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NONLINEAR COMPLEMENTARITY APPROACH TO THE PRICING OF SPINNING RESERVE MARKET

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Abstract: In this paper, the pricing problem of the electric reserve market is addressed. A nonlinear complementarity approach is adopted to analyze the equilibrium price of spinning reserve market. The pricing process is described as optimization of nonlinear program, on the basis of continuous bids of suppliers. For solving it, Karush-Kuhn-Tucker (KKT) condition is utilized to obtain a nonlinear complementarity problem, which can be transformed into a system of nonsmooth equations, and then approximated by a smooth problem. Numerical example shows the validity of the method.

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1. Introduction

As an important ancillary service to the energy, reserve is performed to support the basic services of generating. Besides bidding in the energy market, each supplier may have an interest in submitting its bid to the independent system operator (ISO) in the reserve market. The ISO determines the generating schedule based upon the submitted bids according to the objective of

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procurement cost minimizing. While there are several types of reserve markets, only the issue of building the optimal pricing strategies for suppliers in the spinning reserve market will be addressed in this paper. An interesting body of work has been done on the developing pricing strategies for power suppliers in recent years [5], [6]. Up to now, research works on strategic pricing for competitive suppliers have concentrated on one-period sealed auctions of pool typed electricity in which the uniform clearing price rule is widely utilized. Given this background, it is the objective of this paper to suggest a framework within which optimal pricing strategies for competitive suppliers can be developed in the ISO's spinning reserve market.

The optimal strategy is obtained from the procurement problem which is constructed by the ISO according to the procurement rule. Assuming that each supplier bid a linear supply function into the spinning reserve market. The procurement problem is developed as a nonlinear model with constraints, which has the objective of procurement cost minimizing. The solution of nonlinear problem has been the topic of lots of literatures [1], [3]. In this paper, nonlinear complementarity method is adopted to get a system of nonsmooth equations. [4] utilized developed Levenberg-Marquardt to solve such nonsmooth equations, which has the optimal solution locally optimizing. For solving this problem, we use smoothing techniques to obtain smooth problem. Thus the classical Newton method is applied to approximate the initial problem. The numerical analysis shows the validity of the method.

2. Problem Formulation

Suppose that there are N independent power suppliers participating in the ISO's spinning reserve market, and each supplier is required to bid a linear nondecreasing supply function. It is assumed that the generating cost has the form of quadratic function, which has two parts. One is capacity cost C_{1j_i}, i.e. opportunity cost, meaning the missing income in energy market

$$C_{1j_i}(x_{j_i}) = a_{j_i}x_{j_i}, \quad j = 1, \dots, N, \quad i = 1, \dots, M_j, \tag{2.1}$$

where x_{j_i} is the bidding quantity of the i-th unit of the j-th supplier, $a_{j_i} \geq 0$ the coefficient of capacity cost, M_j the number of units owned by the j-th supplier.

The other is energy cost, which in fact the fuel cost of generating. It only occurs when dispatched

$$C_{2j_i}(x_{j_i}) = b_{j_i}x_{j_i} + c_{j_i}x_{j_i}^2, \quad j = 1, \dots, N, \quad i = 1, \dots, M_j, \tag{2.2}$$

where b_{j_i}, c_{j_i} are the coefficients of energy cost, $c_{j_i} \geq 0$.

Thus the generating cost of the i-th unit of the j-th supplier is

$$\begin{aligned} C_{j_i}(x_{j_i}) &= C_{1j_i}(x_{j_i}) + C_{2j_i}(x_{j_i}) \\ &= a_{j_i}x_{j_i} + b_{j_i}x_{j_i} + c_{j_i}x_{j_i}^2, \quad j = 1, \ldots, N, i = 1, \ldots, M_j. \end{aligned} \tag{2.3}$$

The j-th supplier submits a linear bid to the ISO based upon its generating cost

$$y_{j_i}(x_{j_i}) = \alpha_{j_i} + \beta_{j_i}x_{j_i}, \quad \forall j_i \in I, \tag{2.4}$$

where α_{j_i} and β_{j_i} are the nonnegative bidding coefficients of the i-th unit of the j-th supplier, I be the set of generating units participating in the spinning reserve market.

When the supplier bids according to its marginal cost that really occurs, the following relations are true

$$\alpha_{j_i} = a_{j_i} + b_{j_i}, \quad \beta_{j_i} = 2c_{j_i}, \quad \forall j_i \in I. \tag{2.5}$$

In this paper, we suppose (2.5) hold.

It should be mentioned that the suppliers participating in the spinning reserve market must meet some technical and operating requirements. For example, full response of the spinning reserve capacity is required in 10 minutes. Thus, the spinning reserve capacity of each supplier is dependent on its ramp rate. Based on the generating bids submitted by the suppliers, the ISO constructs the procurement model as

$$\begin{aligned} (\mathrm{P}_1) \qquad & \min_{x_{j_i}} \quad \{\sum_{j=1}^{N}\sum_{i=1}^{M_j}(\alpha_{j_i}x_{j_i} + 0.5\beta_{j_i}x_{j_i}^2)\}, \\ & \text{s.t.} \quad q_{j_{imin}} \leq x_i \leq q_{j_{imax}}, \quad \forall j_i \in I, \\ & \qquad x_i \leq 10q_{j_{irap}}, \qquad \forall j_i \in I, \\ & \qquad \sum_{j=1}^{N}\sum_{i=1}^{M_j} x_i = q_d, \end{aligned}$$

where $q_{j_{imin}}$ and $q_{j_{imax}}$ are the lower and upper output respectively, $q_{j_{irap}}$ the ramp rate, and q_d the requirement of spinning reserve. The relax variable λ of the last constraint is the equilibrium price of the spinning reserve market.

3. Numerical Method

3.1. Nonlinear Complementarity Method

The general nonlinear program with equations and inequations has the form

$$
\begin{aligned}
(\mathrm{P}_2) \qquad & \min_{x\in R^n} && f(x), \\
& \text{s.t.} && g_i(x) \ge 0, \quad i = 1, \ldots, m, \\
& && h_j(x) = 0, \quad j = 1, \ldots, l,
\end{aligned}
$$

where $f, g, h : \Re \to \Re$ are continuously differentiable. Then construct Lagrange function as

$$L(x, \mu, \lambda) = f(x) - \sum_{i=1}^{m} \mu_i g_i(x) - \sum_{j=1}^{l} \lambda_j h_j(x).$$

The KKT optimality condition for (P_2) is

$$\nabla_x L(x^*, \mu^*, \lambda^*) = \nabla f(x^*) - \sum_{i=1}^{m} \nabla \mu_i^* g_i(x^*) - \sum_{j=1}^{l} \nabla \lambda_j^* h_j(x^*) = 0, \tag{3.1a}$$

$$\mu_i^* g_i(x) = 0, \tag{3.1b}$$

$$\mu_i^* \ge 0. \tag{3.1c}$$

Under some constraint qualifications, problem (P_2) is equivalent to the system (3.1).

For solving (3.1), we first introduce nonlinear complementarity function to transform the inequalities (3.1c) into nonsmooth equations. The nonlinear complementarity function has the following form [1]

$$\phi(a, b) = a + b - \sqrt{a^2 + b^2}, \tag{3.2}$$

where, $a, b \in \Re^n$. And $\phi(a, b)$ has the feature of:

(1) $\phi(a, b) = 0 \Leftrightarrow a \ge 0, b \ge 0, ab = 0$,

(2) $\phi^2(a, b)$ is continuously differentiable,

(3) $\phi(a, b)$ is nonsmooth at (0,0).

Substituting (3.1b) and (3.1c) into (3.2), we get

$$\phi(\mu^*, g_i(x^*)) = \mu^* + g_i(x^*) - \sqrt{(\mu)^2 + (g_i(x^*))^2} = 0, \quad i = 1, \ldots, m. \tag{3.3}$$

Equation (3.3) is nonsmooth, which can not apply the classical Newton method. Then, a parameter is introduced to develop the nonlinear complementarity function (3.2) into a smooth function [3]

$$\phi_\varepsilon(a,b) = a + b - \sqrt{a^2+b^2+\varepsilon}, \tag{3.4}$$

where $\varepsilon > 0$ is small enough. It has been proved that $\phi_\varepsilon(a,b) \to \phi(a,b), \forall\varepsilon \to 0^+$. By (3.4), we have

$$\phi_\varepsilon(\mu^*, g_i(x^*)) = \mu^* + g_i(x^*) - \sqrt{(\mu)^2 + (g_i(x^*))^2 + \varepsilon} = 0, \quad i = 1, \dots, m. \tag{3.5a}$$

Combining with

$$\nabla_x L(x^*, \mu^*, \lambda^*) = \nabla f(x^*) - \sum_{i=1}^{m} \nabla \mu_i^* g_i(x^*) - \sum_{j=1}^{l} \nabla \lambda_j^* h_j(x^*) = 0, \tag{3.5b}$$

$$h_j(x) = 0, \quad j = 1, \dots, l, \tag{3.5c}$$

we get a system of smooth equations, which can be solved by the classical Newton method.

However, for solving (P_2), we turn to solve (3.5). When $\varepsilon \to 0^+$, the optimal solution of (3.5) is the approximative solution of (P_2).

3.2. Solution to Procurement Model

The procurement model (P_1) can be solved using above method. Firstly, we construct Lagrange function as follows

$$L(x,\mu,\lambda) = \sum_{j=1}^{N}\sum_{i=1}^{M_j}(\alpha_{j_i}x_{j_i} + 0.5\beta_{j_i}x_{j_i}^2) - [\mu_{1j_i}(-x_{j_i} + q_{j_{imax}}) + \mu_{2j_i}(x_{j_i} - q_{j_{imin}}) + \mu_{3j_i}(-x_{j_i} + 10q_{j_{irap}})] - \lambda(\sum_{j=1}^{N}\sum_{i=1}^{M_j} x_{j_i} - q_d). \tag{3.6}$$

Then, the KKT optimality condition for (P_1) is

$$(x_{j_i}^*, \forall j_i) : (\alpha_{j_i} + \beta_{j_i}x_{j_i}^*) - (-\mu_{1j_i}^* + \mu_{2j_i}^* - \mu_{3j_i}^*) - \lambda^* = 0, \tag{3.7a}$$

$$(\mu_{1j_i}^*, \forall j_i) : \mu_{1j_i}^* \geq 0, (-x_{j_i}^* + q_{j_{imax}}) \geq 0, \mu_{1j_i}^*(-x_{j_i}^* + q_{j_{imax}}) = 0, \tag{3.7b}$$

$$(\mu^*_{2j_i}, \forall j_i) : \mu^*_{2j_i} \geq 0, (x^*_{j_i} - q_{j_{imin}}) \geq 0, \mu^*_{2j_i}(x^*_{j_i} - q_{j_{imin}}) = 0, \quad (3.7c)$$

$$(\mu^*_{3j_i}, \forall j_i) : \mu^*_{3j_i} \geq 0, (-x^*_{j_i} + 10q_{j_{irap}}) \geq 0, \mu^*_{3j_i}(-x^*_{j_i} + 10q_{j_{irap}}) = 0, \quad (3.7d)$$

$$(\lambda^*, \forall j_i) : \sum_{j=1}^{N} \sum_{i=1}^{M_j} x_{j_i} - q_d = 0, \quad (3.7e)$$

where, the lagrange multipliers $\mu^*_{1j_i}, \mu^*_{2j_i}, \mu^*_{3j_i}$ are the shadow prices of the capacity limits and the ramp rate respectively, λ^* the equilibrium price of the spinning reserve market.

Substituting (3.7b)-(3.7d) to (3.5a), then

$$(\mu^*_{1j_i}, \forall j_i) : \mu^*_{1j_i} + (-x^*_{j_i} + q_{j_{imax}}) - \sqrt{(\mu^*_{1j_i})^2 + (-x^*_{j_i} + q_{j_{imax}})^2 + \varepsilon} = 0, \quad (3.8a)$$

$$(\mu^*_{2j_i}, \forall j_i) : \mu^*_{2j_i} + (x^*_{j_i} - q_{j_{imin}}) - \sqrt{(\mu^*_{2j_i})^2 + (x^*_{j_i} - q_{j_{imin}})^2 + \varepsilon} = 0, \quad (3.8b)$$

$$(\mu^*_{3j_i}, \forall j_i) : \mu^*_{3j_i} + (-x^*_{j_i} + 10q_{j_{irap}}) - \sqrt{(\mu^*_{3j_i})^2 + (-x^*_{j_i} + 10q_{j_{irap}})^2 + \varepsilon} = 0. \quad (3.8c)$$

Theorem 3.1. *$\phi_\varepsilon(\mu^*_{1j_i}, -x^*_{j_i} + q_{j_{imax}})$ approximates $\phi(\mu^*_{1j_i}, -x^*_{j_i} + q_{j_{imax}})$ by accuracy $\frac{\varepsilon}{2\mu^*_{1j_i}}$.*

Proof. By calculating, we get

$$\begin{aligned}
&\left|\phi_\varepsilon(\mu^*_{1j_i}, -x^*_{j_i} + q_{j_{imax}}) - \phi(\mu^*_{1j_i}, -x^*_{j_i} + q_{j_{imax}})\right| \\
&\quad = \left|\sqrt{(\mu^*_{1j_i})^2 + (-x^*_{j_i} + q_{j_{imax}})^2 + \varepsilon} - \sqrt{(\mu^*_{1j_i})^2 + (-x^*_{j_i} + q_{j_{imax}})^2}\right| \\
&\quad = \left|\frac{\varepsilon}{\sqrt{(\mu^*_{1j_i})^2 + (-x^*_{j_i} + q_{j_{imax}})^2 + \varepsilon} + \sqrt{(\mu^*_{1j_i})^2 + (-x^*_{j_i} + q_{j_{imax}})^2}}\right| \\
&\quad \leq \frac{\varepsilon}{2\sqrt{(\mu^*_{1j_i})^2 + (-x^*_{j_i} + q_{j_{imax}})^2}} \leq \frac{\varepsilon}{2\mu^*_{1j_i}}.
\end{aligned}$$

This completes the proof of the theorem. □

Similarly, we can obtain Theorem 3.2 and Theorem 3.3.

Theorem 3.2. *$\phi_\varepsilon(\mu^*_{2j_i}, x^*_{j_i} - q_{j_{imin}})$ approximates $\phi(\mu^*_{2j_i}, x^*_{j_i} - q_{j_{imin}})$ by accuracy $\frac{\varepsilon}{2\mu^*_{2j_i}}$.*

Theorem 3.3. *$\phi_\varepsilon(\mu^*_{3j_i}, -x^*_{j_i}+10q_{j_{irap}})$ approximates $\phi(\mu^*_{3j_i}, -x^*_{j_i}+10q_{j_{irap}})$ by accuracy $\frac{\varepsilon}{2\mu^*_{3j_i}}$.*

Combining (3.8) with (3.7a) and (3.7e), we get a system of smooth equations, which can be solved by the classical Newton method. The optimal solution to the smooth equations approximates the initial problem (P_1) under the condition $\varepsilon \to 0^+$.

Consider the following nonlinear equations

$$H(x) = 0, \tag{3.9}$$

where $H(x) : \Re^n \to \Re$ is locally Lipschitzian. The Newton method for solving the smooth equations is given by

$$x^{k+1} = x^k - J_k^{-1} H(x^k), \tag{3.10}$$

where, J_k is Jacobian of H at x^k, see [2].

We now give the algorithm of smoothing Newton method for solving equations (3.7a), (3.7e) and (3.8).

Algorithm 3.1. *Step 0.* Given $\varepsilon > 0$. Set $k = 0$, and choose an initial point x^0. Given termination accuracy ξ.

Step 1. Calculate the value of the function in the smooth equations and J_k^{-1} at x^k, where,

$$J_k = \left| \begin{array}{ccccc} \beta_{j_i} & 1 & -1 & 1 & -1 \\ -1 - \frac{x^*_{j_i} - q_{j_{imax}}}{\Theta(1)} & 1 - \frac{\mu^*_{1j_i}}{\Theta(1)} & 0 & 0 & 0 \\ 1 - \frac{x^*_{j_i} - q_{j_{imin}}}{\Theta(2)} & 0 & 1 - \frac{\mu^*_{2j_i}}{\Theta(2)} & 0 & 0 \\ -1 - \frac{x^*_{j_i} - 10q_{j_{irap}}}{\Theta(3)} & 0 & 0 & 1 - \frac{\mu^*_{3j_i}}{\Theta(3)} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{array} \right|,$$

with $\Theta = \begin{pmatrix} \Theta(1) \\ \Theta(2) \\ \Theta(3) \end{pmatrix} = \begin{pmatrix} \sqrt{(\mu^*_{1j_i})^2 + (-x^*_{j_i} + q_{j_{imax}})^2 + \varepsilon} \\ \sqrt{(\mu^*_{2j_i})^2 + (x^*_{j_i} - q_{j_{imin}})^2 + \varepsilon} \\ \sqrt{(\mu^*_{3j_i})^2 + (-x^*_{j_i} + 10q_{j_{irap}})^2 + \varepsilon} \end{pmatrix}.$

Step 2. Compute x^{k+1} by (3.10).

Step 3. If $|x^{k+1} - x^k| \le \xi$, then stop. Otherwise, set $k = k + 1$ and go to Step 1.

suppliers j_i	a_{j_i}	b_{j_i}	c_{j_i}	$q_{j_{imax}}$ (MW)	$q_{j_{imin}}$ (MW)	$q_{j_{irap}}$ (MW/min)
1_1	0.25	2.0	0.0125	50	0	5
1_2	0.35	1.4	0.0175	20	0	2
1_3	0.5	1.0	0.025	30	0	4
2_1	0.9	1.0	0.0125	30	0	5
3_1	0.5	1.5	0.012	50	0	4
3_2	0.5	1.5	0.012	50	0	4

Table 4.1: Data of units

suppliers j_i	x_{j_i} (MW)	$C_{j_i}(x_{j_i})$ (USD)	π_{j_i} (USD)	λ^* (USD/MW)
1_1	6.772942	15.81253	0.573419	
1_2	19.1237	39.8665	6.399942	
1_3	18.38659	36.03155	8.451585	
2_1	20.77301	44.86269	5.393969	2.419325
3_1	17.47187	38.60693	3.663197	
3_2	17.47189	38.60698	3.663197	
Total	100	213.7872	28.14531	

Table 4.2: Result of bidding

4. Numerical Example

Suppose there are 3 suppliers participating in the spinning reserve market, and the data of their units are listed in Table 4.1. Set the requirement of the market $q_d = 100\,\text{MW}$, and the termination accuracy is fixed at $\xi \leq 10^{-20}$.

Given $\varepsilon = 10^{-20}$, and by the algorithm proposed above, we can get the optimal solution easily, which is shown in Table 4.2. The generating cost $C_{j_i}(x_{j_i})$ and the profit π_{j_i} are also listed in it. From the result, we know that the equilibrium price λ^*=2.419325 USD/MW. According to the market clearing rule, i.e. the uniform clearing pricing, each unit that succeeds will be paid at the uniform price λ^*.

5. Conclusions

A procurement model in the ISO's spinning reserve market is presented and it is constructed as a nonlinear program. In this paper, the nonlinear complementarity method is utilized to transform the KKT optimality conditions for the nonlinear program into a system of nonsmooth equations. For getting a optimum, we use smoothing technique to approximate it. Under some constraint qualifications, we can solve a system of smooth equations to get the approximative solution to the initial problem. From the optimal solution to the procurement model, the equilibrium price of the spinning reserve market can be calculated automatically. The classical Newton method is applied to search for the optimal solution. The numerical test shows the validity of our method.

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