International Journal of Pure and Applied Mathematics
Volume 28 No. 3 2006, 407-416

MULTI-LEVEL OPTIMIZATION MODELS FOR A LARGE-SCALE COMPOUND SYSTEM AND ITS DECOMPOSITION-COORDINATION SOLVING ALGORITHM

Zhongfu Tan[1 §], Xiaojun Li[2]
[1,2]Institute of Electric Power Economics
North China Electric Power University
Zhuxinzhuang Dewai, Beijing, 102206, P.R. CHINA
[1]e-mail: tanzhongfu@sina.com
[2]e-mail: lixiaojun1979@126.com

Abstract: A large-scale compound system can be decomposed into a series of multi-level subsystems, and each subsystem can be decomposed into a series of multi-level subsystems again, until each bottom subsystem needs not to be decomposed. Considering each subsystem's construction cost as coordination parameters of systems from up to down, each subsystem's reliability and its derivative related to construction cost as decomposition parameters of systems from down to up, a decomposition-coordination algorithm for solving reliability optimization models of large-scale compound engineering systems is given.

AMS Subject Classification: 26A33
Key Words: reliability, optimization, decomposition, coordination

1. Introduction

It is known that many engineering systems are complex and large-scale ones which consist of some subsystems, each subsystem can also be decomposed into a group of subsystems, until the bottom subsystems which consist of some components. Hence a large-scale system which is composed of many components

Received: April 18, 2006
§Correspondence author

can be decomposed into multi-level subsystems, [1].

In each level, there exist some subsystems, and the logic relations among them are assumed to be series or parallel [2]. For each bottom subsystem, the logic relations among its components are assumed to e parallel or (1/k) (G) or stand-by. How should components be allocated so that their reliability reach their maximum under a given cost? [3] placed the stress on the repairable system and presented a optimal allocation way. In [4] a units combination and commitment optimization method was presented for the electric power production. Power generation system is a very complex system, high reliability level may result in additional utility cost, and low one may result in additional consumer's cost, [5] constructed four reliability model for power generation system. In this paper, considering cost of each subsystem in its lower level as design variable, by maximizing the subsystem's reliability, the sub-programming problem can be constructed, and its optimal solution can be used for constructing sub-programming for subsystems in its lower level, until the optimal costs for bottom subsystems are attained. Hence, for each bottom subsystem, considering reliability and redundancy of each component as design variables, a mixed programming problem is presented. So the large-scale compound system is transformed into all bottom subsystems which can be solved easily, and from down to up, the reliability and its derivative related to cost are calculated for each subsystem in each level to solve the sub programming problem for its upper subsystem. Therefore, reliability cannot be improved under the cost limitation. Now, the optimal allocation for reliability and redundancy of components can be attained.

2. The Decomposition Optimization Model for a Large Scale Compound System

For a large scale compound system, its multi-level decomposition form is given in Figure 1, where the main system S_0 consists of n_1 subsystem $S_{1,0,i}$.

In the first level, $1 \leq i \leq n_1$,$S_{1,0,i}$ consists of m_{10i} subsystems $S_{2,i,2ij}$ in the second level, $1 \leq j \leq m_{10i}$, where $(..., 2i1, 2i2, ...2im_{10i}, ...) = (1, ..., n_2)$, i.e. there are n_2 subsystems in the second level, $S_{2,i,j}$ consists of m_{2ij} subsystems $S_{3,j,3jk}$.

In the third level, $2i1 \leq j \leq 2im_{10i}$, where $(..., 3j1, 3j2, ...3jm_{2ij}, ...) = (1, ..., n_3)$, i.e. there are n_3 subsystems in the third level, and so on, on the analogy of the above statement, there are n_{l-1} subsystems $S_{l,st}$ in $(l-1)$-th level, $S_{l-1,s,t}$ consists of $m_{(l-1)_{st}}$ subsystems $S_{l,t,ltj}$ is a bottom subsystem

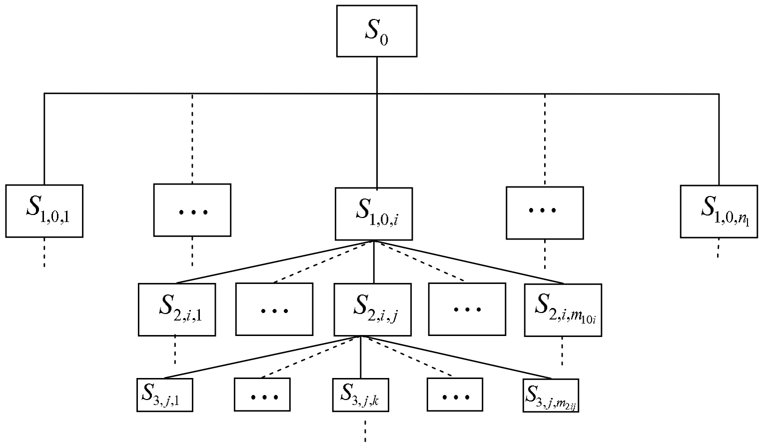

Figure 1: Multi-level decomposition form for a large scale compound system

which is composed of some components in parallel or $(1/k)(G)$ or stand-by logic relation,$1 \leq j \leq m_{(l-1)_{st}}$ and $(..., lt1, lt2, ...ltm_{(l-1)_{st}}, ...) = (1, 2, ..., n_1)$, i.e there are n_1 bottom subsystems in 1-th level. Denoting $R_{i,j,k}$ as reliability of (i, j, k)-th subsystems, $C_{i,j,k}$ as allocated cost of (i, j, k)-th subsystem, which is considered as design variable, where i is the order of the level which includes $S_{i,j,k}$, j is the order of $S_{i,j,k}$ in i-th level. Hence, sub-programming for coordinating subsystems in each level can be constructed as follows.

2.1. The Sub-Programming for Coordinating Subsystems in First Level

$$
\begin{aligned}
(P_0) \quad \max \quad & R_0 = f_0(R_{1,0,1}, R_{1,0,2}, ..., R_{1,0,n_1})\,, \qquad (1)\\
\text{s.t.} \quad & \sum_{i=1}^{\bar{n}_1} C_{1,0,i} = C_0\,,\\
& C_{1,0,i} \geq 0\,, \qquad 1 \leq i \leq n_1\,,
\end{aligned}
$$

where R_0 is system reliability, C_0 is construction cost limitation given for the system.

2.2. The Sub-Programming for Coordinating Subsystems in Second Level

$$(P_{1,0,i}) \quad \max \quad R_{1,0,i} = f_{1,0,i}(R_{2,i,2i1}, R_{2,i,2i2}, ..., R_{2,i,2im_{10i}})\,, \tag{2}$$
$$\text{s.t.} \quad \sum_{j=1}^{m_{10i}} C_{2,i,2ij} = C_{1,0,i},$$
$$C_{2,i,2ij} \geq 0, \qquad 1 \leq j \leq m_{10i},$$

where $C_{1,0,i}$ is given from the optimal result of (P_0), $1 \leq i \leq n_1$.

$$(P_{2,i,j}) \quad \max \quad R_{2,i,j} = f_{2,i,j}(R_{3,j,3j1}, R_{3,j,3j2}, ..., R_{3,j,3jm_{2ij}})\,, \tag{3}$$
$$\text{s.t.} \quad \sum_{k=1}^{m_{2ij}} C_{3,j,3jk} = C_{2,i,j}\,,$$
$$C_{3,j,3jk} \geq 0\,, \qquad 1 \leq k \leq m_{2ij}\,,$$

where $C_{2,i,j}$ is given from the optimal result of $(P_{1,0,i})$, $1 \leq i \leq n_1$, $2i1 \leq j \leq 2im_{10i}$.

On the analogy of the above sub-programming, n_{l-1} sub-programming for coordinating subsystems in l-th level

$$(P_{l-1,s,t}) \quad \max \quad R_{l-1,s,t} = f_{l-1,s,t}(R_{1,t,1t1}, R_{1,t,1t2}, ..., R_{1,t,1tm_{(l-1)st}})\,, \tag{4}$$
$$\text{s.t.} \quad \sum_{k=1}^{m_{(l-1)st}} C_{1,t,1tk} = C_{(l-1),s,t}\,,$$
$$C_{1,t,1tk} \geq 0\,, \qquad 1 \leq k \leq m_{(l-1)st}\,,$$

where $C_{l-1,s,t}$ is given from the optimal result of a sub-programming for coordinating subsystems in $(l-1)$-th level.

For each sub-programming above, its objective function can be constructed by the logic relation among subsystems in lower level, such as series or parallel. For instance, if there exist a series relationships among all subsystems in first level, then the objective function of (P_0) can be expressed as follows

$$R_0 = \prod_{i=1}^{n_1} R_{1,0,i}\,.$$

If there exists a parallel relation among all subsystems in first level, then the objective function of (P_0) can be expressed by

$$R_0 = 1 - \prod_{i=1}^{n_1} (1 - R_{1,0,i})\,.$$

For the other sub-programming, their objective functions can also be expressed analogously to above.

We assume there exists only one class of logic relation among all components in each bottom subsystem, such as parallel or $(1k/mk)(G)$ or stand-by.

Hence, for each bottom subsystem, its sub-programming problem can be given as follows

$$\begin{aligned} (P_{l,t,k}) \quad \max \quad & R_{l,t,k} = f_{l,t,k}(r_k(C_k), m_k), \\ s.t. \quad & m_k C_k = C_{l,t,k}, \end{aligned} \tag{5}$$

where $C_k \geq 0$, $C_{l,t,k}$ is given from the optimal solution of $(P_{l-1,s,t})$,$k = 1, 2, ..., n_1$, and $m_k \geq 1_k$ for the $(1_k/m_k)(G)$ subsystem. There are three function types for $f_{l,t,k}$, for instance, if $S_{l,t,k}$ is a parallel subsystem, then

$$R_{l,t,k} = 1 - (1 - r_k)^{m_k}. \tag{6}$$

If $S_{l,t,k}$ is a $(1_k/m_k)(G)$ subsystem, then

$$R_{l,t,k} = \sum_{q=0}^{m_k - l_k} C^q_{m_k}(1 - r_k)^q r_q^{m_k - q}\,, \tag{7}$$

where 1_k is the minimum number of components, $m_k \geq l_k$, $C^q_{m_k} = \frac{m_k!}{[q!(m_k-q)!]}$. If $S_{l,t,k}$ is a stand-by subsystem, then

$$R_{l,t,k} = \sum_{q=0}^{m_k - 1} \frac{(-\ln r_k)^q r_k}{q!}\,. \tag{8}$$

For each component, without loss of generality, the function relation for its reliability r_k and its construction cost C_k is assumed to be that

$$r_k(C_k) = \exp\,[\frac{\alpha_k}{\beta_k - C_k}]\,, \tag{9}$$

where $\alpha_k \geq 0, \beta_k \geq 0$.

3. An Algorithm for Solving the Decomposition Optimization Model

An algorithm for solving the above decomposition optimization model can be given as follows.

Step 1. For the given construction cost limitation C_0 for the system, from up to down, give initial construction cost allocation for each subsystem i.e., give $C_{1,0,i}, 1 \leq i \leq n_1$, to satisfy (1), $C_{2,i,2ij}, 1 \leq i \leq m_{10i}$, to satisfy (2), $C_{3,j,3jk}, 1 \leq i \leq m_{2ij}$, to satisfy (3),..., $C_{l,t,ltk}, 1 \leq k \leq m_{(l-1)_{st}}$, to satisfy (l). Then from down to up, solve sub-programmings in each level by gradient projection method as follows.

Step 2. Solve the sub-programmings for bottom subsystems, for instance, to $(P_{1,t,k})$:

If $S_{l,t,k}$ is $(1_k/m_k)(G)$ bottom subsystem, take $m_k = l_k, l_k+1, l_k+2, ...$ For each m_k, calculate $C_k = \frac{C_{l,t,k}}{m_k}, r_k(C_k)$ in (9), and the objective of $(P_{l,t,k})$, i.e. $R_{l,t,k} = f_{l,t,k}(r_k(C_k), m_k)$ in (6)or(7)or(8), until $R_(l,t,k)$, starts to decrease. Store $\frac{\partial R_{l,t,k}}{\partial C_{l,t,k}}$ by

$$\frac{\partial R_{l,t,k}}{\partial C_{l,t,k}} = m_k \frac{\partial R_{l,t,k} \partial r_k}{\partial r_k \partial C_k}\,, \tag{10}$$

where $\frac{\partial R_{l,t,k}}{\partial r_k}$ is gotten from (6) or (7), and

$$\frac{\partial r_k}{\partial C_k} = \frac{\alpha_k}{\beta_k} \exp[-\alpha_k(\frac{c_k}{\beta_k} - 1)].$$

Step 3. Solve the sub-programming for coordinating bottom subsystems, for instance, to solve $(P_{l-1,s,t})$:

Calculate $\frac{\partial R_{l-1,s,t}}{\partial C_{l,t,ltk}}$:

$$\frac{\partial R_{l-1,s,t}}{\partial C_{l,t,ltk}} = \frac{\partial R_{l-1,s,t}}{\partial R_{l,t,ltk}} \frac{\partial R_{l,t,ltk}}{\partial C_{l,t,ltk}},$$

$$\nabla_{\perp} R_{l-1,s,t} = (\frac{\partial R_{l-1,s,t}}{\partial C_{l,t,ltl}}, \frac{\partial R_{l-1,s,t}}{\partial C_{l,t,lt2}}, ..., \frac{\partial R_{l-1,s,t}}{\partial C_{l,t,m_{(l-1)_{st}}}}) - (1,1,...,1)^T \sum_{k=1}^{m_{(l-1)_{st}}} \frac{\partial R_{i-1,s,t}}{\partial C_{l,t,ltk}} / m_{(l-1)_{st}}\,,$$

where $\frac{\partial R_{l-1,s,t}}{\partial C_{l,t,ltk}}$ is given according to the specific function form of the objective in $(P_{l-1,s,t}), \frac{\partial R_{l,t,ltk}}{\partial C_{l,t,ltk}}$ is given by Step 2. Let

$$(C_{l,t,lt1}, C_{l,t,lt2}, ..., C_{l,t,m_{(l-1)_{st}}})^T \leftarrow (C_{l,t,lt1}, C_{l,t,lt2}, ..., C_{l,t,m_{(l-1)_{st}}})^T + \lambda \nabla_{\perp} R_{l-1,s,t}\,,$$

where λ is iterate step length determined by one-dimensional search.

If it is satisfied that $\|\nabla R_{l-1,s,t}\| < \varepsilon$, where ε is a given control precision, then store $\frac{\partial R_{l-1,s,t}}{\partial C_{l-1,s,t}}$ as the following

$$\frac{\partial R_{l-1,s,t}}{C_{l-1,s,t}} = \frac{\partial R_{l-1,s,t}}{C_{l,t,ltk}},\ 1 \le k \le m_{(l-1)_{st}} \tag{11}$$

otherwise, go to Step 2.

Step 4. in an analogous manner of Step 3, we may solve sub-programming for coordinating subsystems in $(l-1)th$ level, sub-programming for coordinating subsystems in $(l-2)th$ level, and so on, until the sub-programming for coordinating subsystems in first level, for instance, the solving process of $(P_{1,0,i})$ and (P_0), see the following steps.

Step 5. For $P_{1,0,i}$, calculate $\frac{\partial R_{1,0,i}}{\partial C_{2,i,2ij}}$ as follows

$$\frac{\partial R_{1,0,i}}{C_{2,i,2ij}} = \frac{\partial R_{1,0,i}}{\partial R_{2,i,2ij}} \frac{\partial R_{2,i,2ij}}{\partial C_{2,i,2ij}}, \quad 1 \le j \le m_{10i}\,, \tag{12}$$

where $\frac{\partial R_{2,i,2ij}}{\partial C_{2,i,2ij}}$ is given at the optimal point of $(P_{2,i,j})$, $\frac{\partial R_{1,0,i}}{\partial C_{2,i,2ij}}$ can be given according to the specific form of function $f_{1,0,i}$. Calcualte the projection gradient of $(P_{1,0,i})$ and iterate design variables as follows.

$$\nabla_{\perp} R_{1,0,i} = (\frac{\partial R_{1,0,i}}{\partial C_{2,i,2i1}}, \frac{\partial R_{1,0,i}}{\partial C_{2,i,2i2}}, ..., \frac{\partial R_{1,0,i}}{\partial C_{2,i,2im_{10i}}})^T - (1,1,...,1)^T \sum_{i=1}^{m_{10i}} \frac{\partial R_{1,0,i}}{\partial R_{2,i,ij}} / m_{10i}\,,$$

$$(C_{2,i,2i1}, C_{2,i,2i2}, ..., C_{2,i,2im_{10i}})^T \leftarrow (C_{2,i,2i1}, C_{2,i,2i2}, ..., C_{2,i,2im_{10i}})^T + \lambda \nabla_{\perp} R_{1,0,i}\,,$$

where λ is iterate step length determined by one-dimensional search. If it is satisfied that $\|\nabla R_{1,0,i}\| < \varepsilon$, then store $\frac{\partial R_{1,0,i}}{\partial C_{1,0,i}}$ by

$$\frac{\partial R_{1,0,i}}{\partial C_{1,0,i}} = \frac{\partial R_{1,0,i}}{\partial C_{2,i,2ij}}, \ 1 \le j \le m_{10i}\,. \tag{13}$$

Go to Step 6, otherwise allocate again construction costs of subsystems in the third level, forth level, and so on, until l-th level as follows

$$C_{3,j,3jk} \leftarrow (C_{3,j,3k} / \sum_{q=1}^{m_{2ij}} C_{3,j,3jq}) C_{2,i,j}\,,$$

$$\vdots$$

$$C_{l,t,ltk} \leftarrow (C_{l,t,ltk} / \sum_{q=1}^{m_{(l-1)_{st}}} C_{l,t,3lq}) C_{l-1,s,t}\,,$$

go to Step 2.

Step 6. For (P_0), calculate $\frac{\partial R_0}{\partial C_{1,0,i}}$ by

$$\frac{\partial R_0}{\partial C_{1,0,i}} = \frac{\partial R_0}{\partial C_{1,0,i}} \frac{\partial R_{1,0,i}}{\partial C_{1,0,i}}\,, \tag{14}$$

where $\frac{\partial R_{1,0,i}}{\partial C_{1,0,i}}$ is given form Step 5, $\frac{\partial R_0}{\partial C_{1,0,i}}$ is given according to the specific form of function f_0. If it is satisfied that $\|\nabla R_0\| < \varepsilon$, then output the costs of components and the system reliability R_0, stop. Otherwise, calculate the projection gradient of (P_0) and iterate design variables by

$$\nabla_{\perp} R_0 = (\frac{\partial R_0}{\partial C_{1,0,1}}, \frac{\partial R_0}{\partial C_{1,0,2}}, ..., \frac{\partial R_0}{\partial C_{1,0,n_1}})^T - (1,1,...,1)^T \sum_{i=1}^{n_1} \frac{\partial R-0}{\partial R_{1,0,il}}/n_1 \,,$$

$$(C_{1,0,1}, C_{1,0,2}, ..., C_{1,0,n_1})^T \leftarrow (C_{1,0,1}, C_{1,0,2}, ..., C_{1,0,n_1})^T + \lambda \nabla_{\perp} R_0 \,,$$

where λ is iterate step length determined by one dimensional search. Allocate construction costs of subsystems again in second level, third level, and so on, until l-th level as follows.

$$C_{2,i,2ij} \leftarrow (C_{2,i,2ij} / \sum_{k=1}^{m_{10i}} C_{2,i,2ik}) C_{1,0,i} \,,$$

$$C_{3,j,3jk} \leftarrow (C_{3,j,3k} / \sum_{q=1}^{m_{2ij}} C_{3,j,3jq}) C_{2,i,j} \,,$$

$$\vdots$$

$$C_{l,t,ltk} \leftarrow (C_{l,t,ltk} / \sum_{q=1}^{m_{(l-1)_{st}}} C_{l,t,ltq}) C_{l-1,s,t} \,,$$

go to Step 2. □

Proposition 1. *At the optimal point of each sub-programming problem, one has that*

$$\frac{\partial R_{l,t,k}}{\partial C_{l,t,k}} = m_k \frac{\partial R_{l,t,k}}{\partial r_k} \frac{\partial r_k}{\partial C_k}, \quad 1 \le k \le n_1 \,,$$

$$\frac{\partial R_{l-1,s,t}}{\partial C_{l-1,s,t}} = \frac{\partial R_{l-1,s,t}}{\partial C_{l,t,ltk}}, \quad 1 \le k \le m_{(l-1)_{st}} \,,$$

$$\vdots$$

$$\frac{\partial R_{1,0,i}}{\partial C_{1,0,i}} = \frac{\partial R_{1,0,i}}{\partial C_{2,i,2ij}}, \quad 1 \le k \le m_{10i} \,.$$

Proof. By use of the $K-T$ condition of each sub-programming problem, and (1), (2), (3), (4), (5), we can have (10), (11), (12). □

Proposition 2. *For the above algorithm, its iterate sequences are convergent to the $K-T$ point of the main programming which is expressed as follows*

$$(P) \quad \max \quad R_0 = \phi(r_1(C_1), r_2(C_2), ..., r_{n_1}(C_{n_1}), m_1, m_2, ..., m_{n_1}) \,,$$

$$s.t. \quad \sum_{k=1}^{n_1} m_k C_k = C_0 \,,$$
$$m_k \geq l_k \,, \qquad m_1 + 1 \leq k \leq m_1 + m_2 \,,$$

where $\phi = f_0(f_{1,0,1}(...), ..., f_{1,0,i}(..., f_{2,i,j}(...), ...)..., f_{1,0,n_1}(...))$,*it is a compound function for all objective functions of the above sub-programming.*

Proof. It is easily proved that the composition of $K-T$ conditions for all sub-programming is the $K-T$ condition of the main programming (P). Since the projection gradient method used in solving each sub-programming is convergent, so the iterate sequences are convergent to the $K-T$ point of the main programming (P). □

Proposition 3. *For the above algorithm, its iterate sequences are convergent to the optimal solution of the main programming* (P).

Proof. By the function form of R_0 about $R_{1,0,i}$, it is to know that $\frac{\partial R_0}{\partial C_{1,0,i}} > 0$, on the analogy of the above, the following can be obtained

$$\frac{\partial R_{1,0,i}}{\partial R_{2,i,j}} \geq 0, \quad \frac{\partial R_{2,i,j}}{\partial R_{3,j,3jk}} \geq 0, \quad \frac{\partial R_{l-1,s,t}}{\partial R_{l,i,ltk}} \geq 0, \quad \frac{\partial R_{l,t,ltk}}{\partial r_k} \geq 0.$$

From (9), it can be easily proved that $\frac{\partial^2 r_k}{\partial C_k^2} \leq 0$. So one has that:

$$\frac{\partial R_0}{\partial R_{1,0,i}} \frac{\partial R_{1,0,i}}{\partial R_{2,i,j}} \frac{\partial R_{2,i,j}}{\partial R_{3,j,3jk}} ... \frac{\partial R_{l-1,s,t}}{\partial R_{l,t,ltk}} \frac{\partial R_{l,t,k}}{\partial r_k} \frac{\partial^2 r_k}{\partial^2 C_k^2} \geq 0 \,,$$

i.e.

$$\frac{\partial^2 \phi}{\partial C_k^2} \leq 0 \,.$$

Hence a convex function about C_k, $1 \leq k \leq n_1$, (P) is a convex programming problem. We know that the K-T point of a convex programming problem is also its optimal solution, so from Proposition 2, one has that iterate sequences of the above algorithm are convergent to the optimal solution of the main programming (P).

Example. Assuming that S_0 is composed of 6 parallel subsystems in first level, i.e., $n_1 = 6$, $S_{1,0,i}$ consists of n_{i1} parallel subsystems and n_{i2} $(1/k)(G)$ subsystems and n_{i3} stand-by subsystems in second level, take $n_{i1}(1 \leq i \leq 6)$ as 2, 3, 4, 5, 6, 7, 7, $n_{i3}(1 \leq i \leq 6)$ as 5, 2, 3, 3, 2, 2. So there are 77 to them, from up to down, from left column to right column. Take β_k $(1 \leq k \leq 77)$ as 9*20, 10*23, 13*25, 14*10, 15*17, 16*14, α_k $(1 \leq k \leq 77)$ as 9*16, 10*12, 13*15, 14*10, 15*17, 16*19, l_k for $13 \leq k \leq 17$, $24 \leq k \leq 29$, $38 \leq k \leq 43$, $50 \leq k \leq 56$, $61 \leq k \leq 67$, as 2*2, 5*2, 6*2, 6*2, 7*2, 7*2.

By the above algorithm, the optimal result is obtained as follows: $m_1 =$

$m_7 = m_{13} = m_{19} = 4, m_{25} = 3, m_{31} = m_{37} = m_{43} = m_{40} = 2, m_k = 2$ for $3 \leq k \leq 4, 13 \leq k \leq 17, 24 \leq k \leq 29, 38 \leq k \leq 43, 50 \leq k \leq 56, 61 \leq k \leq 67, m_k = 1$ for other$k, r_1 = r_7 = 0.71$, $r_{13} = r_{19} = r_{25} = r_{31} = r_{37} = r_{43} = 0.896, r_k = 0.04$ for other $k, R_0 = 0.9556$.

Acknowledgements

This work is supported by the National Natural Science Foundation Project of P.R. China (70373017, 70571023) and the Leading Project Pre-Research Foundation of Ncepu.

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