International Journal of Pure and Applied Mathematics
Volume 29 No. 3 2006, 403-411

STF CODING FOR MIMO-OFDM SYSTEMS WITH FOUR OR THREE TRANSMIT ANTENNAS

Peng Zhang[1], Laurie L. Joiner[2 §]
[1,2]Department of Electrical and Computer Engineering
University of Alabama in Huntsville
Huntsville, AL 35899, USA
[1]e-mail: zhangp1@ece.uah.edu
[2]e-mail: ljoiner@ece.uah.edu

Abstract: A space-time-frequency (STF) coding scheme based on quasi-orthogonal space-time block coding (QOSTBC) for MIMO-OFDM systems with four or three transmit antennas is presented. A low-complexity linear decoding algorithm is developed to mitigate the diversity loss caused by inter-symbol couplings in decoding of the block code in a non-quasi-static OFDM channel. Simulation shows that the proposed coding scheme combined with the linear decoder exhibits a significant advantage in BER performance and decoding simplicity over the counterpart space-frequency (SF) scheme in a slow fading channel.

AMS Subject Classification: 94B12
Key Words: quasi-orthogonal space-time block codes, linear decoding, MIMO-OFDM

1. Introduction

Multiple-input-multiple-output (MIMO) systems, which utilize multiple transmit and receive antennas and space-time coding (STC), are capable of exploiting diversity and capacity gain in a wireless channel and potentially satisfy the demands of future wireless communication systems for high data rates. Orthogonal frequency division multiplexing (OFDM), which is another tech-

Received: May 23, 2006 © 2006, Academic Publications Ltd.
§Correspondence author

nique used for high data rate transmission with high bandwidth efficiency and tolerance to dispersive wireless channels, can convert a broadband frequency-selective fading channel into parallel flat-fading subchannels which are favorable to space-time coding. Hence, to build up a MIMO-OFDM system by taking advantage of both techniques, promises a significant boost of system performance for future wireless communication systems.

Coding schemes play an important role in exploiting the diversity resources of wireless channels, which are spatial diversity, temporal diversity and frequency diversity. Orthogonal space-time block codes (OSTBC) were first introduced by Alamouti [1] to provide spatial diversity under a configuration of two transmit antennas, and were generalized by Tarokh [5, 4]. OSTBC achieves full diversity and linear low complexity decoding in a flat fading channel. However, the maximum symbol transmission rate of an OSTBC from complex designs is only 3/4 for three and four transmitter antennas. In [2, 3, 6], so called quasi-orthogonal space-time block codes (QOSTBC) are proposed for systems with more than three transmit antennas. By relaxing the code orthogonality, the QOSTBCs can achieve a unit transmission rate. The QOSTBCs can still be decoded with fast linear processing in flat fading, but in a non-quasi-static channel, which is prevalent in OFDM systems, the simple symbol-pairwise signal coupling can not be achieved. The above research on the QOSTBCs based on flat fading is not applicable under a MIMO-OFDM configuration. In our research, we propose a decoding method to address multiple symbol coupling problems in decoding a space-frequency (SF) implementation of QOSTBC in a MIMO-OFDM system. Furthermore, we use a space-time-frequency (STF) scheme and the slow fading feature of the channel to simplify the decoding process. We focus on the implementation of the QOSTBC proposed in [3] with a STF design and its decoding in slow fading for configurations of three or four transmit antennas. We explore the performance improvement over using linear decoding.

The paper is organized as follows. In Section 2, a space-frequency (SF) scheme of QOSTBC is introduced for a 4×1 configuration. Then, derived from the linear decoding of the SF scheme, a space-time-frequency (STF) coding scheme is presented as well as its simplified linear decoding method. Numerical simulation results are show in Section 3. Section 4 concludes the paper.

2. MIMO-OFDM System

2.1. QOSTBC of Space-Frequency Design

We consider a space-frequency (SF) implementation of the quasi-orthogonal space-time block code (QOSTBC) proposed in [3]. Assuming symbols s_1, s_2, s_3, s_4 are transmitted, a codeword $\mathbf{C}$ is constituted as follows

$$\mathbf{C} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix}, \tag{1}$$

where $(\cdot)^*$ denotes the conjugate operation of a complex number. Symbols along a column of $\mathbf{C}$ are transmitted at the same transmit antenna but at four adjacent OFDM subcarriers, f_1, f_2, f_3, f_4. The symbols along a row are transmitted at the same subcarrier but at four different transmit antennas. Therefore a codeword $\mathbf{C}$ is transmitted through four transmit antennas in four OFDM subcarriers. It forms a space-frequency block code with a unit transmission rate.

For simplicity, we assume there is only one receiver antenna in the system. We describe the received signal corresponding to a codeword above with the following model

$$r_i = \{\mathbf{CH}\}_{ii} + n_i, \quad \text{for} \;\; i = 1, \ldots, 4, \tag{2}$$

where r_i denotes the received signal at the i-th subcarrier. $\mathbf{H}$ is the channel state matrix given by

$$\mathbf{H} = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} & h_{1,4} \\ h_{2,1} & h_{2,2} & h_{2,3} & h_{2,4} \\ h_{3,1} & h_{3,2} & h_{3,3} & h_{3,4} \\ h_{4,1} & h_{4,2} & h_{4,3} & h_{4,4} \end{bmatrix}, \tag{3}$$

where $h_{i,k}$ represents the channel complex gain corresponding to the i-th transmit antenna at the k-th subcarrier, $\{\cdot\}_{ij}$ denotes the matrix entry of the i-th row and j-th column, and $c_{i,k}$ denotes the symbol in a codeword with the corresponding index i and k. We assume the average power transmitted at each transmit antenna and each subcarrier is 1, i.e., $E\{|c_{i,k}|^2\} = 1$, where $E\{\cdot\}$ denotes the expectation operation. The noise sample is n_i, which is complex white Gaussian noise with zero mean and variance σ_n^2 evenly divided by its real and imaginary part. We also assume n_i is independent of the subcarrier index i.

We consider decoding the QOSTBC under the OFDM environment by using linear processing since it has the advantage of simplicity over the complex and time-consuming ML decoding. In [3], a linear decoding method is presented on the basis of a flat-fading assumption. However, under an OFDM configuration, this assumption may not be reasonable. For the SF scheme, four adjacent subcarriers may not have the same channel gains. This consequently removes the possibility of the simple symbol pairwise coupling, which can be achieved under flat fading. In order to achieve partial coupling between symbols, we design the following filtering for the received signals, $\mathbf{r}' = \begin{bmatrix} r_1 & r_2^* & r_3 & r_4^* \end{bmatrix}^T$.

$$\tilde{\mathbf{r}} = \begin{bmatrix} \tilde{r}_{1,\bar{2}} \\ \tilde{r}_{2,\bar{1}} \\ \tilde{r}_{3,\bar{4}} \\ \tilde{r}_{4,\bar{3}} \end{bmatrix} = (\mathbf{F}_1)^H \mathbf{r}', \tag{4}$$

where

$$\mathbf{F}_1 = \begin{bmatrix} h_{1,2} & h_{2,2} & h_{3,2} & h_{4,2} \\ -h_{2,1}^* & h_{1,1}^* & -h_{4,1}^* & h_{3,1}^* \\ -h_{3,4} & h_{4,4} & h_{1,4} & -h_{2,4} \\ -h_{4,3}^* & -h_{3,3}^* & h_{2,3}^* & h_{1,3}^* \end{bmatrix}. \tag{5}$$

From (4), we get the set of combined signals

$$\begin{aligned} \tilde{r}_{1,\bar{2}} &= A_1 s_1 + \alpha_{13} s_3 + \alpha_{14} s_4 + \tilde{\eta}_1 \,, \\ \tilde{r}_{2,\bar{1}} &= A_1 s_2 + \alpha_{24} s_4 + \alpha_{23} s_3 + \tilde{\eta}_2 \,, \\ \tilde{r}_{3,\bar{4}} &= A_2 s_3 + \alpha_{31} s_1 + \alpha_{32} s_2 + \tilde{\eta}_3 \,, \\ \tilde{r}_{4,\bar{3}} &= A_2 s_4 + \alpha_{42} s_2 + \alpha_{41} s_1 + \tilde{\eta}_4 \,, \end{aligned} \tag{6}$$

where

$$\begin{aligned} A_1 &= h_{1,1}h_{1,2}^* + h_{2,1}h_{2,2}^* + h_{3,3}h_{3,4}^* + h_{4,3}h_{4,4}^*, \\ A_2 &= h_{1,3}h_{1,4}^* + h_{2,3}h_{2,4}^* + h_{3,1}h_{3,2}^* + h_{4,1}h_{4,2}^*, \\ \alpha_{13} &= \alpha_{42} = h_{1,2}^*h_{3,1} - h_{1,3}h_{3,4}^* + h_{2,1}h_{4,2}^* - h_{2,4}^*h_{4,3}, \\ \alpha_{14} &= -\alpha_{32} = h_{1,2}^*h_{4,1} - h_{1,4}^*h_{4,3} + h_{2,3}h_{3,4}^* - h_{2,1}h_{3,2}^*, \\ \alpha_{23} &= -\alpha_{41} = h_{1,3}h_{4,4}^* - h_{1,1}h_{4,2}^* + h_{2,2}^*h_{3,1}^* - h_{2,4}^*h_{3,3}, \\ \alpha_{24} &= \alpha_{31} = h_{1,1}h_{3,2}^* - h_{1,4}^*h_{3,3} + h_{2,2}^*h_{4,1} - h_{2,3}h_{4,4}^*. \end{aligned}$$

The combined noise term is defined as $\tilde{\eta}_i$. With the filtering of $\mathbf{F}_1$, each combined signal has only the terms related to three symbols and one symbol vanishes. Hence they are partially decoupled. We can use the combined signals in (6) pairwise to estimate the transmitted symbols. For example, in order to estimate symbols s_1 and s_3, the combined signals $\tilde{r}_{1,\bar{2}}$ and $\tilde{r}_{3,\bar{4}}$ are used. At first the noise terms and undesired coupling terms, $\alpha_{14}s_4$ in $\tilde{r}_{1,\bar{2}}$ and $\alpha_{32}s_2$ in $\tilde{r}_{3,\bar{4}}$, are neglected. By solving the linear equation system of $\tilde{r}_{1,\bar{2}}$ and $\tilde{r}_{3,\bar{4}}$, we get the estimates of s_1 and s_3. Signals s_2 and s_4 can be obtained in the same manner by using $\tilde{r}_{2,\bar{1}}$ and $\tilde{r}_{4,\bar{3}}$. The method is called SF one-stage linear decoding. Furthermore, we can use the results from the one-stage decoder as initial estimates to offset the undesired coupling terms from the signals in (6) and then achieve more accurate estimates by using the rest of the combined signals at the second stage. This method is called SF two-stage linear decoder.

We notice that the undesired symbol coupling in the above decoding affects the estimation accuracy. We can compensate for this intersymbol disturbance by using a two-stage decoding. The accuracy increase of the two-stage decoding is achieved at the cost of complexity. We wish the undesired coupling could be reduced by trivialization of the factors α_{14}, α_{32}, α_{23} and α_{41}, but we cannot achieve this with a SF coding scheme. However, this can be done by rearrangement of the coding scheme.

2.2. STF Scheme for QOSTBC

In order to trivialize the undesired symbol couplings, we redesign the coding scheme and form a space-time-frequency (STF) coding design

$$\mathbf{C} = \{\mathbf{C}_1 \ \mathbf{C}_2\}, \tag{7}$$

where

$$\mathbf{C}_1 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2^* & -s_1^* & s_4^* & -s_3^* \end{bmatrix},$$

$$\mathbf{C}_2 = \begin{bmatrix} s_3 & -s_4 & -s_1 & s_2 \\ s_4^* & s_3^* & -s_2^* & -s_1^* \end{bmatrix}.$$

$\mathbf{C}_1$ and $\mathbf{C}_2$ are symbol sets transmitted at time slot 1 and time slot 2, respectively. Along a column of $\mathbf{C}_1$ or $\mathbf{C}_2$ are the symbols transmitted at the same transmit antenna but at two different OFDM subcarriers, f_1 and f_2. The symbols along a row are transmitted at the same subcarrier (f_1 or f_2) but at different transmit antennas. Therefore a codeword $\mathbf{C}$ is transmitted through four transmit antennas in two time slots, and over two OFDM subcarriers. It forms a space-time-frequency code with a unit rate.

With the above coding scheme, the received signals and the channel coefficients in the previous section have the following changes in notation:

— r_1 and r_2: received signals at f_1 and f_2 at time slot 1,
— r_3 and r_4: received signals at f_1 and f_2 at time slot 2,
— $h_{i,1}$: corresponds to the i-th transmit antenna, time slot 1 and subcarrier f_1,
— $h_{i,2}$: corresponds to the i-th transmit antenna, time slot 1 and subcarrier f_2,
— $h_{i,3}$: corresponds to the i-th transmit antenna, time slot 2 and subcarrier f_1,
— $h_{i,4}$: corresponds to the i-th transmit antenna, time slot 2 and subcarrier f_2.

We further assume the system experiences slow fading and the channel can be characterized by

$$h_{i,1} \approx h_{i,3}, \quad h_{i,2} \approx h_{i,4}. \tag{8}$$

Under this condition, the coefficients of the undesired terms varnish, i.e.,

$$\alpha_{14} = \alpha_{32} = 0, \quad \alpha_{23} = \alpha_{41} = 0.$$

The combined signals are

$$\begin{aligned} \tilde{r}_{1,\bar{2}} &= As_1 + \alpha s_3 + \tilde{\eta}_1, \\ \tilde{r}_{2,\bar{1}} &= As_2 - \alpha s_4 + \tilde{\eta}_2, \\ \tilde{r}_{3,\bar{4}} &= As_3 - \alpha s_1 + \tilde{\eta}_3, \\ \tilde{r}_{4,\bar{3}} &= As_4 + \alpha s_2 + \tilde{\eta}_4, \end{aligned} \tag{9}$$

where

$$A = h_{1,1}h_{1,2}^* + h_{2,1}h_{2,2}^* + h_{3,1}h_{3,2}^* + h_{4,1}h_{4,2}^*, \tag{10}$$

$$\alpha = h_{1,2}^* h_{3,1} - h_{1,1}h_{3,2}^* + h_{2,1}h_{4,2}^* - h_{2,2}^* h_{4,1}. \tag{11}$$

Finally, we simplify the decoding as

$$\begin{aligned} \begin{bmatrix} \hat{s}_1 \\ \hat{s}_3 \end{bmatrix} &= \mathbf{H}_2^{-1} \begin{bmatrix} \tilde{r}_{1,\bar{2}} \\ \tilde{r}_{3,\bar{4}} \end{bmatrix} = \frac{1}{A^2+\alpha^2}\mathbf{H}_2^T \begin{bmatrix} \tilde{r}_{1,\bar{2}} \\ \tilde{r}_{3,\bar{4}} \end{bmatrix}, \\ \begin{bmatrix} \hat{s}_4 \\ \hat{s}_2 \end{bmatrix} &= \mathbf{H}_2^{-1} \begin{bmatrix} \tilde{r}_{4,\bar{3}} \\ \tilde{r}_{2,\bar{1}} \end{bmatrix} = \frac{1}{A^2+\alpha^2}\mathbf{H}_2^T \begin{bmatrix} \tilde{r}_{4,\bar{3}} \\ \tilde{r}_{2,\bar{1}} \end{bmatrix}, \end{aligned} \tag{12}$$

where

$$\mathbf{H}_2 = \begin{bmatrix} A & \alpha \\ -\alpha & A \end{bmatrix}.$$

The proposed decoding algorithm is also suitable for the QOSTBC with three transmit antennas. The code matrix can be formed by removing one column of the codeword matrix. In the decoding, the terms related to the removed channel branch must be deleted.

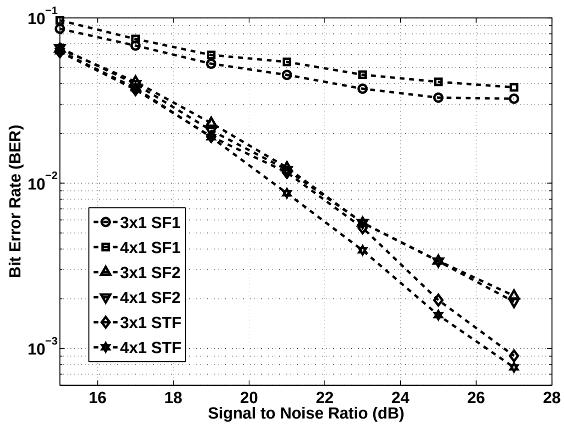

Figure 1: BER performance with gray coded 32 QAM

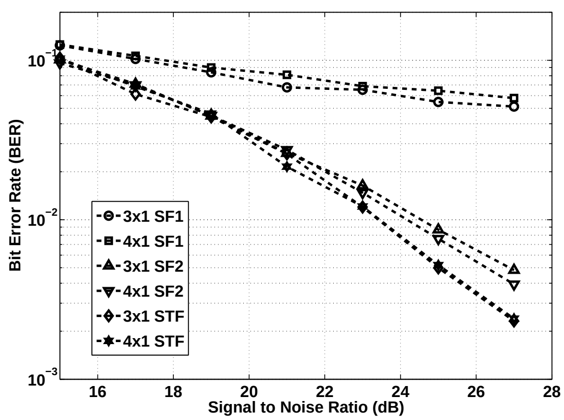

Figure 2: BER performance with gray coded 64 QAM

3. Simulation

In our simulation, we use the following channel profile for channels from the four transmit antennas to the receiver antenna. The channel profile is the same, but the four branches are independent from each other. We assume the channels are Rayleigh fading channels with the follow statistics: path delays = [0, 8×10^{-8}, 2.4×10^{-7}] seconds, average path gains = [0, -5.3, -16] dB, Doppler frequency shift = 200 Hz. In the OFDM modules, we use an IFFT and FFT of size 64.

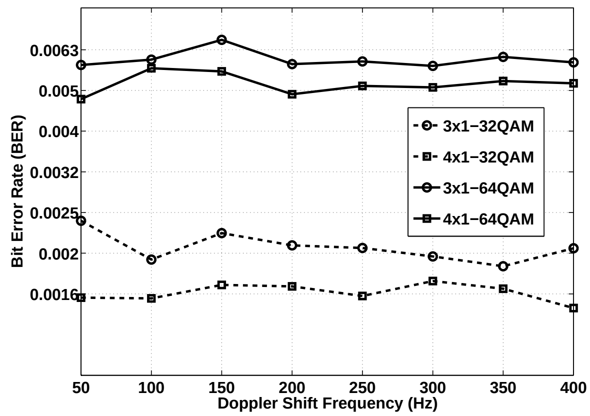

Figure 3: BER performance versus Doppler shift

The CP (cyclic prefix) length is set to 16. The sample time is 4×10^{-8} seconds. We use gray coded 32-QAM and 64-QAM to test the system performance. Figure 1 and Figure 2 show the performance of the proposed coding scheme with 32-QAM and 64-QAM compared with the SF schemes. “SF1” and “SF2” in the legend stand for the SF schemes with one-stage decoding and two-stage decoding, respectively. We can see the SF scheme with one-stage decoding has the worse performance. The proposed STF scheme shows a large advantage over the SF scheme with two-stage decoding. However, for the proposed STF scheme, the complexity of decoding is low compared with the SF two-stage decoding. In Figure 3, we show the system BER performance vs. the Doppler frequency for the proposed STF coding. It shows that the system exhibits a relative stable performance over a wide range of Doppler frequency change with the designed STF coding. Thus, the coding scheme shows a property of robustness to the channel change.

4. Conclusions

We have presented a STF scheme to facilitate the simplification of decoding QOSTBC in MIMO-OFDM systems. With the proposed STF coding scheme in a slow fading environment, proper linear combinations of the received signals can achieve symbol pairwise coupling and a simplified linear decoding with simplicity comparable to the linear decoding method designed for flat fading. The simulations show that the proposed decoding method for the STF scheme improves the system performance greatly compared with the one-stage decoding of the SF scheme and has a performance better than two-stage decoding of the SF scheme.

References

[1] S. Alamouti, A simple transmit diversity technique for wireless communications, *IEEE Journal on Selected Areas in Communications*, **16** (Oct. 1998), 1451-1458.

[2] H. Jafarkhani, A quasi-orthogonal space-time block code, *IEEE Transactions on Communications*, **49** (Jan. 2001), 1-4.

[3] C.B. Papadias, G.J. Foschini, Capacity-approaching space-time codes for systems employing four transmitter antennas, *IEEE Transactions on Information Theory*, **49** (Mar. 2003), 726-733.

[4] V. Tarokh, H. Jafarkhani, A.R. Calderbank, Space-time block codes for wireless communications: performance results, *IEEE Journal on Selected Areas in Communications*, **17** (Mar. 1999), 451-460.

[5] V. Tarokh, H. Jafarkhani, A.R. Calderbank, Space-time block codes from orthogonal designs, *IEEE Transactions on Information Theory*, **45** (July 1999), 1456-1467.

[6] C. Yuen, Y.L. Guan, T.T. Tjhung, Quasi-orthogonal STBC with minimum decoding complexity, *IEEE Transactions on Wireless Communications*, **4** (Sept. 2005), 2089-2094.