + ba_{1k})$$
$$+ [(s_1 + \frac{a}{b})\prod_{k=1}^{n_1}(1 + ba_{1k}) + s_2]\sum_{j=1}^{n_2} w_{2j}\prod_{k=1}^{j}(1 + ba_{2k}) - \frac{a}{b}\sum_{i=1}^{2}\sum_{j=1}^{n_i} w_{ij}.$$

If the order is G_2, G_1. Then

$$\sum_{i=1}^{2}\sum_{j=1}^{n_i} w_{ij}C_{ij}(G_2, G_1) = (s_2 + \frac{a}{b})\sum_{j=1}^{n_2} w_{2j}\prod_{k=1}^{j}(1 + ba_{2k})$$
$$+ [(s_2 + \frac{a}{b})\prod_{k=1}^{n_2}(1 + ba_{2k}) + s_1]\sum_{j=1}^{n_2} w_{1j}\prod_{k=1}^{j}(1 + ba_{1k}) - \frac{a}{b}\sum_{i=1}^{2}\sum_{j=1}^{n_i} w_{ij}.$$

$$\sum_{i=1}^{2}\sum_{j=1}^{n_i} w_{ij}C_{ij}(G_1, G_2) - \sum_{i=1}^{2}\sum_{j=1}^{n_i} w_{ij}C_{ij}(G_2, G_1)$$
$$= \left[(s_1 + \frac{a}{b})\prod_{k=1}^{n_1}(1 + ba_{1k}) - \frac{1}{b}\right]\sum_{j=1}^{n_2} w_{2j}\prod_{k=1}^{j}(1 + ba_{2k})$$

$$- \left[(s_2 + \frac{a}{b}) \prod_{k=1}^{n_2} (1 + ba_{2k}) - \frac{1}{b} \right] \sum_{j=1}^{n_1} w_{1j} \prod_{k=1}^{j} (1 + ba_{1k}) .$$

If

$$\frac{(s_1 + \frac{a}{b}) \prod_{k=1}^{n_1} (1 + ba_{1k}) - \frac{a}{b}}{\sum_{j=1}^{n_1} w_{1j} \prod_{k=1}^{j} (1 + ba_{1k})} \leq \frac{(s_2 + \frac{a}{b}) \prod_{k=1}^{n_2} (1 + ba_{2k}) - \frac{a}{b}}{\sum_{j=1}^{n_2} w_{2j} \prod_{k=1}^{j} (1 + ba_{2k})},$$

then

$$\sum_{i=1}^{2} \sum_{j=1}^{n_i} w_{ij} C_{ij}(G_1, G_2) - \sum_{i=1}^{2} \sum_{j=1}^{n_i} w_{ij} C_{ij}(G_2, G_1) \leq 0.$$

This completes the proof. □

From Theorem 2, we obtain the transitivity, hence for the case of f groups, the order of groups can be obtained similarly. From Theorem 1 and Theorem 2, the problem $1|S, GT, a_{ij} + ka_{ij}t| \sum w_j C_j$ can be solved by the following algorithm:

Algorithm 1. *Step 1.* Jobs in each group scheduled in non-decreasing order of $\frac{a_{ij}}{w_{ij}(1+ba_{ij})}$, $i = 1, 2, \ldots, f; j = 1, 2, \ldots, n_i$.

Step 2. Calculate $\rho(G_i) = \frac{(s_i + \frac{1}{b}) \prod_{j=1}^{n_i} (1+ba_{ij}) - \frac{a}{b}}{\sum_{j=1}^{n_i} w_{ij} \prod_{k=1}^{j} (1+ba_{ik})}$, $i = 1, 2, \ldots, f$.

Step 3. Groups scheduled in non-decreasing order of $\rho(G_i)$, $i = 1, 2, \ldots, f$.

Obviously, the total time for Algorithm 1 is $O(n\log n)$.

4. Conclusions

In this paper we have considered the problem of scheduling jobs with start time dependent processing times (deterioration) for the total weighted completion time minimization. Under the group technology assumption, this problem is proved to be polynomial time solvable.

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