ON THE ADJACENT STRONG EDGE COLORING
OF \( F_n \vee P_n \vee P_n \)

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Abstract: A \( k \)-proper edge coloring of a graph \( G \) is called \( k \)-adjacent strong edge coloring, if it is satisfied with \( C(u) \neq C(v) \) for \( uv \in E(G) \), where \( C(u) = \{ f(uv) | uv \in E(G) \} \), then \( f \) is called \( k \)-adjacent strong edge coloring of \( G \), which is abbreviated \( k \)-ASEC of \( G \), and the adjacent strong edge chromatic number of \( G \), denoted by \( \chi'_{as}(G) \), is the minimal number of colors in an adjacent strong edge coloring of \( G \). In this paper, the adjacent strong edge coloring of \( F_n \vee P_n \vee P_n \) were obtained.

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1. Introduction

The graph coloring is one of the chief topics in graph research. The four-color conjecture is firstly brought up in vertex coloring, which develops the research work in graph theory. Later on, based on many theoretical and practical problems, numbers of mathematical experts began to study total coloring, adjacent vertex distinguishing total coloring, list coloring and vertex distinguishing edge coloring (see [1]-[10]).

Definition 1. (see [1]) For a simple graph \( G(V,E) \), if it exists a mapping \( f : E(G) \to \{ 1, 2, \cdots, k \} \), and it is satisfied with \( f(e) \neq f(e') \) for \( \forall \; e \neq e' \),

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where \(e, e'\) are adjacent edges, then \(f\) is called \(k\)-proper edge coloring of \(G\), which is abbreviated \(k\)-PEC of \(G\).

**Definition 2.** (see [5]) For a simple graph \(G(V, E)\) with no isolated edges, if a proper \(k\)-edge coloring \(f\) satisfies \(C(u) \neq C(v)\) for \(uv \in E(G)\), then \(f\) is called \(k\)-adjacent strong edge coloring of \(G\), which is abbreviated \(k\)-ASEC of \(G\), and

\[
\chi'_{as}(G) = \min\{k \mid G \text{ has a } k \text{-ASEC}\}
\]

is called the adjacent strong edge chromatic number of \(G\), where \(C(u) = \{f(uv) \mid uv \in E(G)\}\).

Obviously, for simple graphs with no isolated edges, \(\chi'_{as}(G)\) exists.

**Conjecture 1.** (see [5]) For simple connected graph \(G\) with \(|V(G)| \geq 3\), if \(G \neq C_5(5\text{-circle})\), then

\[
\chi'_{as}(G) \leq \Delta(G) + 2,
\]

where \(\Delta(G)\) is the maximum degree of \(G\).

Let \(G\) and \(H\) are two simple graphs, the joint graph of \(G\) and \(H\), denote by \(G \lor V\), is obtained from the disjoint union of \(G\) and \(H\) by making all of \(V(G)\) adjacent to all of \(V(H)\).

The other terminology can be found in [6, 8].

### 2. Main Results

**Lemma 1.** (see [5]) Suppose \(n \geq 3\), then

\[
\chi'_{as}(K_n) = \begin{cases} n, & \text{if } n \equiv 1 \pmod{2}, \\ n + 1, & \text{if } n \equiv 0 \pmod{2}. \end{cases}
\]

Here \(K_n\) denotes complete graph with order \(n\).

**Lemma 2.** (see [5]) Suppose \(G\) is connected graph, \(uv \in E(G)\) and \(d(u) = d(v) = \Delta(G) \geq 2\), then \(\chi'_{as}(G) \geq \Delta(G) + 1\).

Because \(F_3 \lor P_1 \lor P_1 = K_4\) and \(F_2 \lor P_2 \lor P_2 = K_7\).

From the Lemma, \(\chi'_{as}(F_1 \lor P_1 \lor P_1) = 5\), \(\chi'_{as}(F_2 \lor P_2 \lor P_2) = 7\).

**Theorem 1.** For \(n=3\), then \(\chi'_{as}(F_3 \lor P_3 \lor P_3) = 10\).

**Proof.** Because \(\Delta(F_3 \lor P_3 \lor P_3) = 9\) and there are connected points with maximum degree, so \(\chi'_{as}(F_3 \lor P_3 \lor P_3) \geq 10\), so we need only prove that \(F_3 \lor P_3 \lor P_3\) has a 10-ASEC.

Suppose \(V(F_3 \lor P_3 \lor P_3) = \{w_0, w_1, w_2, w_3, v_1, v_2, v_3, u_1, u_2, u_3\}; E(F_3) = \{w_0w_i \mid i = 1, 2, 3\} \cup \{w_iw_{i+1} \mid i = 1, 2\}; \) one \(E(P_3) = \{v_iv_{i+1} \mid i = 1, 2\}; \) another \(E(P_3) = \{w_iu_{i+1} \mid i = 1, 2\}.\)
We define a mapping $f$ as follows:

$f(w_iv_j) = i + j - 1, (i, j = 1, 2, 3); f(w_iu_j) = i + j + 2, (i, j = 1, 2, 3); f(v_iu_j) = i + j + 5 (\text{mod } 10), (i, j = 1, 2, 3); f(w_0w_i) = i + 6, (i = 1, 2, 3); f(w_1w_2) = 0; f(w_2w_3) = 1; f(v_1v_2) = 6; f(v_2v_3) = 7; f(u_1u_2) = 2; f(u_2u_3) = 3.$

Obviously, $f$ is $10$-ASEC of $F_3 \lor P_3 \lor P_3$.

**Theorem 2.** For $n \geq 4$, $\chi'_as(F_n \lor P_n \lor P_n) = 3n$.

**Proof.** Suppose $C = \{\alpha_0, \alpha_1, \ldots, \alpha_{n-1}, \beta_1, \ldots, \beta_{n-1}, \gamma_1, \ldots, \gamma_{n-1}\}$.

$$V(F_n) = \{w_i| i = 0, 1, 2, \ldots, n\};$$

$$E(F_n) = \{w_0w_i| i = 0, 1, 2, \ldots, n\} \bigcup \{w_iw_{i+1}| i = 1, 2, \ldots, n-1\};$$

one $V(P_n) = \{v_i| i = 0, 1, 2, \ldots, n\}$; another $V(P_n) = \{u_i| i = 0, 1, 2, \ldots, n\}$;

one $E(P_n) = \{v_iv_{i+1}| i = 1, 2, \ldots, n-1\}$ another $E(P_n) = \{u_iu_{i+1}| i = 1, 2, \ldots, n-1\}$.

Obviously, we need only to prove that $F_n \lor P_n \lor P_n$ has a $3n$-ASEC.

We define a mapping $f$ as follows:

$f(w_iw_j) = \alpha_{i+j-1 (\text{mod } n)}, i = 0, 1, \ldots, n-1; j = 1, 2, \ldots, n.$

$f(w_iv_i) = \beta_i, i = 1, 2, \ldots, n.$

$f(w_iu_j) = \beta_{i+j-1 (\text{mod } n)}, i = 0, 1, \ldots, n-1; j = 1, 2, \ldots, n.$

$f(w_iu_i) = \alpha_i, i = 1, 2, \ldots, n.$

$f(v_iu_j) = \gamma_{i+j-1 (\text{mod } n)}, i, j = 1, 2, \ldots, n.$

$f(w_0w_i) = \gamma_{i-1}, i = 1, 2, \ldots, n.$

$f(w_iw_{i+1}) = \gamma_{i+1 (\text{mod } n)}, i = 1, 2, \ldots, n-1.$

$f(v_iv_{i+1}) = \beta_{i+1 (\text{mod } n)}, i = 1, 2, \ldots, n-1.$

$f(w_iu_{i+1}) = \alpha_{i+1 (\text{mod } n)}, i = 1, 2, \ldots, n-1.$

Obviously, $f$ is a $3n$-ASEC of $F_n \lor P_n \lor P_n$ ($n \geq 4$). So Theorem 2 is true.

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