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PERMANENCE AND EXTINCTION OF A PERIODIC PREDATOR-PREY DELAY SYSTEM WITH HOLLING TYPE FUNCTIONAL RESPONSE AND STAGE STRUCTURE FOR PREY

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Abstract: In this paper, we consider a periodic coefficients predator-prey system with functional response and infinite delay, in which the prey has a history that takes them through two stages, immature and mature. Sufficient conditions which guarantee the permanence and extinction of the system are obtained.

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1. Introduction

It is well known that past history as well as current conditions can influence population dynamics and such interactions has motivated the introduction of delays in population growth. There are several books [11]-[17] devoted to investigations of the dynamic behavior of delay differential equations. Recently, Stage structure models have received much attraction [3]-[20]. This is not only because they are much more simple than the models governed by partial differential equations but also they can exhibit phenomena similar to those of partial differential models [3], and many important physiological parameters can be incorporated. The single species model with stage structure was studied by Aiello and Freedman [1]. Two species models with stage structure were investigated by Wang and Chen [23], and Xiao and Chen [24] and Magnusson [19]. Zhang, Chen and Neumann [25] proposed the following autonomous stage structure predator-prey system:

$$\begin{cases} x_1' = \alpha x_2 - r_1 x_1 - \beta x_1 - \eta x_1^2 - \beta_1 x_1 x_3, \\ x_2' = \beta x_1 - r_2 x_2, \\ x_3' = x_3(-r + k\beta_1 x_1 - \eta_1 x_3), \end{cases} \tag{1}$$

where α β β_1 η η_1 r r_1 r_2 and k are all positive constants, k is a digesting constant. Sufficient conditions which ensure the permanence of two species and extinction of one or two species are obtained.

On the other hand, since biological and environmental parameters are naturally subject to fluctuation in time, the effects of a periodically varying environment are considered as important selective forces on systems in a fluctuating environment. More realistic and interesting models should take into account both the seasonality of the changing environment [8], [16]. This motivated Cui and Song [6] to consider the following periodic nonautonomous predator-prey model with stage structure for prey:

$$\begin{cases} \dot{x_1} = a(t)x_2 - b(t)x_1 - d(t)x_1^2 - p(t)x_1 y, \\ \dot{x_2} = c(t)x_1 - f(t)x_2^2, \\ \dot{y} = y[-g(t) + h(t)x_1 - q(t)y], \end{cases} \tag{2}$$

where $a(t)$, $b(t)$, $c(t)$, $d(t)$, $f(t)$, $g(t)$, $h(t)$, $p(t)$ and $q(t)$ are all continuous positive ω-periodic functions. x_1 and x_2 denote the density of immature and mature population (prey) respectively, and y is the density of the predator that only prey on x_1(immature prey). They obtain a set of sufficient and necessary condition which guarantee the permanence of the above system. Recently, may be stimulated by the works of Teng and Chen [22], Cui and Sun[7] further incorporated infinite delay to system (2) and investigated the following model:

$$\begin{cases} \dot{x_1} = a(t)x_2 - b(t)x_1 - d(t)x_1^2 - p(t)x_1 \int_{-\infty}^{0} k_{12}(s)y(t+s)\,ds, \\ \dot{x_2} = c(t)x_1 - f(t)x_2^2, \\ \dot{y} = y\big[-g(t) + h(t)\int_{-\infty}^{0} k_{21}(s)x_1(t+s)\,ds \\ \qquad\qquad\qquad - q(t)\int_{-\infty}^{0} k_{22}(s)y(t+s)\,ds\big]. \end{cases} \tag{3}$$

Under the assumption that the coefficients in (3) are all ω-periodic and continuous for $t \geq 0$, $a(t), b(t), c(t), d(t)$ and $f(t)$ are all positive, $p(t), h(t)$ and $q(t)$ are nonnegative, and $\int_0^\omega q(t)dt > 0$, $\int_0^\omega g(t)dt \geq 0$. The functions $k_{ij}(s)(i,j = 1,2)$ defined on $R_- = (-\infty, 0]$ are nonnegative and integrable, $\int_{-\infty}^{0} k_{ij}(s) = 1$. By using analysis technique, they obtained a set of sufficient and necessary conditions which guarantee the permanence of the system.

Noticing that (1)-(3) are modified from the classical Lotka-Volterra predator-prey system:

$$\begin{cases} x' = x(a - bx - cy), \\ y' = y(-d + ex - fy). \end{cases} \tag{4}$$

A predators functional response is its per capita feeding rate on prey. Holling [13], [14] suggested that the predator should not be able to consume an unlimited number of prey as the prey population increases. That is, in the Lotka-Volterra equations, the number of prey consumed per predator is unlimited as the prey population increases. The number of prey removed is cxy, so that the number of prey eaten per predator is unlimited as x increases to infinity. Holling proposed 3 models of the rate of prey capture per predator as a function of prey population density: Types I, II, and III. In 2001, Skalski and Gilliam reviewed the literature on functional response curves and presented statistical evidence from 19 predator-prey systems that three predatordependent functional responses (Beddington-DeAngelis[2], [9], Crowley-Martin [4], and Hassell-Varley [12]), i.e., models that are functions of both prey and predator abundance because of predator interference, can provide better descriptions of predator feeding over a range of predator-prey abundances.

To our knowledge, seldom did scholars consider the stage structure predator-prey system with functional response and infinite delay. This paper is largely motivated by the above-mentioned fact. We consider the following system:

$$\begin{cases} \dot{x_1} = a(t)x_2 - b(t)x_1 - d(t)x_1^2 - p_1(t)\frac{x_1^n}{P+x_1^n}\int_{-\infty}^{0} k_{11}(s)y(t+s)ds, \\ \dot{x_2} = c(t)x_1 - f(t)x_2^2 - p_2(t)\frac{x_2^n}{P+x_2^n}\int_{-\infty}^{0} k_{12}(s)y(t+s)ds, \\ \dot{y} = y\big[-g(t) + \sum_{i=1}^{2} h_i(t)\int_{-\infty}^{0} k_{2i}(s)\frac{x_i^n(t+s)}{P+x_i^n(t+s)}ds - y \\ \qquad\qquad\qquad - q(t)(\int_{-\infty}^{0} k_{23}(s)y(t+s)ds)^2\big], \end{cases} \tag{5}$$

where x_1 and x_2 denote the density of immature and mature population (prey) respectively. y is the density of the predator that prey on x_1 and x_2. The coefficients in (5) are all continuous positive ω-periodic for $t \geq 0$. P, n are positive constants. The functions $k_{ij}(s)$ $(i = 1, 2;\ j = 1, 2, 3)$ defined on $R_- = (-\infty, 0]$ are nonnegative and integrable, $\int_{-\infty}^{0} k_{ij}(s) = 1$. $\frac{x_i^n}{P + x_i^n}$ (Holling type) $(i = 1, 2)$ represent the functional response of predator to prey. The biological background for (5) can be found in [10], [6], [22].

Let $C_+ = \{\phi = (\phi_1, \phi_2, \phi_3) : \phi_i(t)$ is continuous and nonnegative on R_- and $\phi_i(0) > 0,\ i = 1, 2, 3\}$. In this paper, we always assume that solutions of (5) satisfy the initial conditions:

$$x_i(s) = \varphi_i(s),\ y(s) = \psi(s), \quad (i = 1, 2),\ (\varphi_1, \varphi_2, \psi) \in C_+,\ s \in (-\infty, 0]. \tag{6}$$

The main purpose of this paper is to find a set of easily verifiable sufficient conditions for the permanence and extinction of the system (5). The present paper is organized as follows. In Section 2, we introduce some notations and definitions, give some preliminary results needed in later sections, and then state the main results of this paper. We then prove, in Section 3, the main results of (5) by using analysis technique. Finally, in Section 4, we work out two examples.

2. Main Results

Let $f(t)$ be a continuous ω-periodic function defined on $[0, +\infty)$, we set

$$A_\omega(f) = \omega^{-1} \int_0^\omega f(t)dt, \quad f^U = \max_{t \in [0, \omega]} f(t),$$

$$f^L = \min_{t \in [0, \omega]} f(t), \quad h(t) = h_1(t) + h_2(t).$$

Definition 1. The system $\dot{x} = F(t, x)$, $x \in R^n$ is said to be permanent if there are constants $M \geq m > 0$ such that every positive solution $x(t) = (x_1(t), \ldots, x_n(t)) \in R^n_+ = \{(x_1, \ldots, x_n) : x_i > 0,\ i = 1, \ldots, n\}$ of this system, satisfies

$$m \leq \liminf_{t \to \infty} x_i(t) \leq \limsup_{t \to \infty} x_i(t) \leq M\,.$$

Lemma 2. (see [5]) *The system*

$$\begin{cases} \dot{x_1} = a(t)x_2 - b(t)x_1 - d(t)x_1^2, \\ \dot{x_2} = c(t)x_1 - f(t)x_2^2, \end{cases} \tag{7}$$

has a positive ω-periodic solution $(x_1^*(t), x_2^*(t))$ which is globally asymptotically stable with respect to $R_{+0}^2 = \{(x_1, x_2) : x_1 > 0, x_2 > 0\}$.

Lemma 3. *For the following nonautonomous differential equation*

$$\dot{u} = u[a(t) - b(t)u - c(t)u^2]\,, \tag{8}$$

where $a(t)$,$b(t)$ and $c(t)$ are $\omega-$periodic continuous functions, $c^L,\ b^L \geq 0$ and $A_\omega(b) > 0$, there is a constant $M > 0$ such that every positive solution $u(t)$ of (3) satisfies $\limsup_{t\to\infty} u(t) \leq M$.

Proof. The proof is obvious, in fact, $\dot{u} = u[a(t) - b(t)u - c(t)u^2] \leq u[a(t) - b(t)u]$. From [21], we have there exist a constant M such that the solution $x(t)$ of the Logistic equation $\dot{x} = x[a(t) - b(t)x]$ satisfies $\limsup_{t\to\infty} x(t) \leq M$. Using comparison theorem, this completes the proof. □

Theorem 4. *Assume $n \geq 3$ and*

$$A_\omega\big(- g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\infty}^{0} k_{2i}(s) \frac{x_i^{*^n}(t+s)}{P + x_i^{*^n}(t+s)} ds\big) > 0\,, \tag{9}$$

where $(x_1^*(t), x_2^*(t))$ is the positive ω-periodic solution of (7). Then system (5) is permanent

Let $\epsilon(\ll 1)$ be some positive constant and

$$\lambda(t) = -g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\infty}^{0} k_{2i}(s) \frac{(x_i^*(t+s) + \epsilon)^n}{P + (x_i^*(t+s) + \epsilon)^n} ds + h(t)\epsilon.$$

Theorem 5. *Assume*

$$A_\omega\big(- g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\infty}^{0} k_{2i}(s) \frac{x_i^{*^n}(t+s)}{P + x_1^{*^n}(t+s)} ds\big) \leq 0 \tag{10}$$

and

$$l = \int_{-\infty}^{0} k_{23}(s) \exp\{\lambda^U s\} ds < \infty\,, \tag{11}$$

where $(x_1^*(t), x_2^*(t))$ is the positive ω-periodic solution of (7). Then for any solution (x_1, x_2, y) of (5), $y(t) \to 0$ as $t \to \infty$.

3. Proof of Main Results

Lemma 6. *There exist positive constants M_x and M_y such that*

$$\limsup_{t\to\infty} x_i(t) \le M_x, \quad \limsup_{t\to\infty} y(t) \le M_y.$$

Proof. Obviously, R_+^3 is a positively invariant set of system (5). Given any positive solution $(x_1(t), x_2(t), y(t))$ of (5) with initial conditions (6), we have

$$\begin{cases} \dot{x_1} \le a(t)x_2 - b(t)x_1 - d(t)x_1^2, \\ \dot{x_2} \le c(t)x_1 - f(t)x_2^2. \end{cases}$$

Next, we consider the following auxiliary equations

$$\begin{cases} \dot{u_1} = a(t)u_2 - b(t)u_1 - d(t)u_1^2, \\ \dot{u_2} = c(t)u_1 - f(t)u_2^2, \end{cases} \tag{12}$$

by Lemma 2, it follows that (12) has a globally asymptotically stable positive ω-periodic solution $(x_1^*(t), x_2^*(t))$. Let $(u_1(t), u_2(t))$ be the solution of (12) with $u_1(0) = x_1(0),\ u_2(0) = x_1(0)$. By the Vector Comparison Theorem [18], we obtain

$$x_i(t) \le u_i(t), \quad i = 1, 2$$

for all $t \ge 0$. From the global asymptotic stability of $(x_1^*(t), x_2^*(t))$, for any positive constant ε, there exists a $T_0 > 0$ such that for all $t \ge T_0$,

$$|u_i(t) - x_i^*(t)| < \varepsilon, \quad i = 1, 2.$$

Hence, for all $t \ge T_0$, we derive

$$x_i(t) \le x_i^*(t) + \varepsilon, \quad i = 1, 2.$$

Let $M_x = \min\{\max_{t\in[0,\omega]}\{x_i^*(t) + \varepsilon\}, Q\}$ where the positive constant Q guarantee that the function $\frac{x_i^{n-2}}{P+x_i^n}(n \ge 3)$ is increasing as $x_i \in (0, Q]$ $(i = 1, 2)$, we get

$$x_i(t) \le M_x, \quad i = 1, 2. \tag{13}$$

Consequently,

$$\limsup_{t\to\infty} x_i(t) \le M_x,\ i = 1, 2.$$

Let $\alpha(t) = -g(t) + h(t)$ and let the constant $\tau > 0$ be such that

$$\int_{-\tau}^{0} k_{23}(s) \exp(\alpha^U s) ds > 0. \tag{14}$$

From $\int_{-\infty}^{0} k_{2i}(s)ds = 1$ $(i = 1, 2)$, for any $t \geq T_0 + \tau$ we have

$$\begin{aligned} \dot{y} &\leq y[-g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\infty}^{0} k_{2i}(s) \frac{x_i^n(t+s)}{P + x_i^n(t+s)} ds] \\ &\leq y\big[-g(t) + h(t)\big] \\ &= y\alpha(t). \end{aligned}$$

Hence, for any $t \geq t + s \geq T_0 + \tau (s \leq 0)$ we obtain

$$y(t+s) \geq y(t) \exp \int_{t}^{t+s} \alpha(\xi) d\xi \geq y(t) \exp(\alpha^U s).$$

It follows from the above inequlity that for any $t \geq T_0 + 2\tau$, we have

$$\begin{aligned} \dot{y} &\leq y[\alpha(t) - y(t) - q(t)(\int_{-\infty}^{0} k_{23}(s) y(t+s) ds)^2] \\ &\leq y[\alpha(t) - y(t) - q(t)(\int_{-\tau}^{0} k_{23}(s) y(t+s) ds)^2] \\ &\leq y[\alpha(t) - y(t) - q(t)(\int_{-\tau}^{0} k_{23}(s) \exp(\alpha^U s) ds)^2 y^2(t)]. \end{aligned}$$

Let $u(t)$ be the solution of the auxiliary equation

$$\dot{u} = u[\alpha(t) - u(t) - q(t)(\int_{-\tau}^{0} k_{23}(s) \exp(\alpha^U s) ds)^2 u^2(t)]$$

with the initial condition $u(T_0 + 2\tau) = y(T_0 + 2\tau)$. Then we derive

$$y(t) \leq u(t) \quad \text{for all} \quad t \geq T_0 + 2\tau. \tag{15}$$

From Lemma 3, we know that there is a constant $M_y > 0$ such that

$$\limsup_{t \to \infty} u(t) \leq M_y.$$

Consequently, by (15) we have

$$\limsup_{t \to \infty} y(t) \leq M_y. \tag{16}$$

This completes the proof. □

Lemma 7. *There is a positive constant ρ_x $(\rho_x < M_x)$ such that*

$$\liminf_{t \to \infty} x_i(t) \geq \rho_x.$$

Proof. By Lemma 6, there exists a positive constant $T_1 > T_0 + 2\tau$ such that

$$0 < x_i(t) \leq M_x (i = 1, 2), \; 0 < y(t) \leq M_y, \quad t \geq T_1. \tag{17}$$

Obviously, there exists a constant $\sigma > 0$ such that

$$H_0 \int_{-\infty}^{-\sigma} k_{1i}(s)\, ds < M_y (i = 1, 2),$$

where $H_0 = \sup\{y(t+s) \mid t \geq 0, s \leq 0\}$. Hence, from (13) and (16), for every $t \geq T_1 + \sigma$, we have

$$\begin{aligned}
\dot{x_1} &= a(t)x_2 - b(t)x_1 - d(t)x_1^2 - p_1(t)\frac{x_1^n}{P + x_1^n}\int_{-\infty}^{-\sigma} k_{11}(s)y(t+s)ds \\
&\quad -p_1(t)\frac{x_1^n}{P + x_1^n}\int_{-\sigma}^{0} k_{11}(s)y(t+s)ds \\
&\geq a(t)x_2 - b(t)x_1 - d(t)x_1^2 - p_1(t)H_0\frac{x_1^n}{P + x_1^n}\int_{-\infty}^{-\sigma} k_{11}(s)ds \\
&\quad -p_1(t)M_y\frac{x_1^n}{P + x_1^n}\int_{-\sigma}^{0} k_{11}(s)ds \\
&\geq a(t)x_2 - b(t)x_1 - d(t)x_1^2 - 2M_y p_1(t)\frac{M_x^{n-2}}{P + M_x^n}x_1^2 \\
\dot{x_2} &= c(t)x_1 - f(t)x_2^2 - p_2(t)\frac{x_2^n}{P + x_2^n}\int_{-\infty}^{0} k_{12}(s)y(t+s)ds \\
&\geq c(t)x_1 - f(t)x_2^2 - 2M_y p_2(t)\frac{M_x^{n-2}}{P + M_x^n}x_2^2.
\end{aligned}$$

Consider the following auxiliary system

$$\begin{cases} \dot{u_1} = a(t)u_2 - b(t)u_1 - [d(t) + \frac{2M_x^{n-2}M_y p_1(t)}{P+M_x^n}]u_1^2, \\ \dot{u_2} = c(t)u_1 - [f(t) + \frac{2M_x^{n-2}M_y p_2(t)}{P+M_x^n}]u_2^2. \end{cases} \tag{18}$$

Let $(u_1(t), u_2(t))$ is the solution of system (18) with the initial condition $(u_1(T_1 + \sigma), u_2(T_1 + \sigma) = (x_1(T_1 + \sigma), x_2(T_1 + \sigma))$, then for all $t \geq T_1 + \sigma$,

$$x_i(t) \geq u_i(t).$$

By Lemma 2, (18) has a positive ω-periodic solution $(\widehat{u_1}^*(t), \widehat{u_2}^*(t))$, which is globally asymptotically stable. By the global asymptotic stability of $\widehat{u_i}^*(t) (i = 1, 2)$, for any a sufficiently small $\varepsilon^* (> 0)$, there exists $T_2 > T_1 + \sigma$ such that for all $t \geq T_2$,

$$u_i(t) \geq \widehat{u_i}^*(t) - \varepsilon^*.$$

Hence, for all $t \geq T_2$, $x_i(t) \geq \rho_x \doteq \max_{0 \leq t \leq \omega}\{\widehat{u_i}^*(t) - \varepsilon^*\}$. So we have

$$\liminf_{t \to \infty} x_i(t) \geq \rho_x.$$

So this completes the proof. □

Lemma 8. *There is a positive constant ϱ_y ($\varrho_y < M_y$) such that*

$$\limsup_{t\to\infty} y(t) > \varrho_y. \tag{19}$$

Proof. By (9), we can choose constant $\varepsilon_0 < \frac{1}{2}\min_{t\in[0,\omega]}\{x_i^*(t), i = 1, 2\}$, where $(x_1^*(t), x_2^*(t))$ is the unique positive solution of system (12) such that

$$A_\omega(\psi_{\varepsilon_0}(t)) > 0 \tag{20}$$

where

$$\psi_{\varepsilon_0}(t) = -g(t) + \sum_{i=1}^{2} h_i(t)\int_{-\infty}^{0} k_{2i}(s)\frac{(x_i^*(t+s)-\varepsilon_0)^n}{P+(x_i^*(t+s)-\varepsilon_0)^n}ds - 4q(t)\varepsilon_0^2 - \varepsilon_0.$$

Consider the following equations with a positive parameter μ,

$$\begin{cases} \dot{x_1} = a(t)x_2 - b(t)x_1 - [d(t) + \frac{2\mu M_x^{n-2}p_1(t)}{P+M_x^n}]x_1^2, \\ \dot{x_2} = c(t)x_1 - [f(t) + \frac{2\mu M_x^{n-2}p_2(t)}{P+M_x^n}]x_2^2. \end{cases} \tag{21}$$

By Lemma 2, (21) has a positive ω-periodic solution $(x_{1\mu}^*(t), x_{2\mu}^*(t))$, which is globally asymptotically stable. Let $(x_{1\mu}(t), x_{2\mu}(t))$ be the solution of (21) with initial condition $x_{i\mu}(0) = x_i^*(0)$, where $x^*(t) = (x_1^*(t), x_2^*(t))$is the positive periodic solution of (7). Hence, for the above ε_0, there exists $T_3 > T_2$ such that

$$|x_{i\mu}(t) - x_{i\mu}^*(t)| < \varepsilon_0/4, \quad \text{for} \quad t \ge T_3,\ i = 1, 2. \tag{22}$$

By the continuity of the solution in the parameter, we have $x_{i\mu}(t) \to x_i^*(t)$ ($i = 1, 2$) uniformly in $[T_3, T_3 + \omega]$ as $\mu \to 0$. Hence for $\varepsilon_0 > 0$, there exists $\mu_0 = \mu_0(\varepsilon_0)$ ($0 < \mu_0 < \varepsilon_0$) such that

$$|x_{i\mu}(t) - x_i^*(t)| < \varepsilon_0/4,;\ 0 \le \mu \le \mu_0,\ t \in [T_3, T_3 + \omega], \quad i = 1, 2. \tag{23}$$

Thus from (22) and (23), we get

$$|x_{i\mu}^*(t) - x_i^*(t)| < \varepsilon_0/2,\ 0 \le \mu \le \mu_0,\ t \in [T_3, T_3 + \omega], \quad i = 1, 2.$$

Since $x_{i\mu}^*(t)$ and $x_i^*(t)$ are all ω-periodic, we have

$$|x_{i\mu}^*(t) - x_i^*(t)| < \varepsilon_0/2,\ 0 \le \mu \le \mu_0,\ t \ge 0, i = 1, 2. \tag{24}$$

Choose a constant $\mu_1 (0 < \mu_1 < \mu_0, \mu_1 < \varepsilon_0)$, from (24), we derive

$$x_{i\mu_1}^*(t) \ge x_i^*(t) - \varepsilon_0/2,\ t \ge 0,\ i = 1, 2. \tag{25}$$

Suppose that the condition (19) is not true, then for the above ε_0, there exists $\phi \in C_+$ such that

$$\limsup_{t\to\infty} y(t, \phi) < \mu_1\,,$$

where $(x_1(t, \phi), x_2(t, \phi), y(t, \phi))$ is the solution of (5) with the initial condition

$(x_1(\theta), x_2(\theta), x_3(\theta)) = \phi(\theta))$. So there exists $T_4(> T_3)$ such that

$$y(t, \phi) < \mu_1, \ t \geq T_4. \tag{26}$$

On the other hand, Lemma 6 shows that there exist an enough large constant $T_5(> T_4)$ such that

$$x_i(t, \phi) < M_x, \ t \geq T_5, \ i = 1, 2. \tag{27}$$

Also, from $\int_{-\infty}^{0} k_{ij}(s)ds = 1(i = 1, 2, j = 1, 2, 3)$, we choose a positive constant τ_0 such that

$$H_0 \int_{-\infty}^{-\tau_0} k(s)ds < \mu_1 \tag{28}$$

where $k(t) = k_{11}(t) + k_{12}(t) + k_{21}(t) + k_{22}(t) + k_{23}(t)$ and H_0 is defined in the proof of Lemma 7. For any $t \geq T_5 + \tau_0$, we have

$$\begin{aligned}
\dot{x_1}(t, \phi) &= a(t)x_2(t, \phi) - b(t)x_1(t, \phi) - d(t)x_1^2(t, \phi) \\
&\quad -p_1(t)\frac{x_1^n(t, \phi)}{P + x_1^n(t, \phi)} \int_{-\infty}^{-\sigma_0} k_{11}(s)y(t + s)ds \\
&\quad -p_1(t)\frac{x_1^n(t, \phi)}{P + x_1^n(t, \phi)} \int_{-\sigma_0}^{0} k_{11}(s)y(t + s)ds \\
&\geq a(t)x_2(t, \phi) - b(t)x_1(t, \phi) - d(t)x_1^2(t, \phi) \\
&\quad -p_1(t)H_0\frac{x_1^n(t, \phi)}{P + x_1^n(t, \phi)} \int_{-\infty}^{-\sigma_0} k_{11}(s)ds \\
&\quad -p_1(t)\mu_1\frac{x_1^n(t, \phi)}{P + x_1^n(t, \phi)} \int_{-\sigma_0}^{0} k_{11}(s)ds \\
&\geq a(t)x_2(t, \phi) - b(t)x_1(t, \phi) - d(t)x_1^2(t, \phi) \\
&\quad -2\mu_1 p_1(t)\frac{M_x^{n-2}}{P + M_x^n}x_1^2(t, \phi) \\
\dot{x_2}(t, \phi) &= c(t)x_1(t, \phi) - f(t)x_2^2(t, \phi) \\
&\quad -p_2(t)\frac{x_2^n(t, \phi)}{P + x_2^n(t, \phi)} \int_{-\infty}^{-\sigma_0} k_{12}(s)y(t + s)ds \\
&\quad -p_2(t)\frac{x_2^n(t, \phi)}{P + x_2^n(t, \phi)} \int_{-\sigma_0}^{0} k_{12}(s)y(t + s)ds \\
&\geq c(t)x_1(t, \phi) - f(t)x_2^2(t, \phi) - 2\mu_1 p_2(t)\frac{M_x^{n-2}}{P + M_x^n}x_2^2(t, \phi).
\end{aligned}$$

Let $(u_{1\mu_1}, u_{2\mu_1})$ be the solution of (21) with $\mu = \mu_1$ and $(u_{1\mu_1}(T_5 + \tau_0), u_{2\mu_1}(T_5 + \tau_0)) = (x_1(T_5 + \tau_0), x_2(T_5 + \tau_0))$, then by the vector comparison theorem, we obtain

$$x_i(t,\phi) \geq u_{i\mu_1}(t), \quad i = 1,2, \quad t \geq T_5 + \tau_0. \tag{29}$$

By the global asymptotic stability of $(x^*_{1\mu_1}(t), x^*_{2\mu_1}(t))$, for the given $\varepsilon_0 > 0$ there exists $T_6 > T_5 + \tau_0$ such that

$$u_{i\mu_1}(t) > x^*_{i\mu_1}(t) - \varepsilon_0/2, \quad t \geq T_6, \quad i = 1,2$$

and hence, by (25), we derive

$$x_i(t,\phi) > x^*_i(t) - \varepsilon_0, \; t \geq T_6, \; i = 1,2. \tag{30}$$

Therefore, for $t \geq T_6 + \tau_0$, we have

$$\begin{aligned}
\dot{y(t,\phi)} &= y(t,\phi)[-g(t) + \sum_{i=1}^{2} h_i(t)\int_{-\infty}^{0} k_{2i}(s)\frac{(x^*_i(t+s,\phi))^n}{P + (x^*_i(t+s,\phi))^n}ds \\
&\quad -y(t,\phi) - q(t)(\int_{-\infty}^{0} k_{23}(s)y(t+s,\phi)ds)^2] \\
&\geq y(t,\phi)[-g(t) + \sum_{i=1}^{2} h_i(t)\int_{-\infty}^{0} k_{2i}(s)\frac{(x^*_i(t+s,\phi)-\varepsilon_0)^n}{P + (x^*_i(t+s,\phi)-\varepsilon_0)^n}ds \\
&\quad -\mu_1 - 4q(t)\mu_1^2] \\
&\geq y(t,\phi)[-g(t) + \sum_{i=1}^{2} h_i(t)\int_{-\infty}^{0} k_{2i}(s)\frac{(x^*_1(t+s,\phi)-\varepsilon_0)^n}{P + (x^*_i(t+s,\phi)-\varepsilon_0)^n}ds \\
&\quad -\varepsilon_0 - 4q(t)\varepsilon_0^2] \geq y(t,\phi)\psi_{\varepsilon_0}(t).
\end{aligned}$$

Integrating the above inequality from $T_6 + \tau_0$ to t yields

$$y(t,\phi) \geq y(T_6+\tau_0)\exp(\int_{T_6+\tau_0}^{t} \psi_{\varepsilon_0}(s)ds).$$

By (20) we know that $y(t,\phi) \to \infty$ as $t \to \infty$, which is a contradiction. This completes the proof. □

Lemma 9. *Assume that (9) holds, there exists a positive constant δ_y ($\delta_y < M_y$) such that any solution (x_1, x_2, y) of system (5) with initial condition satisfies*

$$\liminf_{t\to\infty} y(t) \geq \delta_y. \tag{31}$$

Proof. Suppose that (31) is not true, there must exist a sequence $\{\phi_k\} \subset C_+$ such that

$$\liminf_{t\to\infty} y(t,\phi_k) < \frac{\varrho_y}{(k+1)^2}, \quad k = 1,2,\ldots$$

and by Lemma 8, we have $\limsup_{t\to\infty} y(t,\varphi_k) > \varrho_y$, $k = 1, 2, \dots$. Hence, for each k, we choose two time sequences $\{s_q^{(k)}\} and \{t_q^{(k)}\}$, satisfying $0 < s_1^{(k)} < t_1^{(k)} < s_2^{(k)} < t_2^{(k)} < \cdots < s_q^{(k)} < t_q^{(k)} < \dots$ and $s_q^{(k)} \to \infty$ as $q \to \infty$, and

$$y(s_q^{(k)}, \phi_k) = \frac{\varrho_y}{k+1}, \quad y(t_q^{(k)}, \phi_k) = \frac{\varrho_y}{(k+1)^2}, \tag{32}$$

$$\frac{\varrho_y}{(k+1)^2} < y(t, \phi_k) < \frac{\varrho_y}{k+1}, \quad t \in (s_q^{(k)}, t_q^{(k)}). \tag{33}$$

By Lemma 6, for a given positive integer k, there exists $\widetilde{T}^{(k)} > 0$ such that $x_i(t, \phi_k) \le M_x$ $(i = 1, 2)$ and $y(t, \phi_k) \le M_y$ for all $t \ge \widetilde{T}^{(k)}$. Further, there is a constant $\sigma^{(k)} > 0$ such that

$$H_1^{(k)} \int_{-\infty}^{-\sigma^{(k)}} k(s)\, ds < M_y,$$

where $H_1^{(k)} = \sup\{y(t + s, \phi_k) : t \ge 0, s \le 0\}$. Because of $s_q^{(k)} \to \infty$ as $q \to \infty$, there is a positive integer $K_1^{(k)}$ such that $s_q^{(k)} > \widetilde{T}^{(k)} + \sigma^{(k)}$ as $q \ge K_1^{(k)}$. For any $t \ge \widetilde{T}^{(k)} + \sigma^{(k)}$, we have

$$\begin{aligned}
\dot{y}(t, \phi_k) &\ge y(t, \phi_k)[-g(t) - y(t, \phi_k) - q(t)(\int_{-\infty}^{0} k_{23}(s) y(t + s, \phi_k) ds)^2] \\
&\ge y(t, \phi_k)[-g(t) - y(t, \phi_k) - q(t)(\int_{-\infty}^{-\sigma^{(k)}} k_{23}(s) y(t + s, \phi_k) ds \\
&\quad + \int_{-\sigma^{(k)}}^{0} k_{23}(s) y(t + s, \phi_k) ds)^2] \\
&\ge y(t, \phi_k)[-g(t) - M_y - 4M_y^2 q(t)].
\end{aligned}$$

Integrating the above inequality from $s_q^{(k)}$ to $t_q^{(k)}$, for any $q \ge K_1^{(k)}$ we get

$$y(t_q^{(k)}, \phi_k) \geq y(s_q^{(k)}, \phi_k) \exp(\int_{s_q^{(k)}}^{t_q^{(k)}} [-g(t) - M_y - 4M_y^2 q(t)]dt).$$

Obviously, we derive

$$\int_{s_q^{(k)}}^{t_q^{(k)}} [g(t) + M_y + 4M_y^2 q(t)]dt \geq \ln(k+1) \ for \ q \geq K_1^{(k)}.$$

Hence, in view of the periodicity of $g(t)$ and $q(t)$, we get

$$t_q^{(k)} - s_q^{(k)} \to \infty, \text{ as } k \to \infty, \ q \geq K_1^{(k)}. \tag{34}$$

By (20), (32) and (34), there are positive constants T and N_0 such that

$$y(s_q^{(k)}, \phi_k) = \frac{\varrho_y}{k+1} < \varepsilon_0, \tag{35}$$

$$t_q^{(k)} - s_q^{(k)} > 2T, \tag{36}$$

and

$$\int_0^{\kappa} \psi_{\varepsilon_0}(t)dt > 0, \tag{37}$$

for $k \geq N_0, \ q \geq K_1^{(k)}, \ and \ \kappa > T$. (35) implies that

$$y(t, \phi_k) < \varepsilon_0, \ t \in [s_q^{(k)}, t_q^{(k)}], \tag{38}$$

for $k \geq N_0, \ q \geq K_1^{(k)}$. Noticing that $s_q^{(k)} \to \infty$ as $q \to \infty$ and $\int_{-\infty}^0 k_{ij}(s)ds = 1 (i, j = 1, 2)$, for any k there exists $K_2^{(k)} > K_1^{(k)}$ such that for all $q > K_2^{(k)}$, we obtain

$$H_1^{(k)} \int_{-\infty}^{\widetilde{T}^{(k)} - s_q^{(k)} - \sigma^0} k(s)ds < \frac{1}{2}\varepsilon_0 \tag{39}$$

and

$$M_y \int_{-\infty}^{-\sigma^0} k(s)ds < \frac{1}{2}\varepsilon_0\,, \tag{40}$$

where $\sigma^0 >$ and $k(t) = k_{12} + k_{21}(t) + k_{22}(t)$. By (34), there exists a positive integer N_1 such that

$$t_q^{(k)} - s_q^{(k)} > \sigma^0, \text{ for } k > N_1, \ q \geq K_2^{(k)}.$$

For $k > N_1, \ q \geq K_2^{(k)}$ and $s_q^{(k)} + \sigma^0 \leq t \leq t_q^{(k)}$, from (38)-(40), we have

$$\begin{aligned} \frac{dx_1(t, \phi_k)}{dt} &= a(t)x_2(t, \phi_k) - b(t)x_1(t, \phi_k) - d(t)x_1^2(t, \phi_k) \\ &\quad - \frac{p_1(t)x_1^n(t, \phi_k)}{P + x_1^n(t, \phi_k)} \int_{-\infty}^{\widetilde{T}^{(k)}} k_{11}(u-t)y(u, \phi_k)\,du \end{aligned}$$

$$
\begin{aligned}
& -\frac{p_1(t)x_1^n(t,\phi_k)}{P+x_1^n(t,\phi_k)}\int_{\widetilde{T}^{(k)}}^{s_q^{(k)}} k_{11}(u-t)y(u,\phi_k)\,du \\
& -\frac{p_1(t)x_1^n(t,\phi_k)}{P+x_1^n(t,\phi_k)}\int_{s_q^{(k)}}^{t} k_{11}(u-t)y(u,\phi_k)du \\
\geq\ & a(t)x_2(t,\phi_k)-b(t)x_1(t,\phi_k)-d(t)x_1^2(t,\phi_k) \\
& -\frac{p_1(t)x_1^n(t,\phi_k)}{P+x_1^n(t,\phi_k)}H_1^{(k)}\int_{-\infty}^{\widetilde{T}^{(k)}-t} k_{11}(s)\,ds \\
& -\frac{p_1(t)x_1^n(t,\phi_k)}{P+x_1^n(t,\phi_k)}M_y\int_{-\infty}^{s_q^{(k)}-t} k_{11}(s)\,ds \\
& -\frac{p_1(t)x_1^n(t,\phi_k)}{P+x_1^n(t,\phi_k)}\varepsilon_0\int_{-\infty}^{0} k_{11}(s)\,ds \\
\geq\ & a(t)x_2(t,\phi_k)-b(t)x_1(t,\phi_k)-d(t)x_1^2(t,\phi_k) \\
& -\frac{p_1(t)M_x^{n-2}x_1^2(t,\phi_k)}{P+M_x^n}H_1^{(k)}\int_{-\infty}^{\widetilde{T}^{(k)}-t} k_{11}(s)\,ds \\
& -\frac{p_1(t)M_x^{n-2}x_1^2(t,\phi_k)}{P+M_x^n}M_y\int_{-\infty}^{s_q^{(k)}-t} k_{11}(s)\,ds \\
& -\frac{p_1(t)_1M_x^{n-2}x_1^2(t,\phi_k)}{P+M_x^n}\varepsilon_0\int_{-\infty}^{0} k_{11}(s)\,ds \\
=\ & a(t)x_2(t,\phi_k)-b(t)x_1(t,\phi_k)-[d(t)+\frac{2\varepsilon_0p_1(t)M_x^{n-2}}{P+M_x^n}]x_1^2(t,\phi_k), \\
\frac{dx_2(t,\phi_k)}{dt} \geq\ & c(t)x_1(t,\phi_k)-[f(t)+\frac{2\varepsilon_0p_2(t)M_x^{n-2}}{P+M_x^n}]x_2^2(t,\phi_k).
\end{aligned}
$$

Let $(u_{1\varepsilon_0},u_{2\varepsilon_0})$ be the solution of (21) with $\mu=\varepsilon_0$ and $(u_{1\mu_1}(s_q^{(k)}+\tau^0), u_{2\mu_1}(s_q^{(k)}+\tau^0))=(x_1(s_q^{(k)}+\tau^0),x_2(s_q^{(k)}+\tau^0))$, then by the vector comparison theorem, we obtain

$$x_i(t,\phi_k)\geq u_{i\varepsilon_0}(t),\quad i=1,2,\ t\in[s_q^{(k)}+\tau^0,t_q^{(k)}]. \tag{41}$$

From $\lim_{q\to\infty}s_q^{(k)}=\infty$ and Lemma 6 and Lemma 7, we obtain that for any k there is a $K_3^{(k)}>K_2^{(k)}$ such that for any $q\geq K_3^{(k)}$,

$$\rho_x\leq x_i(s_q^{(k)}+\sigma^0,\phi_k)\leq M_x,\quad i=1,2.$$

For $\mu=\varepsilon_0$, equation (21) has a globally asymptotically stable positive ω-periodic solution $(x_{1\mu}^*(t),x_{2\mu}^*(t))$. From the periodicity of (21) we know that the periodic solution $(x_{1\mu}^*(t),x_{2\mu}^*(t))$ also is globally uniformly asymptotically stable. Hence, there is a $T_7 > T$, and T_7 is independent of any k and q, such that

$$u_{i\varepsilon_0}(t) > x^*_{i\mu}(t) - \frac{\varepsilon_0}{2}$$

for all $t \geq T_7 + s_q^{(k)} + \sigma^0$ and $q \geq K_3^{(k)}$. Consequently, by (25),

$$u_{i\varepsilon_0}(t) > x^*_i(t) - \varepsilon_0,\ i = 1,2 \tag{42}$$

for all $t \geq T_7 + s_q^{(k)} + \sigma^0$ and $q \geq K_3^{(k)}$. By (34), there is a $N_2 \geq N_1$ such that $t_q^{(k)} - s_q^{(k)} \geq 2T$ for all $k \geq N_2$ and $q \geq K_3^{(k)}$, where $T \geq T_7 + \sigma^0$. Hence, from (41) and (42) we obtain

$$x_i(t, \phi_k) \geq x^*_i(t) - \varepsilon_0, \tag{43}$$

for all $t \in [T + s_q^{(k)}, t_q^{(k)}]$, $k \geq N_2$ and $q \geq K_3^{(k)}$. Since, for any $t \in [T + s_q^{(k)} + \sigma_0, t_q^{(k)}]$, $k \geq N_2$ and $q \geq K_3^{(k)}$, by (5), (39) and (40), we have

$$\begin{aligned}
\frac{dy(t,\phi_k)}{dt} \geq\ & y(t,\phi_k)[-g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\sigma^0}^{0} k_{2i}(s) \frac{x_i^n(t+s,\phi_k)}{P + x_1^n(t+s,\phi_k)}\, ds \\
& -y(t,\phi_k) - q(t)(\int_{-\infty}^{\widetilde{T}^{(k)}} k_{23}(u-t) y(u,\phi_k) du \\
& + \int_{\widetilde{T}^{(k)}}^{s_q^{(k)}} k_{23}(u-t) y(u,\phi_k) du \\
& + \int_{s_q^{(k)}}^{t} k_{23}(u-t) y(u,\phi_k) du)^2] \\
\geq\ & y(t,\phi_k)[-g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\sigma^0}^{0} k_{2i}(s) \frac{x_i^n(t+s,\phi_k)}{P + x_i^n(t+s,\phi_k)}\, ds \\
& -y(t,\phi_k) - q(t)(H_1^{(k)} \int_{-\infty}^{\widetilde{T}^{(k)}-t} k_{23}(s)\, ds + M_y \int_{-\infty}^{s_q^{(k)}-t} k_{23}(s)\, ds \\
& +\varepsilon_0 \int_{-\infty}^{0} k_{23}(s)\, ds)^2] \\
\geq y(t,\phi_k)[-g(t) & + \sum_{i=1}^{2} h_i(t) \int_{-\sigma^0}^{0} k_{2i}(s) \frac{(x_i^*(t+s,\phi_k) - \varepsilon_0)^n}{P + (x_1^*(t+s,\phi_k) - \varepsilon_0)^n}\, ds \\
& -\varepsilon_0 - 4q(t)\varepsilon_0^2] = y(t,\phi_k)\psi_{\varepsilon_0}(t)\,.
\end{aligned}$$

Integrating from $T+s_q^{(k)}+\sigma^0$ to $t_q^{(k)}$ for any $k \geq N_2$ and $q \geq K_3^{(k)}$ we obtain

$$y(t_q^{(k)},\phi_k) \geq y(T+s_q^{(k)}+\sigma^0,\phi_k)\exp\int_{T+s_q^{(k)}+\sigma^0}^{t_q^{(k)}} \psi_{\varepsilon_0}(t)dt.$$

Hence, by (32) and (33) we finally have

$$\frac{\varrho_y}{(k+1)^2} \geq \frac{\varrho_y}{(k+1)^2}\exp\int_{T+s_q^{(k)}+\sigma_0}^{t_q^{(k)}} \psi_{\varepsilon_0}(t)dt > \frac{\varrho_y}{(k+1)^2}.$$

This leads to a contradiction. This completes the proof. □

Proof of Theorem 4. This theorem now follows from Lemmas 6, 7, 8, 9. □

Proof of Theorem 5. Assume

$$\int_0^\omega [-g(t)+\sum_{i=1}^2 h_i(t)\int_{-\infty}^0 k_{2i}(s)\frac{x_i^{*^n}(t+s)}{P+x_i^{*n}(t+s)}ds]dt \leq 0.$$

We will show that $\lim_{t\to\infty} y(t)=0$. In fact, we know that for any given $0<\varepsilon<1$, there exist $\epsilon<\varepsilon$ and $\epsilon_0>0$ such that

$$\int_0^\omega [-g(t)+\sum_{i=1}^2 h_i(t)\int_{-\infty}^0 k_{2i}(s)\frac{(x_i^*(t+s)+\epsilon)^n}{P+(x_i^*(t+s)+\epsilon)^n}ds+h(t)\epsilon - \frac{1}{2}q(t)l\epsilon^2]dt < -\epsilon_0, \quad (44)$$

where $l=\int_{-\infty}^0 k_{22}(s)\exp(\lambda^U s)\,ds$. Since

$$\begin{cases} \dot{x}_1 \leq a(t)x_2-b(t)x_1-d(t)x_1^2, \\ \dot{x}_2 \leq c(t)x_1-f(t)x_2^2, \end{cases}$$

for all $t\geq 0$. Let $(\overline{x}_1(t),\overline{x}_2(t))$ be the solution of (7) with initial condition $\overline{x}_i(0)=x_i(0)$ $(i=1,2)$. By the vector comparison theorem we obtain $x_i(t)\leq \overline{x}_i(t)$ $(i=1,2)$, $t\geq 0$. Obviously, by the global asymptotic stability of $x^*(t)$, there is a $\overline{T}$, for all $t\geq\overline{T}$, we have

$$x_i(t) \leq x_i^*(t)+\epsilon \quad (i=1,2). \quad (45)$$

Choose a constant $\tau_1>0$ such that

$$\int_{-\infty}^{-\tau_1} k(s)\,ds < \epsilon, \quad (46)$$

$$\int_{-\tau_1}^0 k_{23}(s)\exp(\lambda^U s)\,ds > \sqrt{\frac{l}{2}}. \quad (47)$$

For any $t \geq \overline{T} + \tau_1$, by (45) and (46) we have

$$\begin{aligned}
\dot{y} &\leq y[-g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\infty}^{0} k_{2i}(s) \frac{(x_i^*(t+s))^n}{P + (x_i^*(t+s))^n} ds + h(t)\epsilon] \\
&\leq y[-g(t) + \sum_{i=1}^{2} h_i(t) \int_{-\tau_1}^{0} k_{2i}(s) \frac{(x_i^*(t+s)+\epsilon)^n}{P + (x_i^*(t+s)+\epsilon)^n} ds + h(t)\epsilon] \\
&\leq y\lambda(t).
\end{aligned} \tag{48}$$

Hence, by (47), for any $t \geq t + s \geq \overline{T} + \tau_1$ we obtain

$$\begin{aligned}
\dot{y} &\leq y\big[\lambda(t) - q(t)(\int_{-\tau_1}^{0} k_{23}(s) y(t+s)\, ds)^2\big] \\
&\leq y\big[\lambda(t) - q(t)(\int_{-\tau_1}^{0} k_{23}(s) \exp(\lambda^U s)\, ds)^2\, y^2\big] \\
&\leq y\big[\lambda(t) - \frac{1}{2} l q(t) y^2\big].
\end{aligned}$$

If $y(t) \geq \epsilon$ for all $t \geq \overline{T} + 2\tau_1$, then we have

$$\dot{y} \leq y[\lambda(t) - \frac{1}{2} l q(t) \epsilon^2]. \tag{49}$$

Consequently, by (44) we obtain

$$y(t) \leq y(\overline{T} + 2\tau_1) \exp \int_{\overline{T}+2\tau_1}^{t} [\lambda(s) - \frac{1}{2} l q(s) \epsilon^2] ds \to 0$$

as $t \to \infty$, which leads to a contradiction. Hence, there is a $t_1 \geq \overline{T} + 2\tau_1$ such that $y(t_1) < \epsilon$.

Let $M(\epsilon) = \max_{t \geq 0}\{|\lambda(t)| + \frac{1}{2} l q(t) \epsilon^2\}$. We know that $M(\epsilon)$ is bounded for $\epsilon \in [0, 1]$. We then show that

$$y(t) \leq \epsilon \exp(M(\epsilon)\omega), \quad t \geq t_1. \tag{50}$$

Otherwise, there are $t_3 > t_2 > t_1$ such that $y(t_3) > \epsilon \exp(M(\epsilon)\omega)$, $y(t_2) = \epsilon$ and $y(t) > \epsilon$ for all $t \in (t_2, t_3]$. Let $\theta \geq 0$ be an integer such that $t_3 \in (t_2 + \theta\omega, t_2 + (\theta+1)\omega]$. Then from (49) we have

$$\begin{aligned}
\epsilon \exp(M(\epsilon)\omega) &< y(t_3) \\
&\leq y(t_2) \exp \int_{t_2}^{t_3} [\lambda(t) - \frac{1}{2} l q(t) \epsilon^2] dt \\
&= \epsilon \exp(\int_{t_2}^{t_2+\theta\omega} + \int_{t_2+\theta\omega}^{t_3})[\lambda(t) - \frac{1}{2} l q(t) \epsilon^2] dt
\end{aligned}$$

$$
\begin{aligned}
&< \epsilon \exp\Big(\int_{t_2+\theta\omega}^{t_3} [\lambda(t) - \frac{1}{2} l q(t)\epsilon^2] dt\Big) \\
&\leq \epsilon \exp(M(\epsilon)\omega).
\end{aligned}
$$

This leads to a contradiction. Hence, inequality (50) holds. Further, by the arbitrariness of ϵ we obtain $y(t) \to 0$ as $t \to \infty$. This completes the proof. □

4. Example

Example 1.1.

$$
\begin{cases}
\dot{x_1} = \frac{2}{2+\sin t} x_2 - x_1 - x_1^2 - \frac{5x_1^3}{1+x_1^3} \int_{-\infty}^0 k_{11}(s) y(t+s) ds, \\
\dot{x_2} = 2x_1 - \frac{2-\cos t}{(2+\sin t)^2} x_2^2 - \frac{x_1^3}{1+x_1^3} \int_{-\infty}^0 k_{12}(s) y(t+s) ds, \\
\dot{y} = y\big[-(2 + \frac{\cos t}{2}) + (4+\cos t) \int_{-\infty}^0 k_{21}(s) \frac{x_1^3(t+s)}{1+x_1^3(t+s)} ds \\
+ (\frac{2-\sin t}{3}) \int_{-\infty}^0 k_{22}(s) \frac{x_2^3(t+s)}{1+x_2^3(t+s)} ds - y - (\int_{-\infty}^0 k_{23}(s) y(t+s) ds)^2 \big].
\end{cases}
\qquad (51)
$$

Example 1.2

$$
\begin{cases}
\dot{x_1} = \frac{2}{2+\sin t} x_2 - x_1 - x_1^2 - \frac{5x_1^3}{1+x_1^3} \int_{-\infty}^0 k_{11}(s) y(t+s) ds, \\
\dot{x_2} = 2x_1 - \frac{2-\cos t}{(2+\sin t)^2} x_2^2 - \frac{2x_1^3}{1+x_1^3} \int_{-\infty}^0 k_{12}(s) y(t+s) ds, \\
\dot{y} = y\big[-3 + 4 \int_{-\infty}^0 k_{21}(s) \frac{x_1^3(t+s)}{1+x_1^3(t+s)} ds + \int_{-\infty}^0 k_{22}(s) \frac{x_2^3(t+s)}{1+x_2^3(t+s)} ds \\
\qquad - y - (\int_{-\infty}^0 k_{23}(s) y(t+s) ds)^2 \big].
\end{cases}
\qquad (52)
$$

We consider the subsystem of (51) and (52):

$$
\begin{cases}
\dot{x_1} = \frac{2}{2+\sin t} x_2 - x_1 - x_1^2, \\
\dot{x_2} = 2x_1 - \frac{2-\cos t}{(2+\sin t)^2} x_2^2.
\end{cases}
\qquad (53)
$$

Obviously, (53) admits an unique positive 2π-periodic solution $(1, 2+\sin t)$. By simple computation, we derive system (51) is permanent, and $y(t) \to 0$ *as* $t \to \infty$ in system (52).

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