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ON THE ADJACENT STRONG EDGE COLORING OF $F_n \bigvee P_n \bigvee P_n$

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Abstract: A k-proper edge coloring of a graph G is called k-adjacent strong edge coloring, if it is satisfied with $C(u) \neq C(v)$ for $uv \in E(G)$, where $C(u) = \{f(uv)|uv \in E(G)\}$, then f is called k-adjacent strong edge coloring of G, which is abbreviated $k-$ASEC of G, and the adjacent strong edge chromatic number of G, denoted by $\chi'_{as}(G)$, is the minimal number of colors in an adjacent strong edge coloring of G. In this paper, the adjacent strong edge coloring of $F_n \bigvee P_n \bigvee P_n$ were obtained.

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1. Introduction

The graph coloring is one of the chief topics in graph research. The four-color conjecture is firstly brought up in vertex coloring, which develops the research work in graph theory. Later on, based on many theoretical and practical problems, numbers of mathematical experts began to study total coloring, adjacent vertex distinguishing total coloring, list coloring and vertex distinguishing edge coloring (see [1]-[10]).

Definition 1. (see [1]) For a simple graph $G(V, E)$, if it exists a mapping $f : E(G) \to \{1, 2, \cdots, k\}$, and it is satisfied with $f(e) \neq f(e')$ for $\forall \;\; e \neq e'$,

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where $e, e^{'}$ are adjacent edges, then f is called k-proper edge coloring of G, which is abbreviated $k-$PEC of G.

Definition 2. (see [5]) For a simple graph $G(V, E)$ with no isolated edges, if a proper k-edge coloring f satisfies $C(u) \neq C(v)$ for $uv \in E(G)$, then f is called k-adjacent strong edge coloring of G, which is abbreviated $k-$ASEC of G, and

$$\chi'_{as}(G) = \min\{k \big| \ G \text{ has a } k - \text{ASEC}\}$$

is called the adjacent strong edge chromatic number of G, where $C(u) = \{f(uv) \big| uv \in E(G)$.

Obviously, for simple graphs with no isolated edges, $\chi'_{as}(G)$ exists.

Conjecture 1. (see [5]) *For simple connected graph G with $\mid V(G) \mid \geq 3$, if $G \neq C_5$(5-circle), then*

$$\chi'_{as}(G) \leq \Delta(G) + 2\,,$$

where $\Delta(G)$ is the maximum degree of G.

Let G and H are two simple graphs, the joint graph of G and H, denote by $G \vee V$, is obtained from the disjoint union of G and H by making all of $V(G)$ adjacent to all of $V(H)$.

The other terminology can be found in [6, 8].

2. Main Results

Lemma 1. (see [5]) *Suppose $n \geq 3$, then*

$$\chi'_{as}(K_n) = \begin{cases} n\,, & \text{if } n \equiv 1 \pmod 2\,, \\ n+1\,, & \text{if } n \equiv 0 \pmod 2\,. \end{cases}$$

Here K_n denotes complete graph with order n.

Lemma 2. (see [5]) *Suppose G is connected graph, $uv \in E(G)$ and $d(u) = d(v) = \Delta(G) \geq 2$, then $\chi'_{as}(G) \geq \Delta(G) + 1$.*

Because $F_1 \bigvee P_1 \bigvee P_1 = K_4$ and $F_2 \bigvee P_2 \bigvee P_2 = K_7$.

From the Lemma, $\chi'_{as}(F_1 \bigvee P_1 \bigvee P_1) = 5$, $\chi'_{as}(F_2 \bigvee P_2 \bigvee P_2) = 7$.

Theorem 1. *For n=3, then $\chi'_{as}(F_3 \bigvee P_3 \bigvee P_3) = 10$.*

Proof. Because $\Delta(F_3 \bigvee P_3 \bigvee P_3) = 9$ and there are connected points with maximum degree, so $\chi'_{as}(F_3 \bigvee P_3 \bigvee P_3) \geq 10$, so we need only prove that $F_3 \bigvee P_3 \bigvee P_3$ has a 10-ASEC.

Suppose $V(F_3 \bigvee P_3 \bigvee P_3) = \{w_0, w_1, w_2, w_3, v_1, v_2, v_3, u_1, u_2, u_3\}$; $E(F_3) = \{w_0 w_i | i = 1, 2, 3\} \cup \{w_i w_{i+1} | i = 1, 2\}$; one $E(P_3) = \{v_i v_{i+1} | i = 1, 2\}$; another $E(P_3) = \{u_i u_{i+1} | i = 1, 2\}$.

We define a mapping f as follows:

$f(w_iv_j) = i + j - 1, (i, j = 1, 2, 3)$; $f(w_iu_j) = i + j + 2, (i, j = 1, 2, 3)$; $f(v_iu_j) = i + j + 5(\text{mod}\,(10)), (i, j = 1, 2, 3)$; $f(w_0w_i) = i + 6, (i = 1, 2, 3)$; $f(w_1w_2) = 0$; $f(w_2, w_3) = 1$; $f(v_1v_2) = 6$; $f(v_2v_3) = 7$; $f(u_1u_2) = 2$; $f(u_2u_3) = 3$.

Obviously, f is 10-ASEC of $F_3 \bigvee P_3 \bigvee P_3$. □

Theorem 2. *For $n \geq 4$, $\chi'_{as}(F_n \bigvee P_n \bigvee P_n) = 3n$.*

Proof. Suppose $C = \{\alpha_0, \alpha_1, \cdots, \alpha_{n-1}, \beta_1, \cdots, \beta_{n-1}, \gamma_1, \cdots, \gamma_{n-1}\}$.

$$V(F_n) = \{w_i | i = 0, 1, 2, \cdots, n\};$$

$$E(F_n) = \{w_0w_i | i = 0, 1, 2, \cdots, n\} \bigcup \{w_iw_{i+1} | i = 1, 2, \cdots, n-1\};$$

one $V(P_n) = \{v_i | i = 0, 1, 2, \cdots, n\}$; another $V(P_n) = \{u_i | i = 0, 1, 2, \cdots, n\}$; one $E(P_n) = \{v_iv_{i+1} | i = 1, 2, \cdots, n-1\}$ another $E(P_n) = \{u_iu_{i+1} | i = 1, 2, \cdots, n-1\}$.

Obviously, we need only to prove that $F_n \bigvee P_n \bigvee P_n$ has a $3n$-ASEC.

We define a mapping f as follows:

$f(w_iv_j) = \alpha_{i+j-1(\text{mod }n)}, i = 0, 1, \cdots, n-1; j = 1, 2, \cdots, n$.

$f(w_nv_i) = \beta_i, i = 1, 2, \cdots, n$.

$f(w_iu_j) = \beta_{i+j-1(\text{mod }n)}, i = 0, 1, \cdots, n-1; j = 1, 2, \cdots, n$.

$f(w_nu_i) = \alpha_i, i = 1, 2, \cdots, n$.

$f(v_iu_j) = \gamma_{i+j-1(\text{mod }n)}, i, j = 1, 2, \cdots, n$.

$f(w_0w_i) = \gamma_{i-1}, i = 1, 2, \cdots, n$.

$f(w_iw_{i+1}) = \gamma_{i+1(\text{mod }n)}, i = 1, 2, \cdots, n-1$.

$f(v_iv_{i+1}) = \beta_{i+1(\text{mod }n)}, i = 1, 2, \cdots, n-1$.

$f(u_iu_{i+1}) = \alpha_{i+1(\text{mod }n)}, i = 1, 2, \cdots, n-1$.

Obviously, f is a $3n$-ASEC of $F_n \bigvee P_n \bigvee P_n$ $(n \geq 4)$. So Theorem 2 is true. □

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References

[1] J.A. Bondy, U.S.R. Murty, *Graph Theory with Applications*, The Macmillan Press, New York (1976).

[2] Li Jingwen, Yao Bing, Cheng Hui, Zhang Zhongfu, Adjacent vertex distinguishing edge chromatic number of C_mVK_n, *Journal of Lanzhou University*, **41**, No. 1 (2005), 96-98.

[3] Bao Shitang, Ren Zhiguo, Zhao Chuanchen, Zhang Zhongfu, The edge chromatic number and adjacent strong edge chromatic number of graph $F_m\nabla W_n$, *Journal of Northwest Normal University*, **41**, No. 3 (2005), 22-23.

[4] Chen Xiang'en, Zhang Zhongfu, Adjacent vertex distinguishing total coloring on P_mVPn, *Journal of Northwest Normal University*, **41**, No. 1 (2005), 13-15.

[5] Zhang Zhongfu, Liu Linzhong, Wang Jianfang, Adjacent strong edge coloring of graph, *Applied Mathematics Letters*, **15**, No. 5 (2002), 623-626.

[6] Zhang Zhongfu, Chen Xiang'en, Li Jingwen, On the adjacent vertex-distinguishing total coloring of graphs, *Science in China*, **34**, No. 5 (2004), 574-583.

[7] Qiang Huiying, On the adjacent strong edge coloring of $C_n^2, C_n^3(n \equiv 0 (\mathrm{mod}\, 5))$ and $C_n^{(3)}(n \equiv 0(\mathrm{mod}\, 7))$ graph, *International Journal of Pure and Applied Mathematics*, **26**, No. 1 (2006), 127-133.

[8] Zhang Zhongfu, Li Jingwen, et al, $D(\beta)$-vertex distinguishing proper edge colorings of graphs, *Acta Mathematica Sinica, Chinese Series*, **49**, No. 3 (2006), 703-708.

[9] A. C. Burres, R. H. Schelp, Vertex distinguishing proper edge colourings, *J. Graph Theory*, **26**, No. 2 (1997), 70-82.

[10] Wang Weifan, Lih Ko-Wei, Equitable list coloring of graphs, *Taiwanese Journal of Mathematics*, **8**, No. 4 (2004), 747-759.