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WEAKLY SEMI-θ-CONTINUOUS FUNCTIONS

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Abstract: The purpose of this paper is to introduce and investigate a new class of continuity, called weakly semi-θ-continuous functions, which contains the class of θ-continuous functions and is contained in the class of θ-semi continuous functions. Also it is shown that hyperconnectedness is preserved under weakly semi-θ-continuous.

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1. Introduction

Fomin [3] has introduced and investigated the concept of θ-continuous functions. After, Arya and Bhamini [1] have studied the concept of θ-semi continuous functions, which contains the class of θ-continuous functions. The purpose of the this paper is to introduce and investigate a new class of continuity, called weakly semi-θ-continuous functions, which contains the classes of θ-continuous and θ-irresolute functions and is contained in the class of θ-semi continuous functions.

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2. Preliminaries

In this paper, spaces (X,τ) and (Y,σ) (or simply X and Y) are always assumed to be topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of X. A subset A is said to be regular open (regular closed) if $A = \mathrm{Int}(\mathrm{Cl}A)$ ($A = \mathrm{Cl}(\mathrm{Int}A)$), where $\mathrm{Cl}A$ ($\mathrm{Int}A$) denotes the closure (interior) of A. A subset A is called semi-open [8] (resp. pre-open [9]) if $A \subset \mathrm{Cl}(\mathrm{Int}A)$ (resp. $A \subset \mathrm{Int}(\mathrm{Cl}(A))$). The complement of a semi-open (resp. pre-open) set is called semi-closed (resp. pre-closed). A point x of a space X is said to be in the θ-semiclosure [6] (resp. θ-closure [17]) of a subset A of X, denoted by $\theta - s\mathrm{Cl}A$ (resp. $\mathrm{Cl}_\theta A$), if $A \cap \mathrm{Cl}U \neq \emptyset$ for every semi-open (resp. open) set containing x. A point x of a space X is said to be in the θ-semiinterior of a subset A of X, denoted by $\theta - s\mathrm{Int}A$, if there exists a semi-open set U containing x such that $x \in \mathrm{Cl}U \subseteq A$. A subset A is said to be θ-semiclosed (resp. θ-closed, θ-semiopen) if $A = \theta - s\mathrm{Cl}A$ (resp. $A = \mathrm{Cl}_\theta A$, $A = \theta - s\mathrm{Int}A$). The complement of a θ-semiclosed (resp. θ-closed) set is called θ -semiopen (resp. θ-open) set. The family of all semi-open (resp. pre-open, θ-semiopen) subsets of X is denoted by $SO(X)$ (resp. $PO(X)$, $\theta SO(X)$).

The following basic facts will be needed in the sequel. The proofs of them are obvious.

Proposition 1. *Let (X,τ) be a topological space and let $A \subseteq X$. Then:*

(1) $\theta - s\mathrm{Cl}(X - A) = X - (\theta - s\mathrm{Int}A)$.

(2) $X - (\theta - s\mathrm{Cl}A) = \theta - s\mathrm{Int}(X - A)$.

3. Weakly Semi-θ-Continuous Functions

Definition 1. A function $f : X \to Y$ is said to be weakly semi-θ-continuous if for each $x \in X$ and each open set V containing $f(x)$, there exists a semi-open set U containing x such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$.

Theorem 1. *For a function $f : X \to Y$ the followings are equivalent:*

(1) f is weakly semi-θ-continuous;

(2) For each $x \in X$ and each open set V containing $f(x)$, there exists a regular closed set F containing x such that $F \subseteq f^{-1}(\mathrm{Cl}V)$.

(3) $f(\theta - s\mathrm{Cl}A) \subseteq \mathrm{Cl}_\theta f(A)$ for every subset A of X;

(4) $\theta - s\mathrm{Cl}f^{-1}(B) \subseteq f^{-1}(\mathrm{Cl}_\theta B)$ for every subset B of Y;

(5) $f^{-1}(\mathrm{Int}_\theta B) \subseteq \theta - s\mathrm{Int}f^{-1}(B)$ for every subset B of Y.

(6) $\theta - s\mathrm{Cl}f^{-1}(V) \subseteq f^{-1}(\mathrm{Cl}V)$ for every open subset V of Y;

(7) $f^{-1}(V) \subseteq \theta - s\mathrm{Int}f^{-1}(\mathrm{Cl}V)$ for every open subset V of Y;

(8) $\theta - s\mathrm{Cl}f^{-1}(\mathrm{Int}F) \subseteq f^{-1}(F)$ *for every closed subset* F *of* Y*;*

Proof. (1)⇒(2). This is obvious.

(2)⇒(3). Let A be any subset of X and $y \notin \mathrm{Cl}_\theta f(A)$. If $f^{-1}(y) = \emptyset$, then (3) is trivial. Therefore, let $f^{-1}(y) \neq \emptyset$ and x be an arbitrary point of $f^{-1}(y)$. Then there exists an open set V of y such that $f(A) \cap \mathrm{Cl}V = \emptyset$. By (2), there exists a regular closed set F containing x such that $f(F) \subseteq \mathrm{Cl}V$. Thus we have $f(A) \cap f(F) = \emptyset$ and $F \cap A = \emptyset$. This implies that $x \notin \theta - s\mathrm{Cl}A$. Therefore, we have $y \notin f(\theta - s\mathrm{Cl}A)$. Consequently, $f(\theta - s\mathrm{Cl}A) \subseteq \mathrm{Cl}_\theta f(A)$.

(3)⇒(4). Let B be a subset of Y. Then by (3), $f(\theta - s\mathrm{Cl}f^{-1}(B)) \subseteq \mathrm{Cl}_\theta f(f^{-1}(B)) \subseteq \mathrm{Cl}_\theta B$. Therefore, we obtain, $\theta - s\mathrm{Cl}f^{-1}(B) \subseteq f^{-1}(\mathrm{Cl}_\theta B)$.

(4)⇔(5). This follows from the fact that $X\backslash(\theta - s\mathrm{Cl}A) = \theta - s\mathrm{Int}(X\backslash A)$ and $X\backslash(\mathrm{Cl}_\theta A) = \mathrm{Int}_\theta(X\backslash A)$ for each subset A of X.

(4)⇒(6). This is obvious, because $\mathrm{Cl}V = \mathrm{Cl}_\theta V$ for an open subset V of Y.

(6)⇒(7). Let V be any open set of Y. Then $Y\backslash\mathrm{Cl}V$ is open in Y and by (6), we have $\theta - s\mathrm{Cl}f^{-1}(Y\backslash\mathrm{Cl}V) \subseteq f^{-1}(\mathrm{Cl}(Y\backslash\mathrm{Cl}V))$. Now, we have $\theta - s\mathrm{Cl}f^{-1}(Y\backslash\mathrm{Cl}V) = \theta - s\mathrm{Cl}(X\backslash f^{-1}(\mathrm{Cl}V)) = X\backslash(\theta - s\mathrm{Int}f^{-1}(\mathrm{Cl}V))$ and $f^{-1}(\mathrm{Cl}(Y\backslash\mathrm{Cl}V)) = f^{-1}(Y\backslash\mathrm{Int}\,(\mathrm{Cl}V)) \subseteq X\backslash f^{-1}(V)$. Therefore, we obtain $f^{-1}(V) \subseteq \theta - s\mathrm{Int}f^{-1}(\mathrm{Cl}V)$.

(7)⇔(8). This proof is similar to that of (4)⇔(5).

(7)⇒(1). Let V be any open set and $f(x) \in V$. By (7), we have $x \in f^{-1}(V) \subseteq \theta - s\mathrm{Int}f^{-1}(\mathrm{Cl}V)$. Then there exists a semi-open set U containing x such that $x \in U \subseteq \mathrm{Cl}U \subseteq f^{-1}(\mathrm{Cl}V)$. This shows that f is weakly semi-θ-continuous. □

Theorem 2. *If* $f : X \to Y$ *is a weakly semi-θ-continuous function, then the followings are hold:*

(1) $f^{-1}(V)$ *is θ-semiopen for every θ-open subset* V *of* Y*;*

(2) $f^{-1}(V)$ *is θ-semiclosed for every θ-closed subset* V *of* Y*;*

Proof. Let K be a θ-closed set of Y. By (4) of Theorem 1, we have $\theta - s\mathrm{Cl}f^{-1}(K) \subseteq f^{-1}(K)$. Hence $f^{-1}(K)$ is θ -semiclosed. It is obvious that (1) and (2) are equivalent. □

Remark 1. The converse of Theorem 2 is not true in general as the following example.

Example 1. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. Let $f : (X, \tau) \to (X, \sigma)$ be the identity function. Then f is satisfies (1) of Theorem 2 but it is not weakly semi-θ-continuous.

Definition 2. A net (x_i) is said to be θ-converges [17] (resp. θ-semi converges) to a point x if for each open (resp. semi-open) set U containing x, there exists i_0 such that $x_i \in \mathrm{Cl}U$ for all $i \geq i_0$.

It is obvious that if the net (x_i) θ-semi converges to x, then (x_i) θ-converges to x. It is easy to show that the following proposition hold.

Proposition 2. *A net (x_i) θ-semi converges to a point x if and only if for each regular closed set F containing x, there exists i_0 such that $x_i \in F$ for all $i \geq i_0$.*

Theorem 3. *For a function $f : X \to Y$ the followings are equivalent:*

(1) f is weakly semi-θ-continuous;

(2) For each $x \in X$ and for every net (x_i) which θ-semi converges to x, the net $(f(x_i))$ θ-converges to $f(x)$.

Proof. (1)$\Rightarrow$(2). Let $x \in X$ and (x_i) be a net in X such that (x_i) θ-semi converges to x. Let V be an open set containing $f(x)$. Since f is weakly semi-θ-continuous, there exists a semi-open set U such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$. Since (x_i) θ-semi converges to x, there is a i_0 such that $x_i \in \mathrm{Cl}U$ for all $i \geq i_0$. Hence $f(x_i) \in \mathrm{Cl}V$ for all $i \geq i_0$. Thus $(f(x_i))$ θ-converges to $f(x)$.

(2)$\Rightarrow$(1) Suppose (1) is not true, i.e. f is not weakly semi-θ-continuous. Then, there exist $x \in X$ and an open set V containing $f(x)$ such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$ for all semi-open set U containing x. Choose $x_U \in U$ such that $f(x_U) \notin \mathrm{Cl}V$. Let us consider the net (x_U). Obviously (x_U) θ-semi converge to x but $(f(x_U))$ not converges to $f(x)$. This is a contradiction. □

4. Basic Properties

In [10], it is shown that for a topological space (X, τ), if $U \in SO(X)$ and $A \in PO(X)$, then $U \cap A \in SO(A)$.

Proposition 3. *Let (X, τ) be a topological space. If $U \in \theta SO(X)$ and $A \in PO(X)$, then $U \cap A \in \theta SO(A)$.*

Proof. Let $x \in U \cap A$. Then $x \in U$ and $x \in A$. Since $U \in \theta SO(X)$, there exists a semi-open subset T of X such that $x \in T \subseteq \mathrm{Cl}T \subseteq U$. On the other hand, since $A \in PO(X)$, $T \cap A \in SO(A)$ and $x \in T \cap A \subseteq \mathrm{Cl}_A(T \cap A) = \mathrm{Cl}(T \cap A) \cap A \subseteq \mathrm{Cl}T \cap \mathrm{Cl}A \cap A = \mathrm{Cl}T \cap A \subseteq U \cap A$. Thus we have $x \in \theta - s\mathrm{Int}_A(U \cap A)$. This implies that $U \cap A$ is θ-semiopen in the relative topology of A. □

Theorem 4. *Let $f : X \to Y$ be a weakly semi-θ-continuous function and let A be a pre-open subset of X. Then $f|_A : A \to Y$ is weakly semi-θ-continuous.*

Proof. This follows from the fact that if $U \in SO(X)$ and $A \in PO(X)$, then $U \cap A \in SO(A)$. □

Let $\{X_\alpha : \alpha \in I\}$ be family of spaces. We simply denote the product space $\Pi\{X_\alpha : \alpha \in I\}$ by ΠX_α. The following theorems are easily obtained and the proofs are omitted.

Theorem 5. *Let $f : X \to \Pi X_\alpha$ be given. Then f is weakly semi-θ-continuous if and only if the composition with each projection p_α is weakly* *semi-θ-continuous.*

Theorem 6. *Let $f : X \to Y$ be a function and let $g : X \to X \times Y$, given by $g(x) = (x, f(x))$ for each $x \in X$, be graph function. Then f is weakly semi-θ-continuous if and only if g is weakly semi-θ-continuous.*

5. Comparisions

Definition 3. A function $f : X \to Y$ is called θ-irresolute [7] (resp. θ-quasi irresolute [18], weakly θ-irresolute [4], almost s-continuous [11]) if for each $x \in X$ and each semi-open set V containing $f(x)$, there is a semi-open (resp. open, semi-open, open) set U containing x such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$ (resp. $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$, $f(U) \subseteq \mathrm{Cl}V$, $f(\mathrm{Cl}U) \subseteq s\mathrm{Cl}V$).

Definition 4. A function $f : X \to Y$ is called θ-continuous [3] (resp. θ-semi continuous [1], weakly semi continuous [1]) if for each $x \in X$ and each open set V containing $f(x)$, there is an open (resp. semi-open, semi-open) set U containing x such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$ (resp. $f(s\mathrm{Cl}U) \subseteq \mathrm{Cl}V$, $f(U) \subseteq \mathrm{Cl}V$).

It is obvious that θ-irresoluteness and θ-continuity implies weakly semi-θ-continuity and weakly semi-θ-continuity implies θ-semicontinuity as follows:

$$\begin{array}{ccccccc}
almost\ s-cont. & & & & & & \\
\Downarrow & & & & & & \\
\theta-quasi\ irr. & \Rightarrow & \theta-irr. & & \Rightarrow & & weak\ \theta-irr \\
\Downarrow & & \Downarrow & & & & \Downarrow \\
\theta-continuity & \Rightarrow & weak\ semi-\theta-c. & \Rightarrow & \theta-semi-c. & \Rightarrow & weak\ semi-c.
\end{array}$$

None of these implications are reverseble as the following examples show.

Example 2. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and $\sigma = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}\}$. The identity function $f : (X, \tau) \to (X, \sigma)$ is θ-semicontinuous. But it is not weakly semi-θ-continuous at point b.

Example 3. Let $X = \{a, b, c\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Define the function $f : (X, \tau) \to (X, \tau)$ by $f(a) = f(c) = a$, $f(b) = c$. Then f is weakly semi-θ-continuous, but it is not θ-irreducible at point b.

Example 4. Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, d\}, \{a, b, d\}\}$ and $\sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$. Define the function $f : (X, \tau) \to (X, \sigma)$ by $f(a) = f(c) = a$, $f(b) = b$, $f(d) = d$. Then f is weakly semi-θ-continuous on X, but it is not θ-continuous at point $c \in X$.

Recall that a space (X, τ) is said to be extremally disconnected (abbreviated as e.d.) if $\mathrm{Cl}V \in \tau$ for each $V \in \tau$. A space is called locally indiscrete if every open subset is closed.

Since semi-open subsets are open in locally indiscrete space, proofs of the following two theorems are clear.

Theorem 7. *If $f : X \to Y$ is weakly semi-θ-continuous and X is locally indiscrete, then f is θ-continuous.*

Theorem 8. *If $f : X \to Y$ is weakly semi-θ-continuous and Y is locally indiscrete space, then f is θ-irreducible.*

Theorem 9. *If $f : X \to Y$ is θ-semi-continuous X is e.d space, then f is weakly semi-θ-continuous.*

Proof. This follows from the fact that in e.d space X, $\mathrm{Cl}U = s\mathrm{Cl}U$ for all $U \in SO(X)$. □

Theorem 10. *Let $f : X \to Y$ and $g : Y \to Z$ be two functions. Then:*

(1) If f is θ-irresolute and g is weakly semi-θ -continuous, then $g \circ f$ is weakly semi-θ-continuous.

(2) If f is θ-quasi irresolute and g is weakly semi-θ -continuous, then $g \circ f$ is θ-continuous.

(3) If f is weakly θ-irresolute and g is weakly semi-θ -continuous, then $g \circ f$ is weakly semi continuous.

(4) If f is weakly semi-θ-continuous and g is θ-quasi irresolute, then $g \circ f$ is θ-irresolute.

(5) If f is weakly semi-θ-continuous and g is θ -continuous, then $g \circ f$ is weakly semi-θ-continuous.

(6) If f and g are weakly semi-θ-continuous and Y is locally indiscrete space, then $g \circ f$ is weakly semi-θ-continuous.

Proof. Proofs are obvious from the definitions. □

6. Some Preservation Properties

Definition 5. A space (X, τ) is called:

(1) S-closed [6] if every regular closed cover of X has a finite subcover.

(2) quasi H-closed [13] if every open cover of X has a finite subcollection whose closures cover X.

Theorem 11. *Let $f : X \to Y$ be a weakly semi-θ-continuous surjection. If X is S-closed, then Y is quasi H-closed.*

Proof. Let $\{V_i\}_{i\in I}$ be an open cover of Y. If $x \in X$, $f(x) \in V_j$ for some $j \in I$. Since f is weakly semi-θ-continuous, there exists a semi-open set U_x such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$. Thus $\{U_x\}_{x\in X}$ is an semi-open cover of X. Since X is S-closed, there exist $x_1, x_2, ..., x_k \in X$ such that $X \subseteq \cup\{\mathrm{Cl}U_{x_i} : x_i \in X, i = 1, 2, ..., k\}$.

So we obtain $f(X) \subseteq f(\cup\{\mathrm{Cl}U_{x_i} : x_i \in X, i = 1, 2, ..., k\}) \subseteq \cup\{\mathrm{Cl}V_{x_i} : x_i \in X, i = 1, 2, ..., k\}$. Since f is surjective, then Y is quasi H-closed. □

A space is called Urysohn (resp. s-Urysohn [2]) for each pair of points x_1, $x_2 \in X$, where $x_1 \neq x_2$, there exist open (resp. semi-open) sets U_1 and U_2 containing x_1 and x_2, respectively, such that $\mathrm{Cl}U_1 \cap \mathrm{Cl}U_2 = \emptyset$.

By a weakly semi-θ-continuous retraction we mean a weakly semi-θ-continuous function $r : X \to A$, where $A \subseteq X$, and $r\,|_A$ is the identity function on A.

Theorem 12. *Let A be a subset of X and $r : X \to A$ be a weakly semi-θ-continuous retraction. If X is Urysohn, then A is θ-semiclosed subset of X.*

Proof. Suppose that A is not θ-semiclosed. Then there exists a point x in X such that $x \in \theta - s\mathrm{Cl}A$ but $x \notin A$. It follows that $r(x) \neq x$ because r is retraction. Since X is Urysohn, there exist open sets U and V of x and $r(x)$ respectively, such that $\mathrm{Cl}U \cap \mathrm{Cl}V = \emptyset$. By hypothesis $r : X \to A$ be a weakly semi-θ-continuous and $r(x) \in V$. Then there exist a semi-open set $W \subset X$ containing x such that $r(\mathrm{Cl}W) \subset \mathrm{Cl}V$. Since $W \cap U$ is an semi-open set containing x and $x \in \theta - s\mathrm{Cl}A$, $\mathrm{Cl}(W \cap U) \cap A \neq \emptyset$. Therefore, there exist a point $y \in \mathrm{Cl}(W \cap U) \cap A \subset \mathrm{Cl}(W) \cap \mathrm{Cl}(U) \cap A$. So, $y \in A$ and $r(y) = y \in \mathrm{Cl}U$, $\mathrm{Cl}U \cap \mathrm{Cl}V = \emptyset$ gives $r(y) \notin \mathrm{Cl}V$. On the other hand $y \in \mathrm{Cl}W$ this implies $r(\mathrm{Cl}W) \subseteq \mathrm{Cl}V$. This contradicts the hypothesis that r is weakly semi-θ-continuous. Thus A is θ-semiclosed as claimed. □

Theorem 13. *If $f : X \to Y$ is a weakly semi-θ-continuous injection and Y is Urysohn, then X is s-Urysohn.*

Proof. Let x_1, $x_2 \in X$ and $x_1 \neq x_2$. Then since f is injective and Y is Urysohn, $f(x_1) \neq f(x_2)$ and there exist open sets V_1 and V_2 such that $f(x_1) \in V_1$ and $f(x_2) \in V_2$ and $\mathrm{Cl}V_1 \cap \mathrm{Cl}V_2 = \emptyset$. Since f is weakly semi-θ-continuous, There exist semi-open sets U_1 and U_2 such that $x_1 \in U_1$ and $x_2 \in U_2$ and $f(\mathrm{Cl}U_1) \subseteq \mathrm{Cl}V_1$ and $f(\mathrm{Cl}U_2) \subseteq \mathrm{Cl}V_2$. Therefore, we obtain that semi-open sets U_1 and U_2 containing x_1 and x_2 respectively, such that $\mathrm{Cl}U_1 \cap \mathrm{Cl}U_2 = \emptyset$. Thus X is s-Urysohn. □

A space is called weakly Hausdorff if and only if each point in X is the intersection of regular closed subset of X [14]. It follows from the definition that a space is weakly Hausdorff if and only if for each pair of points $x_1, x_2 \in X$ where $x_1 \neq x_2$, there exists a regular open set G containing x_1 and regular closed set F containing x_2 such that $G \cap F = \emptyset$.

Corollary 1. *If $f : X \to Y$ is a weakly semi-θ-continuous injection and Y is Urysohn, then X is weakly Hausdorff.*

A topological space X is said to be hyperconnected [15] if every pair nonempty open sets of X has nonempty intersection. A space is said to be θ-irreducible [5] every pair nonempty regular closed sets of X has nonempty intersection.

It is pointed out in [5] that hyperconnectedness implies θ-irreducibility and θ-irreducibility implies connectedness.

In [5], Jankovic and Long proved that θ-irreducibility is preserved under θ-continuous surjection. Next, we will some analogous results.

Lemma 1. *Let (X, τ) be a topological space. Then: X is θ-irreducible iff $\theta - s\mathrm{Cl}(\mathrm{Cl}U) = X$ for every nonempty $U \in SO(X)$, see [12].*

Theorem 14. *If X is θ-irreducible, and $f : X \to Y$ is weakly semi-θ-continuous and surjective, then Y is hyperconnected.*

Proof. Assume that X is θ-irreducible. Let V be any nonempty open set in Y. Since f is surjective, there exist a point x in X such that $f(x) \in V$. By weakly semi-θ-continuity of f, there exists a semi-open set U containing x such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$. Since X is θ-irreducible, $\theta - s\mathrm{Cl}(\mathrm{Cl}U) = X$. On the other hand, $\mathrm{Cl}U$ is a regular closed set and hence is a θ-semiclosed set. Therefore, we obtain $Y = f(X) = f(\theta - s\mathrm{Cl}(\mathrm{Cl}U)) = f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V$. This shows that Y is hyperconnected. □

Corollary 2. *If X is hyperconnected, and $f : X \to Y$ is weakly semi-θ-continuous and surjective, then Y is hyperconnected.*

Theorem 15. *If X is θ-irreducible, Y is Urysohn and $f : X \to Y$ is weakly semi-θ-continuous, then f is constant.*

Proof. Suppose that there exist two point x and y of X such that $f(x) \neq f(y)$. Since Y is Urysohn, there exists open sets V_x and V_y in Y containing $f(x)$ and $f(y)$, respectively, such that $\mathrm{Cl}V_x \cap \mathrm{Cl}V_y = \emptyset$. By weakly semi-$\theta$-continuous of f, there exist semi-open sets U_x and U_y containing x and y respectively, such that $f(\mathrm{Cl}U_x) \subseteq \mathrm{Cl}V_x$ and $f(\mathrm{Cl}U_y) \subseteq \mathrm{Cl}V_y$. Therefore, we obtain $\mathrm{Cl}U_x \cap \mathrm{Cl}U_y = \emptyset$. This contradicts the assumption that X is θ-irreducible. □

7. Functions with Weakly Semi-θ-Closed Graphs

Definition 6. The graph $G(f)$ of a function $f : X \to Y$ is said to be weakly semi-θ-closed if for each $(x, y) \in (X \times Y) - G(f)$, there exist a semi-open set U containing x and an open set V containing y such that $(\mathrm{Cl}U \times \mathrm{Cl}V) \cap G(f) = \emptyset$.

Lemma 2. *The graph $G(f)$ of $f : X \to Y$ is weakly semi-θ-closed in $X \times Y$ if and only if for each $(x, y) \in (X \times Y) - G(f)$, there exist a semi-open set U containing x and an open set V containing y such that $f(\mathrm{Cl}U) \cap \mathrm{Cl}V = \emptyset$.*

Proof. It follows immediately from the definition. □

Theorem 16. *If $f : X \to Y$ is θ-quasi irresolute and Y is Urysohn, then the graph $G(f)$ of f is weakly semi-θ-closed in $X \times Y$.*

Proof. Let $(x, y) \notin G(f)$, then $y \neq f(x)$. Since Y is Urysohn, there exist open sets V_1 and V_2 containing $f(x)$ and y, respectively, such that $\mathrm{Cl}V_1 \cap \mathrm{Cl}V_2 = \emptyset$. Since f is weakly semi-θ -continuous, there exists a semi-open set U containing x such that $f(\mathrm{Cl}U) \subseteq \mathrm{Cl}V_1$. Therefore, $f(\mathrm{Cl}U) \cap \mathrm{Cl}V_2 = \emptyset$ and $G(f)$ is weakly semi-θ-closed in $X \times Y$. □

Theorem 17. *If $f : X \to Y$ is a surjection with a weakly semi-θ-closed graph, then Y is weakly Hausdorff.*

Proof. Let y, y_1 be any distict points of Y. Since f is surjective, there exists $x \in X$ such that $f(x) = y_1$. Then $(x, y) \notin G(f)$, and by Lemma 2, there exist a semi-open set U containing x and an open set V containing y such that $f(\mathrm{Cl}U) \cap \mathrm{Cl}V = \emptyset$. Since $y_1 \in f(U)$, $y_1 \notin \mathrm{Cl}V$ and $\mathrm{Cl}V$ is a regular closed set containing y. This shows that Y is weakly Hausdorff. □

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