International Journal of Pure and Applied Mathematics

Volume 30 No. 4 2006, 467-474

ON FUZZY PARTITIONS

M.A.H. Shakhatreh[1 §], W.A.A. Shatanawi[2]
M.M.M. Jaradat[3], A.A.A. Fora[4]

[1,3,4]Department of Mathematics
Yarmouk University
Irbid, JORDAN
[1]e-mail: mali@yu.edu.jo
[3]e-mail: mmjst4@yu.edu.jo
[4]e-mail: afora_jo@yahoo.com

[2]Department of Mathematics
The Hashemite University
Zarqa, JORDAN
[2]e-mail: swasfi@hu.edu.jo

Abstract: Several definitions of fuzzy partitions are introduced. The relations between them are studied. The weakest and the strongest one are indicated. The relations between our definitions of fuzzy partition on X and a partition of X are studied.

AMS Subject Classification: 26E50, 03E72, 46S40
Key Words: fuzzy partitions, fuzzy sets, fuzzy analysis

1. Basic Concepts

The study of fuzzy sets was largely motivated by the theory of distributions founded by Zadeh in 1965 [5]. In some sense, fuzzy sets play a major role in many topics in mathematics such as fuzzy topology, fuzzy algebra, fuzzy graph theory and fuzzy analysis. Thus the attraction of fuzzy sets to a large number of annalist is understandable, for fuzzy sets one can refer to [1] and [2]. Let $\mathcal{A}$ be a family of fuzzy sets on X, we say that $\mathcal{A}$ is locally finite if for each $x \in X$ there is a finite subset $A_0 = \{f_1, f_2, \dots, f_n\}$ of $\mathcal{A}$ such that $f(x) = 0$ for all $f \notin A_0$ and $\sum_{i=1}^{n} f_i(x) = 1$. A family $\mathcal{A}$ of fuzzy sets on X is said to be a fuzzy

Received: June 8, 2006

§Correspondence author

partition of a set X if $\mathcal{A}$ is locally finite and $f \neq 0$ for all $f \in \mathcal{A}$ (see [3] and [4]). In the present paper, the authors introduced several definitions of fuzzy partitions and they studied the relations between them.

Let X be a universal set and P subset of X. The characteristic function χ_P of the set P is defined to be the function $\chi_P : X \to [0,1]$ such that $\chi_P(x) = 1$ if $x \in P$ and 0 otherwise. By a fuzzy set λ in X we mean a function from X to [0,1]. Let λ and μ be two fuzzy sets on X by $\lambda \leq \mu$ we mean $\lambda(x) \leq \mu(x)$ for all $x \in X$. Let $\mathcal{A} = \{\lambda_\gamma : \gamma \in \Lambda\}$ be a collection of fuzzy sets. Then the union and the intersection are defined as follows: $(\bigcup_{\gamma\in\Lambda} \lambda_\gamma)(x) = \sup_{\gamma\in\Lambda} \lambda_\gamma(x)$ and $(\bigcap_{\gamma\in\Lambda} \lambda_\gamma)(x) = \inf_{\gamma\in\Lambda} \lambda_\gamma(x)$. The support of the fuzzy set λ in X; denoted by suppλ, is defined by supp$\lambda = \lambda^{-1}(0,1] = \{x \in X : \lambda(x) > 0\}$.

Definition 1.1. A fuzzy set λ is called a weakly empty if $\lambda < 1/2$, a weakly universal if $\lambda > 1/2$.

Definition 1.2. Let $\mathcal{A} = \{\lambda_\gamma : \gamma \in \Lambda\}$ be a collection of fuzzy sets on the set X. The fuzzy sum, $f\sum_{\gamma\in\Lambda} \lambda_\gamma$, is defined to be $(f\sum_{\gamma\in\Lambda} \lambda_\gamma)(x) = \inf\{\sum_{\gamma\in\Lambda} \lambda_\gamma(x), 1\}$

2. Main Results

In order to facility our subsequent arguments, the letters A_1, A_2, B_1, B_2, C_1, C_2, C_3, and C_4 stand for the following statements:

A_1 : Each element of $\mathcal{A}$ is a non weakly empty.

A_2 : Each element of $\mathcal{A}$ is a nonzero.

B_1 : For any two distinct λ_γ and λ_α of $\mathcal{A}$, $\lambda_\gamma \cap \lambda_\alpha$ is weakly empty.

B_2 : For any two distinct λ_γ and λ_α of $\mathcal{A}$, $\lambda_\gamma \cap \lambda_\alpha = 0$.

C_1 : The union of all fuzzy sets of $\mathcal{A}$ is 1; that is, $\bigcup_{\gamma\in\Lambda} \lambda_\gamma = 1$.

C_2 : The fuzzy sum of all fuzzy sets of $\mathcal{A}$ is 1; that is, $f\sum_{\gamma\in\Lambda} \lambda_\gamma = 1$.

C_3 : $f\sum_{\gamma\in\Lambda} \lambda_\gamma$ is weakly universal.

C_4 : $\bigcup_{\gamma\in\Lambda} \lambda_\gamma$ is weakly universal.

Now we are in the position to present our fuzzy partitions.

Definition 2.1. Let $\mathcal{A}$ be a collection of fuzzy sets on X. Then $\mathcal{A} = \{\lambda_\gamma : \gamma \in \Lambda\}$ is called:

1. F_1 fuzzy partition of X iff A_1, B_1, and C_4 are satisfied.
2. F_2 fuzzy partition of X iff A_1, B_1, and C_3 are satisfied.
3. F_3 fuzzy partition of X iff A_1, B_1, and C_1 are satisfied.
4. F_4 fuzzy partition of X iff A_1, B_1, and C_2 are satisfied.
5. F_5 fuzzy partition of X iff A_2, B_1, and C_4 are satisfied.

6. F_6 fuzzy partition of X iff A_2, B_1, and C_3 are satisfied.
7. F_7 fuzzy partition of X iff A_2, B_1, and C_1 are satisfied.
8. F_8 fuzzy partition of X iff A_2, B_1, and C_2 are satisfied.
9. F_9 fuzzy partition of X iff A_1, B_2, and C_4 are satisfied.
10. F_{10} fuzzy partition of X iff A_1, B_2, and C_1 are satisfied.

Lemma 2.1. *Let $\mathcal{A}$ be a family of fuzzy sets on X. If $\mathcal{A}$ satisfies C_1, then $\mathcal{A}$ satisfies C_2 and so C_3.*

Proof. Given $x \in X$. If there is $\gamma \in \Lambda$ such that $\lambda_\gamma(x) = 1$, then $(f\sum_{\gamma\in\Lambda}\lambda_\gamma)(x) = \inf\{\sum_{\gamma\in\Lambda}\lambda_\gamma(x), 1\} = 1$. If $\lambda_\gamma(x) < 1$ for all $\gamma \in \Lambda$, then there is $\gamma_1, \gamma_2 \in \Lambda$ such that $1 > \lambda_{\gamma_1}(x), \lambda_{\gamma_2}(x) > 0.9$ because $(\bigcup_{\gamma\in\Lambda}\lambda_\gamma)(x) = 1$. Thus, $(f\sum_{\gamma\in\Lambda}\lambda_\gamma)(x) = \inf\{\sum_{\gamma\in\Lambda}\lambda_\gamma(x), 1\} = 1$. So $\mathcal{A}$ satisfies C_2. □

Following the same arguments as in the proof of Lemma 2.1, we can prove the next results.

Lemma 2.2. *Let $\mathcal{A}$ be a family of fuzzy sets on X. If $\mathcal{A}$ satisfies C_4, then $\mathcal{A}$ satisfies C_3.*

Theorem 2.1. *Let $\mathcal{A}$ be a family of fuzzy sets on X. Then F_1 fuzzy partition of X implies F_2 fuzzy partition of X implies F_6 fuzzy partition of X*

Proof. The proof of this theorem follows from Lemma 2.2 and the following fact: "every non empty weakly empty fuzzy set is a nonzero fuzzy set". □

The converse of the implications in Theorem 2.1 are not true.

Example 2.1. Let $X = [0, 1]$, and let

$$\lambda_1(x) = (\frac{1}{4}\chi_{\{0\}} + \frac{1}{3}\chi_{(0,1/2)} + \frac{1}{4}\chi_{\{1/2\}} + \frac{1}{3}\chi_{(1/2,1)} + \frac{1}{2}\chi_{\{1\}})(x),$$
$$\lambda_2(x) = (\frac{1}{4}\chi_{\{0\}} + \frac{1}{3}\chi_{(0,1/2)} + \frac{1}{2}\chi_{\{1/2\}} + \frac{1}{3}\chi_{(1/2,1)} + \frac{1}{4}\chi_{\{1\}})(x),$$

and

$$\lambda_3(x) = (\frac{1}{2}\chi_{\{0\}} + \frac{1}{3}\chi_{(0,1/2)} + \frac{1}{4}\chi_{\{1/2\}} + \frac{1}{3}\chi_{(1/2,1)} + \frac{1}{4}\chi_{\{1\}})(x).$$

Then $\{\lambda_1, \lambda_2, \lambda_3\}$ is F_2 fuzzy partition of X which is not F_1 fuzzy partition of X.

Example 2.2. Let $X = [0, 1]$, and let $\lambda_1 = \frac{5}{7}$, $\lambda_2 = \frac{2}{7}$. Then $\{\lambda_1, \lambda_2\}$ is F_6 fuzzy partition of X which is not F_2 fuzzy partition of X.

Theorem 2.2. *Let $\mathcal{A}$ be a family of fuzzy sets on X. Then F_3 fuzzy partition of X implies F_4 fuzzy partition of X implies F_8 fuzzy partition of X implies F_6 fuzzy partition of X*

Proof. The proof of this theorem follows by Lemma 2.1, then by noting that every non empty weakly empty fuzzy set is a nonzero fuzzy set and noting that if C_2 holds, then C_3 holds. □

The converse of the implication in Theorem 2.2 are not true.

Example 2.3. Let $X = [0, 1]$, and let $\lambda_1(x) = (0.9\chi_{[0,1/2]} + 0.1\chi_{(1/2,1]})$ (x), $\lambda_2(x) = (0.1\chi_{[0,1/2]} + 0.9\chi_{(1/2,1]})(x)$. Then $\{\lambda_1, \lambda_2\}$ is F_4 fuzzy partition of X which is not F_3 fuzzy partition of X.

Example 2.4. Let $X = [0, 1]$, and let $\lambda_1(x) = \frac{1}{4}$, $\lambda_2(x) = \frac{1}{3}$, and $\lambda_3 = \frac{2}{3}$. Then $\{\lambda_1, \lambda_2, \lambda_3\}$ is F_8 fuzzy partition of X which is not F_4 fuzzy partition of X.

Example 2.5. Let $X = [0, 1]$, and let $\lambda_1(x) = (\frac{1}{3}\chi_{[0,1/2]} + \frac{1}{2}\chi_{(1/2,1]})(x)$, $\lambda_2(x) = (\frac{1}{2}\chi_{[0,1/2]} + \frac{1}{3}\chi_{(1/2,1]})(x)$. Then $\{\lambda_1, \lambda_2\}$ is F_6 fuzzy partition of X which is not F_8 fuzzy partition of X.

Theorem 2.3. *Let $\mathcal{A}$ be a family of fuzzy sets on X. Then F_3 fuzzy partition of X implies F_7 fuzzy partition of X implies F_8 fuzzy partition of X.*

Proof. The proof of this theorem follows by noting that every non empty weakly empty fuzzy set is a nonzero fuzzy set and Lemma 2.1. □

The converse of the implications of Theorem 2.3 are not true.

Example 2.6. Let $X = [0, 1]$, and let $\lambda_1 = 0.2$, $\lambda_2 = 1$. Then $\{\lambda_1, \lambda_2\}$ is F_7 fuzzy partition of X which is not F_3 fuzzy partition of X.

Example 2.7. Let $X = [0, 1]$, and let $\lambda_1 = 0.1$, $\lambda_2 = 0.2$, $\lambda_3 = 0.3$, and $\lambda_4 = 0.4$. Then $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\}$ is F_8 fuzzy partition of X which is not F_7 fuzzy partition of X.

Theorem 2.4. *Let $\mathcal{A}$ be a family of fuzzy sets on X. Then F_{10} fuzzy partition of X implies F_9 fuzzy partition of X implies F_1 fuzzy partition of X implies F_5 fuzzy partition of X implies F_6 fuzzy partition of X.*

Proof. This theorem follows from the definition of C_1, C_2, B_1, B_2, A_1, and A_1 and by Lemma 2.2. □

The converse of the implications of Theorem 2.4 are not true.

Example 2.8. Let $X = [0, 1]$, and let $\lambda_1(x) = (\frac{3}{4}\chi_{[0,1/2]})(x)$, $\lambda_2(x) = (\frac{3}{4}\chi_{(1/2,1]})(x)$. Then $\{\lambda_1, \lambda_2\}$ is F_9 fuzzy partition of X which is not F_{10} fuzzy partition of X.

Example 2.9. Let $X = [0, 1]$, and let $\lambda_1(x) = (\frac{3}{4}\chi_{[0,1/2]} + \frac{1}{4}\chi_{(1/2,1]})(x)$, $\lambda_2(x) = (\frac{1}{4}\chi_{[0,1/2]} + \frac{3}{4}\chi_{(1/2,1]})(x)$. Then $\{\lambda_1, \lambda_2\}$ is F_1 fuzzy partition of X which is not F_9 fuzzy partition of X.

Example 2.6 is F_5 fuzzy partition of X which is not F_1 fuzzy partition of X.

Example 2.10. Let $X = [0,1]$, and let $\lambda_1(x) = 0.3$, $\lambda_2(x) = 0.4$. Then $\{\lambda_1, \lambda_2\}$ is F_6 fuzzy partition of X which is not F_5 fuzzy partition of X.

Theorem 2.5. *Let $\mathcal{A}$ be a family of fuzzy sets on X. Then F_{10} fuzzy partition of X implies F_3 fuzzy partition of X implies F_4 fuzzy partition of X implies F_2 fuzzy partition of X.*

Proof. This Result follows from the definition of B_1, B_2, C_2, C_3, and Lemma 2.1. □

The converse of Theorem 2.5 is not true.

Example 2.11. Let $X = [0,1]$, and let $\lambda_1(x) = (\chi_{[0,1/2]} + 0.1\chi_{(1/2,1]})(x)$, $\lambda_2(x) = (0.1\chi_{[0,1/2]} + \chi_{(1/2,1]})(x)$. Then $\{\lambda_1, \lambda_2\}$ is F_3 fuzzy partition of X which is not F_{10} fuzzy partition of X.

Example 2.9 is F_4 fuzzy partition of X which is not F_3 fuzzy partition of X. While Example 2.8 is F_2 fuzzy partition of X which is not F_4 fuzzy partition of X; that is, F_4 does not imply F_3 fuzzy partition of X and F_2 does not imply F_4 fuzzy partition of X.

Theorem 2.6. *Let $\mathcal{A}$ be a family of fuzzy sets on X. Then:*
1. F_7 fuzzy partition of X implies F_5 fuzzy partition of X.
2. F_3 fuzzy partition of X implies F_1 fuzzy partition of X.

Proof. This result follows by noting that if $\bigcup_{\gamma\in\Lambda} \lambda_\gamma = 1$, then $\bigcup_{\gamma\in\Lambda} \lambda_\gamma$ is weakly universal. □

The converse of the implications of Theorem 2.6 are not true.

Example 2.12. Let $X = [0,1]$, and let $\lambda_1(x) = \frac{1}{4}$, $\lambda_2(x) = \frac{3}{4}$. Then $\{\lambda_1, \lambda_2\}$ is F_5 fuzzy partition of X which is not F_7 fuzzy partition of X.

Example 2.3 is F_1 fuzzy partition of X which is not F_3 fuzzy partition of X.

The next result indicates the independency of the introduced fuzzy partitions of X.

Theorem 2.7. *The following two pairs of fuzzy partitions of X are independent:*
1. F_9 and F_3 are fuzzy partitions of X.
2. F_1 and F_4 are fuzzy partitions of X.
3. F_1 and F_7 are fuzzy partitions of X.
4. F_4 and F_7 are fuzzy partitions of X.

5. F_4 *and* F_5 *are fuzzy partitions of* X.
6. F_9 *and* F_4 *are fuzzy partitions of* X.
7. F_9 *and* F_7 *are fuzzy partitions of* X.
8. F_2 *and* F_7 *are fuzzy partitions of* X.
9. F_2 *and* F_5 *are fuzzy partitions of* X.
10. F_2 *and* F_8 *are fuzzy partitions of* X.
11. F_9 *and* F_8 *are fuzzy partitions of* X.

Proof. Example 2.8 is an example on F_9, F_1, F_5, and F_2 fuzzy partitions of X which is not F_3, F_4, F_7, and F_8 fuzzy partitions of X; that is the following implications are not hold, $F_9 \to F_3$, $F_1 \to F_4$, $F_1 \to F_7$, $F_5 \to F_4$, $F_9 \to F_4$, $F_9 \to F_7$, $F_2 \to F_7$, $F_2 \to F_8$, and $F_9 \to F_8$. While Example 2.12 is a F_3 fuzzy partition of X which is not F_9 fuzzy partition of X, Example 2.1 is a F_4 fuzzy partition of X which is not F_1, F_5, and F_9 fuzzy partitions of X, Example 2.6 a is F_7 fuzzy partition of X which is not F_1 and F_2 fuzzy partitions of X, Example 2.7 is a F_8 fuzzy partition of X which is not F_2 and F_9 fuzzy partitions of X, and Example 2.11 is a F_7 fuzzy partition of X which is not F_9 fuzzy partition of X. This proves *1, 2, 3, 5, 6, 7, 8, 10,* and *11* of our theorem. Example 2.1 is a F_2 fuzzy partition of X which is not F_5 fuzzy partition of X, while Example 2.2 is a F_5 fuzzy partition of X which is not F_2 fuzzy partition of X; that is F_5 and F_2 fuzzy partitions of X are independent on each other which proves *9* of our theorem. Example 2.6 is F_7 fuzzy partition of X which is not F_4 fuzzy partition of X. On the other hand Example 2.1 is a F_4 fuzzy partition of X which is not F_7 fuzzy partition of X; that is F_4 does not imply F_7 and F_7 does not imply F_4 which proves *4* of our theorem. □

Our next results show that the weakest fuzzy partitions of our definitions is F_{10} fuzzy partition and the strongest fuzzy partitions of our definitions is F_6 fuzzy partition.

Theorem 2.8. *Let* $\mathcal{A}$ *be a family of fuzzy sets on* X*. Then* F_{10} *fuzzy partition of* X *implies* F_i *fuzzy partition of* X *for all* $i = 1, 2, \ldots, 9$.

Theorem 2.9. *Let* $\mathcal{A}$ *be a family of fuzzy sets on* X*, and let* $i = 1, 2, \ldots, 10$*. Then* F_i *fuzzy partition of* X *implies* F_6 *fuzzy partition of* X.

The next results give the relations between the partition of X and the fuzzy partition of X.

Theorem 2.10. *Let* $\{D_i : i \in \Lambda\}$ *be a partition of* X*. Then* $\mathcal{A} = \{\chi_{D_i} : i \in \Lambda\}$ *is a* F_{10} *fuzzy partition of* X.

Proof. Given $i \in \Lambda$. Since $D_i \neq \emptyset$, there is $x_i \in D_i$. So $\chi_{D_i}(x_i) = 1$. Hence χ_{D_i} is non weakly empty. So $\mathcal{A}$ satisfies A_1. For $i \neq j$ in Λ, we have $D_i \cap D_j = \emptyset$

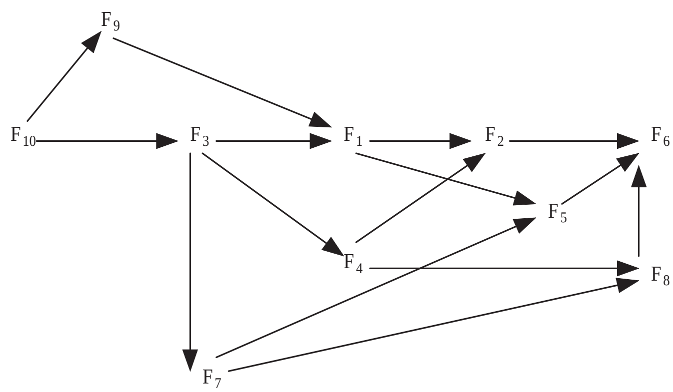

Figure 1: This diagram shows the implications between the fuzzy partitions.

and hence $\chi_{D_i} \cap \chi_{D_j} = 0$. So $\mathcal{A}$ satisfies B_2. Finally, for $x \in X$, there is unique $j \in \Lambda$ such that $x \in D_j$. So $\chi_{D_j}(x) = 1$ and $\chi_{D_i}(x) = 0$ for all $i \neq j$. Therefore $(\bigcup_{i\in\Lambda} \chi_{D_i})(x) = 1$ and $\mathcal{A}$ satisfies C_1. Therefore $\mathcal{A}$ is F_{10} fuzzy partition of X. □

Applying Theorem 2.8 and Theorem 2.10 we have the following result.

Corollary 2.1. *Let $\{D_i : i \in \Lambda\}$ be a partition of X. Then $\mathcal{A} = \{\chi_{D_i} : i \in \Lambda\}$ is a F_i fuzzy partition of X, where $i = 1, 2, \ldots, 10$.*

Theorem 2.11. *Let $D_i \subseteq X$ for all $i \in \Lambda$ be such that $\mathcal{A} = \{\chi_{D_i} : i \in \Lambda\}$ is a F_6 fuzzy partition of X. Then $\{D_i : i \in \Lambda\}$ is a partition of X.*

Proof. Given $i \in \Lambda$. Since $\chi_{D_i} \neq 0$, we have $D_i \neq \emptyset$. For $i \neq j$ in Λ, we have $\chi_{D_j} \cap \chi_{D_i}$ is weakly empty; that is, $\inf\{\chi_{D_j}(x), \chi_{D_i}(x)\} < \frac{1}{2}$ for all $x \in X$ and hence $\inf\{\chi_{D_j}(x), \chi_{D_i}(x)\} = 0$. Therefore $D_j \cap D_i = \emptyset$. If there is $x \in X$ such that $x \notin D_i$ for all $i \in \Lambda$, we have $\chi_{D_i}(x) = 0$ for all $i \in \Lambda$. Therefore $(f\sum_{i\in\Lambda} \chi_{D_i})(x) = 0$ which is a contradiction. So $X = \bigcup_{i\in\Lambda} D_i$. Therefore $\{D_i : i \in \Lambda\}$ is a partition of X. □

Applying Theorem 2.9 and Theorem 2.11, we have the following result.

Corollary 2.2. *Let $D_i \subseteq X$ for all $i \in \Lambda$ be such that $\mathcal{A} = \{\chi_{D_i} : i \in \Lambda\}$ is a F_i fuzzy partition of X where $i = 1, 2, \ldots, 10$. Then $\{D_i : i \in \Lambda\}$ is a partition of X.*

The next result presents a way to construct a partition on a set X from a fuzzy partition of X by using the support of the of the fuzzy sets in X.

Theorem 2.12. *Let $\mathcal{A} = \{\lambda_\gamma : \gamma \in \Lambda\}$ be an F_9 fuzzy partition of X. Then $\{\mathrm{supp}\lambda_\gamma : \gamma \in \Lambda\}$ is a partition on X.*

Proof. Since $\mathcal{A}$ is non weakly empty, we have $\mathrm{supp}\lambda_\gamma \neq \emptyset$ for all $\gamma \in \Lambda$. If $\mathrm{supp}\lambda_{\gamma_1} \cap \mathrm{supp}\lambda_{\gamma_2} \neq \emptyset$ for some $\gamma_1 \neq \gamma_2$ in Λ, then there is $x \in X$ such that $\lambda_{\gamma_1}(x) > 0$ and $\lambda_{\gamma_2}(x) > 0$. So $(\lambda_{\gamma_1} \cap \lambda_{\gamma_2})(x) > 0$ which is a contradiction. Therefore for $\lambda_{\gamma_1} \neq \lambda_{\gamma_2}$ in Λ, we have $\mathrm{supp}\lambda_{\gamma_1} \cap \mathrm{supp}\lambda_{\gamma_2} = \emptyset$. Given $x \in X$. Since $(\bigcup_{\gamma \in \Lambda} \lambda_\gamma)(x) > \frac{1}{2}$, there is $\gamma_0 \in \Lambda$ such that $\lambda_{\gamma_0}(x) > 0$ and hence $x \in \mathrm{supp}\lambda_{\gamma_0}$. Hence $X = \bigcup_{\gamma \in \Lambda} \mathrm{supp}\lambda_\gamma$. Therefore $\{\mathrm{supp}\lambda_\gamma : \gamma \in \Lambda\}$ is a partition on X. □

Applying Theorem 2.8 and Theorem 2.12, we have the following result.

Corollary 2.3. *Let $\mathcal{A} = \{\lambda_\gamma : \gamma \in \Lambda\}$ be F_{10} fuzzy partition of X. Then $\{\mathrm{supp}\lambda_\gamma : \gamma \in \Lambda\}$ is a partition on X.*

Remark 2.1. Let $\mathcal{A} = \{\lambda_\gamma : \gamma \in \Lambda\}$ be F_i $(i = 1, 2, \ldots, 8)$ fuzzy partition of X. Then it is not necessary $\{\mathrm{supp}\lambda_\gamma : \gamma \in \Lambda\}$ to be a partition on X.

Example 2.13. Let $\lambda_1 = \chi_{[0,\frac{1}{2}]} + 0.1\chi_{(\frac{1}{2},1]}$ and $\lambda_2 = 0.1\chi_{[0,\frac{1}{2}]} + \chi_{(\frac{1}{2},1]}$. Then $\{\lambda_1, \lambda_2\}$ is F_i $(i = 1, 2, \ldots, 8)$ fuzzy partition of X but $\{\mathrm{supp}\lambda_1, \mathrm{supp}\lambda_2\}$ in not a partition on X.

References

[1] G.J. Klir, T.A. Folger, *Fuzzy Sets, Uncertainties and Information*, Prentice Hall (1988).

[2] G.J. Klir, Bo Yuan, *Fuzzy Sets and Fuzzy Logic: Theory and Applications*, Prentice Hall (1995).

[3] F.G. Lupiáñez, Fuzzy partitions of unity, *Matematicki Vesink*, **56** (2004), 13-15.

[4] B. Riečan, Entropy of fuzzy Dynamical Systems, *Robust 2000*, 244-246.

[5] L.A. Zadeh, Fuzzy Sets, *Inform. and Control*, **8** (1965), 338-353.